CS5446 Assignment 1

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1 Classical Planning: Shakey's World

1.1 Define Predicates

Predicates for Shakey's World:

- $Location(x)$: x is a location within a room, $room$
- $In(x, room)$: location x is in room, $room$
- $At(obj, x)$: object, obj is at location x
- $On(robot, obj)$: robot, $robot$ is on object obj
- $Box(box)$: box is a box
- $Floor(floor)$: $floor$ is the floor in the room where the robot is located
- $Room(room)$: $room$ is a room
- $Door(door)$: $door$ is a door
- $Robot(robot)$: $robot$ is a robot
- $Switch(switch)$: $switch$ is a switch
- $Light(room, status)$: the light in room $room$ is in status $status$ i.e. On or Off
- $Pushable(obj)$: obj can be pushed by robot. In Shakey's world, this obj is the box.
- $Climbable(obj)$: obj can be climbed by robot. In Shakey's world, this obj is the box.
- $Flipable(obj)$: obj can be flipped by robot. In Shakey's world, this obj is the switch.

Along with defining the predicates, here are the assumptions made:

- Following the assumption above, the doors and switches of each room are located at the same location. Please refer Section 1.3 for more details on how the locations were declared.
- A door exists in both the room and the corridor.
- The position $On(robot, obj)$ implies the height of Shakey, if on $Floor$, Shakey is in low height and able to perform actions Go or $Move$; if on Box, Shakey is in high height and unable to perform actions Go or $Move$.

1.2 Shakey's Action Schema

There are a total of 6 action schema by Shakey:

1. Go from Location, $from$ to Location, to in Room, r
2. $Push$ box, b from Location, x to Location, y in Room, r

3. Climb up a box, b at Location, x
4. Climb down a box, b at Location, x
5. Turn on a light switch, s in room, r
6. Turn off a light switch, s in room, r

The action schema are written as follows:

$$
\begin{aligned}
&Action(Go(from, to, r),\\
&\qquad PRECOND: && Location(from) \land Location(to) \land Room(r) \land Robot(Shakey)\\
&&& At(Shakey, from) \land In(from, r) \land In(to, r) \land On(Shakey, Floor)\\
&&& \land Light(r, On)\\
&\qquad EFFECT: && \neg At(Shakey, from) \land At(Shakey, to))
\end{aligned}
$$

$$
\begin{aligned}
&Action(Push(b, x, y, r),\\
&\qquad PRECOND: && Box(b) \land Location(x) \land Location(y) \land Room(r) \land Robot(Shakey)\\
&&& \land At(Shakey, x) \land At(b, x) \land In(x, r) \land In(y, r) \land On(Shakey, Floor)\\
&&& \land Light(r, On) \land Pushable(b)\\
&\qquad EFFECT: && \neg At(Shakey, x) \land \neg At(b, x) \land At(Shakey, y)) \land At(b, y)
\end{aligned}
$$

$$
\begin{aligned}
&Action(ClimbUp(b, x),\\
&\qquad PRECOND: && Box(b) \land Location(x) \land Robot(Shakey) \land At(Shakey, x) \land At(b, x)\\
&&& \land On(Shakey, Floor) \land Climbable(b)\\
&\qquad EFFECT: && \neg On(Shakey, Floor) \land On(Shakey, b))
\end{aligned}
$$

$$
\begin{aligned}
&Action(ClimbDown(b),\\
&\qquad PRECOND: && Box(b) \land Robot(Shakey) \land On(Shakey, b) \land Climbable(b)\\
&\qquad EFFECT: && \neg On(Shakey, b) \land On(Shakey, Floor))
\end{aligned}
$$

$$
\begin{aligned}
&Action(TurnOn(s, b, x, r),\\
&\qquad PRECOND: && Switch(s) \land Box(b) \land Location(x) \land Room(r) \land On(Shakey, b)\\
&&& \land At(s, x) \land At(b, x) \land In(x, r) \land Light(r, Off) \land Flipable(s)\\
&\qquad EFFECT: && \neg Light(r, Off) \land Light(r, On)
\end{aligned}
$$

