Grace Miguel HW2

I pledge my honor that I've abided by the Stevens Honor system.

4) a) Mystery(n) $n=4$

$S=0$

$S=1$

$S=1+4=5$

$S=5+9=14$

$S=14+16=30$

This algorithm computes the sum of squares up to and including n.

b) Basic operation is setting S = itself + i^2

c) Operation is executed n times

d) $\Theta(n)$

e) $\frac{[n(n+1)(2n+1)]}{6}$ new algorithm w/ time complexity of $\Theta(1)$

1) a) $x(n)=x(n-1)+5$ for $n>1$ $x(1)=0$

Step 1

$x(n-1)=x(n-2)+5$

$x(n)=x(n-2)+10$

Step 2

$x(n-2)=x(n-3)+5$

$x(n)=x(n-3)+15$

Step 3

$x(n-i+1)+5i$

Step 4

IC $n-i=1$

$i=n-1$

Step 5

$x(n-(n-1))+5(n-1)$

$+5n-5$

$\boxed{5n-5}$

b) $x(n) = 3x(n-1)$ for $n > 1$ $x(1) = 4$

Step 1

$x(n-1) = 3x(n-2)$

$x(n) = 3[3x(n-2)] = 9x(n-2)$

Step 2

$x(n-2) = 3x(n-3)$

$x(n) = 9(3x(n-3) = 27x(n-3)$

Step 3

$x(n) = 3^i(n-i)$

Step 4

$3^x(n-i=1) = 4$

$i = i = n-1$

Step 5

$x(n) = 3^{n-1}(n-(n-1))$

$= \boxed{3^{n-1} \cdot 4}$

c) $x(n) = x(n-1) + n$ for $n > 0$ $x(0) = 0$

Step 1

$x(n-1) = x(n-2) + n-1$

$x(n) = x(n-2) + 2n-1$

Step 2

$x(n-2) = x(n-3) + n-1$

$x(n) = x(n-3) + 3n-2$

Step 3

$x(n-i) + (n-i+1) + (n-i+2) \ldots + n$

Step 4

$n-i = 0$

$i = n$

Step 5

$x(n-n) + (n-n+1) + (n-n+2) \ldots$

$\boxed{\frac{n(n+1)}{2}}$

d) $x(n) = x(\frac{n}{2}) + n$ for $n > 1$ $x(1) = 1$ solve for $n = 2^k$

Step 1

$x(2^k) = x(\frac{2^k}{2}) + 2^k \rightarrow x(2^{k-1}) + 2^k$

Step 2

$x(2^{k-1}) = x(2^{k-2}) + 2^{k-1}$

$x(2^k) = x(2^{k-2}) + 2^{k-1} + 2^k$

~~Step~~ 2.5

$x(2^{k-2}) = x(2^{k-3}) + 2^{k-2}$

$x(2^k) = x(2^{k-3}) + 2^{k-2} + 2^{k-1} + 2^k$

Step 3

General form:

$x(2^{k-i}) + 2^{k-i} + 2^{k-i+1} + 2^{k-i+2} \dots 2^k$

Step 4

IC $= x(1) = 1$

$2^{k-i} = 1 \quad 2^{k-i} = 2^0 \quad k-i=0 \quad \boxed{i=k}$

Step 5

$x(2^{0\,k}) + 2^{k-k} + 2^{k-k+1} + 2^{k-k+2} \dots 2^k$

$1 + 2 + 4 + \dots 2^k$

$\boxed{x(n) = 2n - 1}$

e) $x(n) = x(n/3) + 1 \quad n > 1 \quad x(1) = 1$ solve for $n = 3^k$

$x(3^k) = x\left(\frac{3^k}{3}\right) + 1 = x(3^{k-1}) + 1$

Step 1
$x(3^{k-1}) = x(3^{k-2}) + 1$
$x(3^k) = x(3^{k-2}) + 2$

Step 2
$x(3^{k-2}) = (3^{k-3}) + 1$
$x(3^k) = 3^{k-3} + 3$

Step 3
$x(3^{k-i}) + i$

Step 4
IC = $k \cdot 3^{k-i} = 1 \qquad 3^{k-i} = 3^0$

$\boxed{k = i}$

Step 5
$x(3^{k-k}) + k \qquad n = 3^k$
$1 + k \qquad \log_3 n = k$

$\boxed{x = 1 + \log_3 n}$

3) a) $S(n-1) + 2$

1. $S(n-1) = S(n-2) + 2$
   $S(n) = S(n-2) + 4$
2. $S(n-2) = S(n-3) + 2$
   $S(n) = S(n-3) + 6$
3. $S(n) = S(n-i) + 2i$
4. $n - i = 1 \quad i = n-1$
5. $S(n-(n-1)) + 2(n-1) \quad \boxed{x(n) = 2(n-1)}$

b) This has the same number of multiplications as the recursive, but the nonrecursive will take less storage and time.