

Introduction:

In this lab we examine the wiener filters for restoring degraded images, either blur image with zero additive noise or blur image with non-zero additive noise. We also look for the optimal filter design for recovering an image.

Part A: Wiener Filtering (with zero additive noise):

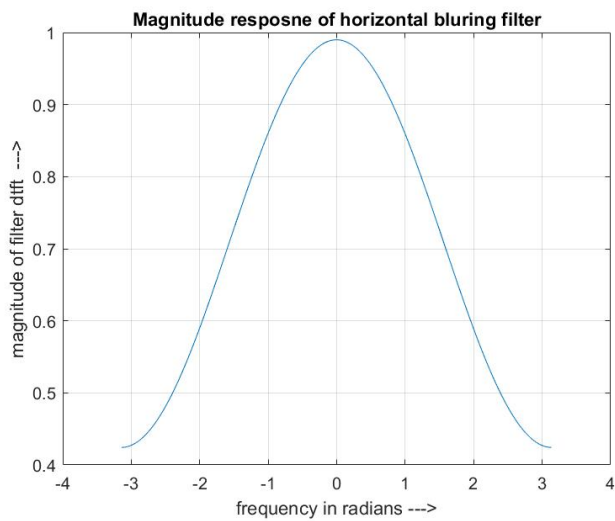
In partA, we apply wiener filtering to the zero additive noise, in which wiener filter is actually the same with inverse filter in this setting.

Frequency coefficient of horizontal filter (which is same as vertical one): [0.1414 0.7071 0.1414]

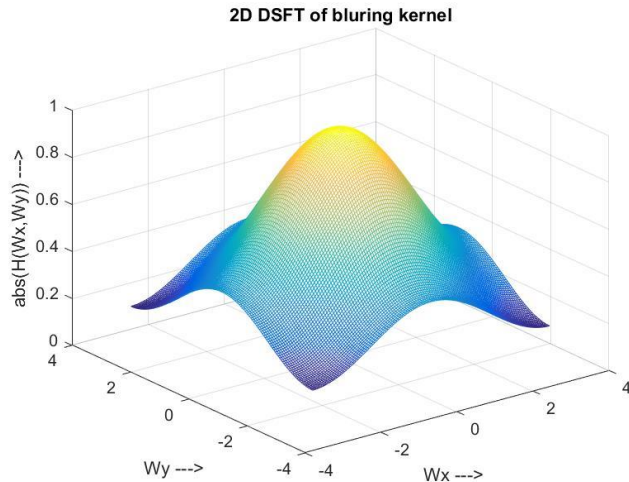
Frequency coefficient of 2d filter:

0.0200	0.1000	0.0200
0.1000	0.5000	0.1000
0.0200	0.1000	0.0200

Frequency response of row filter:



Frequency response of 2d filter:



The MSE after reconstruction of this zero-noise additive blurring image is 0.

Part B: Wiener Filtering (with additive noise)

First we added Gaussian random noise to the lena_blur image and we call the obtained image Y. Then apply 3x3 wiener filter obtained from part A to Y. The resulting image not fully constructed as the one from part A.

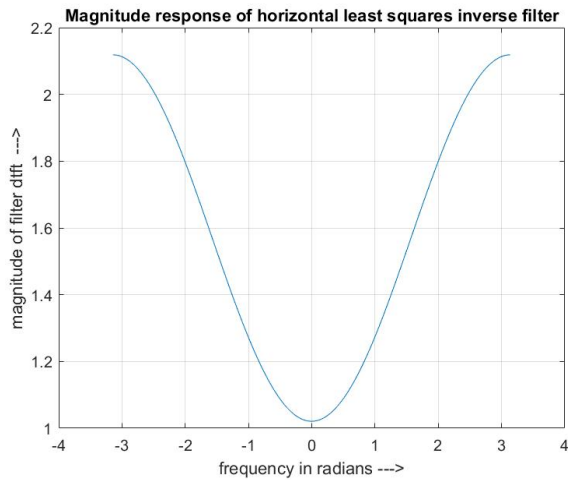
MSE between X and the restored image is: 56.4717 which is larger than the mse in part A when there is zero additive noise. Because when there is additive noise, the inverse filter will no longer be the ideal reconstruction filter. We expect that the restoration algorithm should filter out high frequencies, which are more likely due to noise than to the signal. We need another filter in place of inverse filter when there is non-zero additive noise.