$$
\begin{aligned}
&Action(TurnOff(s, b, x, r),\\
&\qquad PRECOND: && Switch(s) \land Box(b) \land Location(x) \land Room(r) \land On(Shakey, b)\\
&&& \land At(s, x) \land At(b, x) \land In(x, r) \land Light(r, On) \land Flipable(s)\\
&\qquad EFFECT: && \neg Light(r, On) \land Light(r, Off)
\end{aligned}
$$

1.3 Shakey's Initial State

The initial states provided by the problem statement include:

- Shakey is in $Room3$
- Light is on in $Room3$ and $Corridor$
- There is a location p in $Room2$

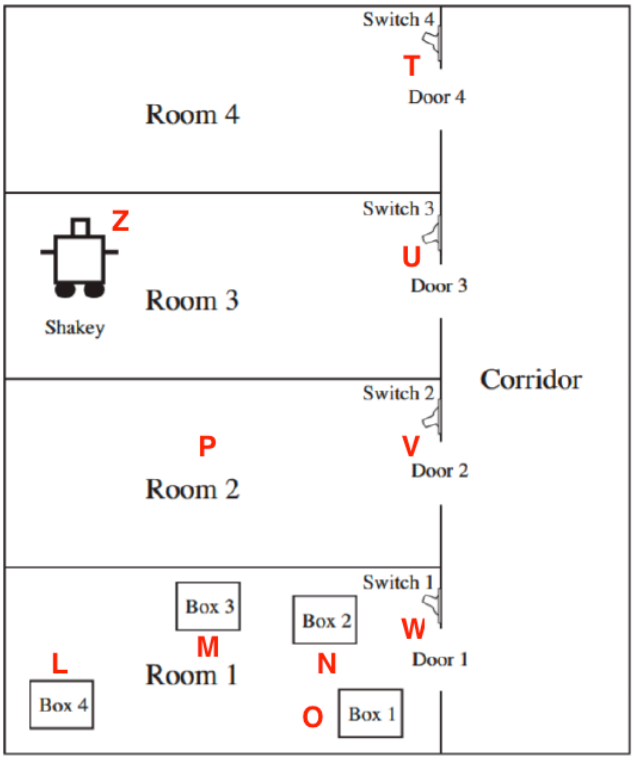

Figure 1: Shakey's World

To define the locations of the objects within the room, we have annotated the location as shown in Figure 1.

The initial state of Shakey is given by:

$Robot(Shakey) \wedge Box(Box1) \wedge Box(Box2) \wedge Box(Box3) \wedge Box(Box4) \wedge Room(Room1)$
$\wedge\ Room(Room2) \wedge Room(Room3) \wedge Room(Room4) \wedge Room(Corridor) \wedge Switch(Switch1)$
$\wedge\ Switch(Switch2) \wedge Switch(Switch3) \wedge Switch(Switch4) \wedge Location(Z) \wedge Location(O)$
$\wedge\ Location(N) \wedge Location(M) \wedge Location(L) \wedge Location(W) \wedge Location(V) \wedge Location(U)$
$\wedge\ Location(T) \wedge Location(P) \wedge At(Shakey, Z) \wedge At(Box1, O) \wedge At(Box2, N) \wedge At(Box3, M)$
$\wedge\ At(Box4, L) \wedge At(Door1, W) \wedge At(Door2, V) \wedge At(Door3, U) \wedge At(Door4, T) \wedge At(Switch1, W)$
$\wedge\ At(Switch2, V) \wedge At(Switch3, U) \wedge At(Switch4, T) \wedge In(Z, Room3) \wedge In(O, Room1)$
$\wedge\ In(N, Room1) \wedge In(M, Room1) \wedge In(L, Room1) \wedge In(W, Room1) \wedge In(V, Room2)$
$\wedge\ In(U, Room3) \wedge In(T, Room4) \wedge In(W, Corridor) \wedge In(V, Corridor) \wedge In(U, Corridor)$
$\wedge\ In(T, Corridor) \wedge In(P, Room2) \wedge On(Shakey, Floor) \wedge Light(Room1, On) \wedge Light(Room2, Off)$
$\wedge\ Light(Room3, On) \wedge Light(Room4, Off) \wedge Pushable(Box1) \wedge Pushable(Box2) \wedge Pushable(Box3)$
$\wedge\ Pushable(Box4) \wedge Climbable(Box1) \wedge Climbable(Box2) \wedge Climbable(Box3) \wedge Climbable(Box4)$
$\wedge\ Flipable(Switch1) \wedge Flipable(Switch2) \wedge Flipable(Switch3) \wedge Flipable(Switch4)$

