MS&E 346 Assignment 12

Junting Duan

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1 Problem 3

Proof. By definition, the return G_t can be calculated as

$$\begin{aligned} G_t &= R_{t+1} + \gamma \cdot R_{t+2} + \gamma^2 \cdot R_{t+3} + \cdots + \gamma^{T-t-1} \cdot R_T + \gamma^{T-t} \cdot V(S_T) \\ &= \sum_{u=t}^{T-1} \gamma^{u-t} \cdot R_{u+1} + \gamma^{T-t} \cdot V(S_T). \end{aligned}$$

Additionally, the RHS can be rewritten as

$$\begin{aligned} &\sum_{u=t}^{T-1} \gamma^{u-t} \cdot R_{u+1} + \sum_{u=t}^{T-1} \gamma^{u-t+1} \cdot V(S_{u+1}) - \sum_{u=t}^{T-1} \gamma^{u-t} \cdot V(S_u) \\ =&\sum_{u=t}^{T-1} \gamma^{u-t} \cdot R_{u+1} + \sum_{u=t+1}^{T} \gamma^{u-t} \cdot V(S_u) - \sum_{u=t}^{T-1} \gamma^{u-t} \cdot V(S_u) \\ =&\sum_{u=t}^{T-1} \gamma^{u-t} \cdot R_{u+1} + \gamma^{T-t} \cdot V(S_T) - V(S_t), \end{aligned}$$

which is exactly the same with the formula on the LHS. □