① ?

$5C_1$

ways to choose empathetic & innovative in any order

ways to choose innovative & analytical to get admitted.

$3 \times 5 \, C_1 =$
$3 \times 5 = 15$

**OR**

↓ 6 order $\times \, 5C_1 = 30$

$45 - 3 = 42$

↓ intersection between

total
$7 \, C_3 = 210$

$\dfrac{42}{210} = .2\cancel{88}$   $2(3.8\%)$.

$\boxed{\dfrac{1}{5}}$

b) **Binomial distribution**

$10 C_4 \times (.2)^4 \times (.8)^6 = \boxed{0.088}$

② a) events a & b are independent if

A) $P(A \cap B) = P(A) \cdot P(B)$

2 consecutive flips yield heads

HHTT  THHT  TTHH
HHTH  (HHH)  HTHH
$P(A) = \frac{8}{16}$ (HHH)  (HHHH)  (THHH)
HHHH

B) T☺☺☺ = 8
☺☺☺T = 8

$16 - 4 = 12$

$P(B) \frac{12}{16}$

• THHH
THTT
TTHT  →
TTTH
• TH HT →
THTH
• T THH
TTTT  →
• HHH T
HTHT
HTTT
• HTTT

**KEPEATS**

b)
⭐

$P(A \cap B)$

All the comb with HH is overlap $\frac{5}{16}$

$P(A) \times P(B)$
$= \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

NOT INDEPENDANT

$3(\frac{1}{16}) + 2(\frac{2}{16})$
$+ \frac{5}{16}$
$= \frac{12}{16}$
$\boxed{= \frac{3}{4}}$

2 pair
$\frac{1}{16}$
4 H H H H

2 pair
$\frac{2}{16}$

1 pair
$\frac{5}{16}$

1 pair
doesn't matter, expected value is 0

$\frac{5}{16}$

③

NASTY

$$\frac{1}{5}(1) + \frac{1}{5}(2) + \frac{1}{5}\left(\frac{2+3}{2}\right) + \frac{1}{5}\left(\frac{2+3}{2}\right) + \frac{1}{5}(2)$$

$$\frac{3}{5} + \frac{5}{10} + \frac{5}{10} + \frac{2}{5} = 2$$

HASTY

$$\frac{1}{5}(1) + \left(\frac{1}{5}\right)(2) + \left(\frac{1}{5}\right)(1) + \left(\frac{1}{5}\right)(2) + \left(\frac{1}{5}\right)(2)$$

hasty  nasty        board        board      maths

$$\frac{1}{5} + \frac{2}{5} + \frac{1}{5} + \frac{2}{5} + \frac{2}{5} = \frac{8}{5}$$

④

4.1]  $\frac{1}{100}$        99% likely innocent

4.2]

$$\frac{99}{99999} \leftarrow innocent$$

$$= \frac{11}{11111}$$        | 0.634

★ 4.3]  $1 - P$ (all are not innocent) complement

$$P(\text{none matches}) = \frac{(100000-100)C_{1000}}{100000C_{1000}} = 0.366$$

gas
nuth
kbbz

⑤ Conditional probability

★ Baye Theorem | ← USE THIS

2 giant exoplanet
probability star has rocky Earth-sized planet that isn't detected

↳ expansion of conditional

30% of stars → 2 planet
70% of stars → 3 planet
01% of stars < 2 or > 3 planets

$\frac{1}{2}$ gas planet   $\frac{1}{2}$ rocky plant

100% Jupiter     20% Earth

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$$

↓
2 giant gas
↓ 0 rocky observed

exists
1 rocky
planet

2 giant gas

$P(A) =$
2·2·2 = 8

○ ○ ⊗   $P(A) = \frac{3}{8}$
○ ⊗ ○
⊗ ○ ○

$P(B|A) = .8$

$$\frac{(.8)(.2625)}{.285}$$

$$\boxed{0.7368}$$

$P(B) = $  2 planet
2 plane

○○
⊗○
⊗⊗
○⊗

$\frac{3}{8}(.7)(.8)$

$\frac{1}{4}(.3)$

$= \frac{168}{800} + \frac{3}{40}$

$= 0.21 + 0.075$

$= 0.285$

$\frac{3}{8}(.7) = \frac{21}{80} = .2625$