1.4 Shakey Move Box, b to Location, p in Room 2, $Room2$

The execution steps for Shakey to push box from it's location in $Room1$ to p location in $Room2$ following the defined action schema would be:

1. Go($Z, U, Room3$)

2. Go($U, W, Corridor$)
3. Go($W, N, Room1$)
4. Push($Box2, N, W, Room1$)
5. Push($Box2, W, V, Corridor$)
6. ClimbUp($Box2, V$)
7. TurnOn($Switch2, Box2, V, Room2$)
8. ClimbDown($Box2$)
9. Push($Box2, V, P, Room2$)

2 Hierarchical Planning

HLA 1 $NavigateRoom2Room(room1, room2, door\_of\_room1\_pos, door\_of\_room2\_pos, startlocation, destination)$
Refinement: $GoRoom(room1, room2, door\_of\_room1\_pos, door\_of\_room2\_pos, startlocation, destination)$
Execution Steps:

1. $Go(startlocation, door\_of\_room1\_pos, room1)$
2. $Go(door\_of\_room1\_pos, door\_of\_room2\_pos, Corridor)$
3. $Go(door\_of\_room2\_pos, destination, room2)$

HLA 2 $SwitchLight(box, switch, room\_of\_shakey, room\_of\_box, room\_of\_switch, door\_of\_shakey\_room\_pos, door\_of\_box\_room\_pos, startlocation, box\_pos, switch\_pos)$
Refinement 1: $SwitchOn(box, switch, room\_of\_shakey, room\_of\_box, room\_of\_switch, door\_of\_shakey\_room\_pos, door\_of\_box\_room\_pos, startlocation, box\_pos, switch\_pos)$
Execution Steps:

1. $GoRoom(room\_of\_shakey, room\_of\_box, door\_of\_shakey\_room\_pos, door\_of\_box\_room\_pos, startlocation, box\_pos)$
2. $Push(box, box\_pos, door\_of\_box\_room\_pos, room\_of\_box)$
3. $Push(box, door\_of\_box\_room\_pos, switch\_pos, Corridor)$
4. $ClimbUp(box, switch\_pos)$
5. $TurnOn(switch, box, switch\_pos, room\_of\_switch)$
6. $ClimbDown(box)$

Refinement 2: $SwitchOff(box, switch, room\_of\_shakey, room\_of\_box, room\_of\_switch, door\_of\_shakey\_room\_pos, door\_of\_box\_room\_pos, startlocation, box\_pos, switch\_pos)$
Execution Steps:

1. $GoRoom(room\_of\_shakey, room\_of\_box, door\_of\_shakey\_room\_pos, door\_of\_box\_room\_pos, startlocation, box\_pos)$

2. $Push(box, box\_pos, door\_of\_box\_room\_pos, room\_of\_box)$
3. $Push(box, door\_of\_box\_room\_pos, switch\_pos, Corridor)$
4. $ClimbUp(box, switch\_pos)$
5. $TurnOff(switch, box, switch\_pos, room\_of\_switch)$
6. $ClimbDown(box)$

For HLA 2, we follow the same setting as Question 1, which did not assume that Shakey is with the box. We also use the refinement from HLA 1 as part of the procedure to get to the location of the box.