Secondly, we find the optimal filter for Y. Same procedures were carried out as part A, only replacing the input degrade image with Y.

When $\sigma_{\text{noise}} = 0$ (standard deviation of noise is 0):

MSE after row filtering: 3.1375

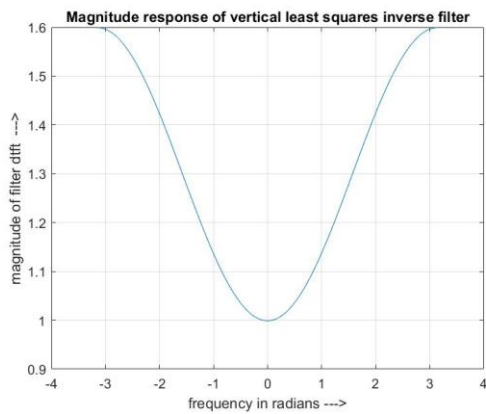
Frequency response of row filter:



Filter coefficient: [-0.2748 1.5697 -0.2740]

MSE after column filtering + row filtering: 0.7491

Frequency response of column filter:



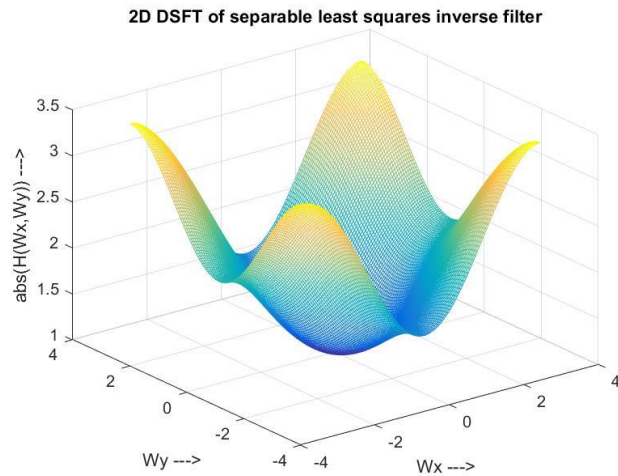
Filter coefficient: [-0.1497 1.2990 -0.1503]

MSE after reconstruction with inverse filter: 0.9778

Filter coefficient of inverse filter is :

0.0411	-0.2350	0.0410
-0.3570	2.0390	-0.3560
0.0413	-0.2359	0.0412

The frequency response of 2D-inverse filter:



Compare with the MSE from previous one when we directly using the inverse filter as the reconstruction filter, the MSE reduce greatly.

Third, we apply `wiener2()` function to `Y` with three different window sizes (3x3,5x5,7x7).

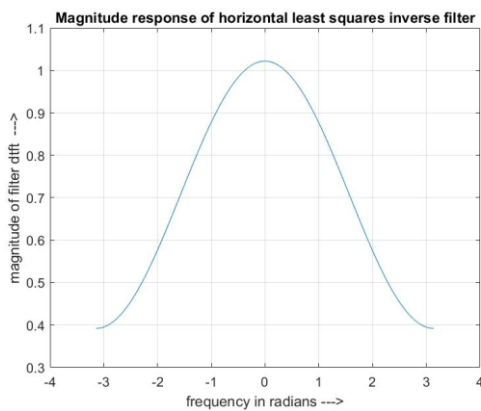
Performance of 3x3,5x5,7x7

MSE = 17.7112

Fourth, we generate the noise with standard deviation 5 (Low noise case), $\sigma_{\text{noise}} = 5$

