

$$\begin{array}{c}
\boxed{\tau <: \tau} \\
\text{(S-REFL)} \quad \frac{}{\tau <: \tau} \quad \text{(S-TRANS)} \quad \frac{\tau_3 <: \tau_2 \quad \tau_2 <: \tau_1}{\tau_3 <: \tau_1} \quad \text{(S-STRUCT)} \quad \frac{\tau \ni \bar{\varsigma}}{\tau <: \mathbf{type}\{\bar{\varsigma}\}} \\
\text{(S-AND)} \quad \frac{\tau_3 <: \tau_1 \quad \tau_3 <: \tau_2}{\tau_3 <: (\tau_1 \& \tau_2)} \quad \text{(S-OR)} \quad \frac{\tau_2 <: \tau_1 \quad \tau_3 <: \tau_1}{(\tau_2 \mid \tau_3) <: \tau_1} \quad \text{(S-UNFOLD/L)} \quad \frac{[\tau_2/X]\tau_2 <: \tau_1}{\mu X.\tau_2 <: \tau_1} \quad \text{(S-UNFOLD/R)} \quad \frac{\tau_2 <: [\tau_1/X]\tau_1}{\tau_2 <: \mu X.\tau_1} \\
\text{(S-AND/L)} \quad \frac{\tau_2 <: \tau_1}{(\tau_2 \& \tau_3) <: \tau_1} \quad \text{(S-AND/R)} \quad \frac{\tau_3 <: \tau_1}{(\tau_2 \& \tau_3) <: \tau_1} \quad \text{(S-OR/L)} \quad \frac{\tau_3 <: \tau_1}{\tau_3 <: (\tau_1 \mid \tau_2)} \quad \text{(S-OR/R)} \quad \frac{\tau_3 <: \tau_2}{\tau_3 <: (\tau_1 \mid \tau_2)} \\
\text{(S-BOT)} \quad \frac{\tau_2 \ni m(x : \square) \rightarrow \_ \quad \tau_2 \ni m(y : \square) \rightarrow \_ \quad |\bar{x}| \neq |\bar{y}|}{\tau_2 <: \tau_1} \\
\boxed{\varsigma <: \varsigma} \quad \text{(SS-REFL)} \quad \frac{}{\varsigma <: \varsigma} \quad \text{(SS-TYPE)} \quad \frac{}{\mathbf{type}\ X = \tau <: \mathbf{type}\ X} \quad \text{(SS-SIG)} \quad \frac{\bar{\tau}'_1 <: \bar{\tau}'_2 \quad \tau_2 <: \tau_1}{m(\bar{y} : \bar{\tau}'_2) \rightarrow \tau_2 <: m(\bar{x} : \bar{\tau}'_1) \rightarrow \tau_1} \\
\boxed{\tau \ni \varsigma} \quad \text{(M-SIG)} \quad \frac{\tau <: \mathbf{type}\{\bar{\varsigma}\} \quad \varsigma_2 \in \bar{\varsigma} \quad \varsigma_2 <: \varsigma_1}{\tau \ni \varsigma_1} \quad \text{(M-BOTH)} \quad \frac{\tau_1 \ni m(\bar{x} : \bar{\tau}'_3) \rightarrow \tau_3 \quad \tau_2 \ni m(\bar{y} : \bar{\tau}'_4) \rightarrow \tau_4}{m(\bar{x} : (\bar{\tau}'_3 \mid \bar{\tau}'_4)) \rightarrow (\tau_3 \& \tau_4) <: \varsigma} \\
\frac{}{(\tau_1 \& \tau_2) \ni \varsigma}
\end{array}$$

■ **Figure 20** Subtyping, signature subtyping, and type-name/signature membership