HLA 3 $PushObject(box, switch, startlocation, destination, door\_of\_shakey\_room\_pos, door\_of\_box\_room\_pos, switch\_pos, box\_pos, room\_of\_shakey, room\_of\_box, room\_destination)$
Refinement: $PushBox(box, switch, startlocation, destination, door\_of\_shakey\_room\_pos, door\_of\_box\_room\_pos, switch\_pos, box\_pos, room\_of\_shakey, room\_of\_box, room\_destination)$
Execution Steps:

1. $GoRoom(room\_of\_shakey, room\_of\_box, door\_of\_shakey\_room\_pos, door\_of\_box\_room\_pos, startlocation, box\_pos)$
2. $Push(box, box\_pos, door\_of\_box\_room\_pos, room\_of\_box)$
3. $Push(box, door\_of\_box\_room\_pos, switch\_pos, Corridor)$
4. $ClimbUp(box, switch\_pos)$
5. $TurnOn(switch, box, switch\_pos, room\_destination)$
6. $ClimbDown(box)$
7. $Push(box, switch\_pos, destination, room\_destination)$

For HLA 3, we follow the assumption provided in Question 1 that a box can only be pushed to a location accurately when the light is on in that particular room, so this will be included as part of the HLA. We could probably replace Step 1 to 6 with HLA 2 Refinement 1, but to demonstrate completeness, we left the primitive actions as is.

3 Decision Theory

3.1 Expected value of Lottery Ticket

For this question, we assume that the person already has the lottery ticket and the cost of the ticket is not considered in the expected value of the lottery ticket. The expected value of the lottery ticket can be formulated as follows:

$$E[T] = (Payoff_1 \cdot Probability_1) + (Payoff_2 \cdot Probability_2)$$

With the formulation above and the values provided, the expected value of lottery ticket is:

$$\begin{aligned} E[T] &= 12 \cdot \frac{1}{25} + 1.2 \cdot 10^6 \cdot \frac{1}{3 \cdot 10^6} \\ &= 0.192 \end{aligned}$$

3.2 When is it Rational to Buy the Lottery Ticket?

This is essentially asking when the following holds true:

$$\frac{1}{25}U(S_{k+12}) + \frac{1}{3\cdot 10^6}U(S_{k+1.2\cdot 10^6}) + U(S_{k-1}) > U(S_k)$$

Since we can assume that $U(S_k) = 0$, we can derive $U(S_{k-1}) = -U(S_{k+1})$, hence the above can be rewritten as:

$$\frac{1}{25}U(S_{k+12}) + \frac{1}{3\cdot 10^6}U(S_{k+1.2\cdot 10^6}) > U(S_{k+1})$$

Since we can assume that $U(S_{k+12}) = 12 \cdot U(S_{k+1})$, we have:

$$\begin{aligned}\frac{12}{25}U(S_{k+1}) + \frac{1}{3\cdot 10^6}U(S_{k+1.2\cdot 10^6}) &> U(S_{k+1})\\ \frac{25}{3.9\cdot 10^7}U(S_{k+1.2\cdot 10^6}) &> U(S_{k+1})\end{aligned}$$

In this case, it would be rational to buy a ticket.

Conclusion: The decision to purchase a ticket becomes rational if the combined utilities of the potential wins exceed the utility of the current state S_k. This depends on the individual utility function, particularly how the individual value the potential payoff vs. cost and the smaller win. For someone with a high-risk tolerance, they may place a significant utility $U(S_{k+1.2\cdot 10^6})$ on the potential of a 1.2 million jackpot, buying a lottery ticket can be "rational" to the person despite the low expected monetary value.