MSE after row filtering: 28.5495

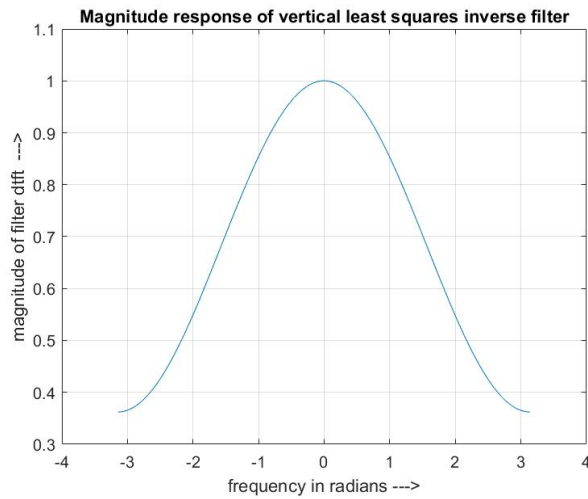
Frequency response of row filter:



Filter coefficient: [0.1578 0.7069 0.1572]

MSE after column filtering + row filtering: 24.3624

Frequency response of column filter:



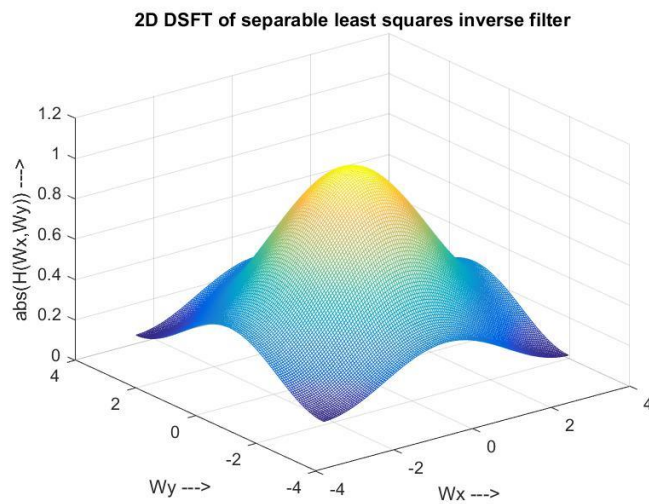
Filter coefficient: [0.1595 0.6812 0.1595]

MSE after reconstruction with inverse filter: 32.8021

Filter coefficient of inverse filter is :

0.0252	0.1128	0.0251
0.1075	0.4815	0.1071
0.0252	0.1128	0.0251

The frequency response of 2D-inverse filter:



When we apply `wiener2()` function to `Y` with three different window sizes (3x3,5x5,7x7).

Compare matlab's wiener-filtered image to the one obtained on ourself: the MSE becomes larger when applying wiener2() .

Performance of 3x3,5x5,7x7

MSE of one with window size 3x3: 45.0108

MSE of one with window size 5x5: 46.3975

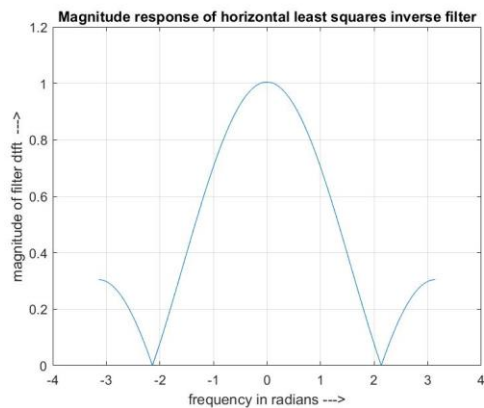
MSE of one with window size 7x7: 48.4144

70.6226

Then we generate the noise with standard deviation 30 (high noise case), $\sigma = 30$

MSE after row filtering: 337.2844

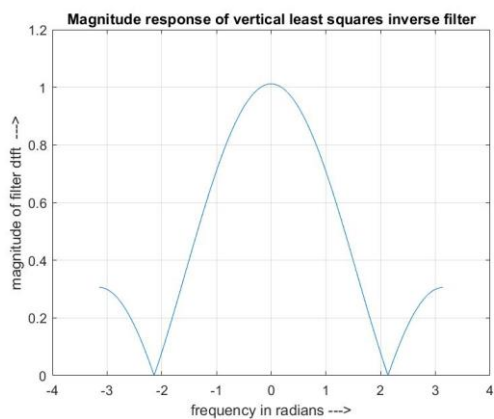
Frequency response of row filter:



Filter coefficient: [0.3271 0.3504 0.3277]

MSE after column filtering + row filtering: 137.2039

Frequency response of column filter:



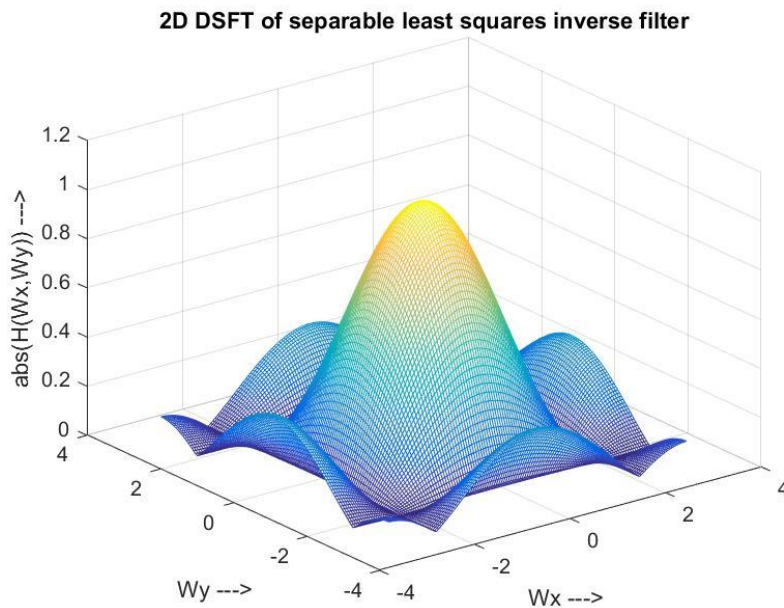
Filter coefficient: [0.3285 0.3531 0.3302]

MSE after reconstruction with inverse filtering: 156.1473

Filter coefficient of inverse filter is :

0.1074	0.1151	0.1076
0.1155	0.1237	0.1157
0.1080	0.1157	0.1082

Frequency response of inverse filter:



When we apply `wiener2()` function to `Y` with three different window sizes (3x3,5x5,7x7).

Compare matlab's wiener-filtered image to the one obtained on ourself: the MSE becomes larger when applying `wiener2()`

Performance of 3x3,5x5,7x7 becomes worse than when sigma value is lower.

MSE of one with window size 3x3: 681.8215

MSE of one with window size 5x5: 601.9590

MSE of one with window size 7x7: 604.3642

Q1: The blurring filter will blur the image without additive noise while wiener filter will also add noise to the image after blurring with blurring filter. The inverse blurring filter will be the inverse filter which performs better when recovering image with low noise than high noise, but not the optimal filter for recovering the blurred image with additive non-zero noise. Inverse wiener filter will perform better in the low noise case than the high noise case.

Q2: The image recovers pretty well after applying the optimal filter for blurring image with additive noise. When additive noise is non-zero, we will need to find the optimal filter which can remove the high-frequency component which is due to the noise while preserving the high frequency component due to the original signal. After comparing the resulting recovered image for $\sigma = 0, 5, 30$, we could find that as the standard deviation of gaussian noise is larger, the recovering is worse. The smoothing effect of filter increase as the noise level (standard deviation of gaussian noise) increase.

Q3: The inverse wiener filter in part A is same with inverse filter. It amplify the high-frequency noise components to preserve the high-frequency components of the original signal since there is no high-frequency noise exists. So we find a high pass filter in the end. The inverse wiener filter in part B need to preserve low frequency component of blurred image which are mostly given by the original signal. So we find a low pass filter in this end. Comparing 2 different noise realizations ($\sigma = 5$ and $\sigma = 30$), the frequency response of the one with $\sigma = 30$ has shape of sinc function while the one with $\sigma = 5$ is not, with magnitude of frequency response decaying on both side y axis.

Conclusion:

In this lab, we examine the wiener filters for both blurred image with zero noise and blurred image with non-zero noise. We also find the optimal filters for both situations and compare the resulting reconstructed image with different noise level.