Scenario taken from **A First in Mathematical Modeling: Third Edition** by Frank R. Giordano, Maurice D. Weir, and William P. Fox, specifically Chapter 5 Section 5 Project 4, parts C and D.

In the Los Angeles County School District, substitute teachers are placed in a pool and paid whether they teach or not. It is assumed that if the need for substitutes exceeds the size of the pool, classes can be covered by regular teachers, but at a higher pay rate. Letting x represent the number of substitutes needed on a given day, S the pool size, p the amount of pay for pool members, and r the daily overtime rate, we have for the cost

$$C(x,S) = \begin{Bmatrix} pS & if\ x < S \\ pS + (x-S)r & if\ x \geq S \end{Bmatrix}$$

Here, we assume $p < r$.

Use the data provided for the number of substitutes needed on Tuesdays to simulate the situation in an attempt to optimize the pool size. The optimized pool will be the one with the lowest expected cost to the school district. Use for pay rates, $p = \$45$, $r = \$81$. Assume that the data are distributed uniformly.

I. Make 500 simulations at each value of S from $S = 100$ to $S = 900$ in steps of 100 using the averages of the 500 runs to estimate the cost for each value of S.
II. Narrow the search for the best value of S to an interval of length 200 and make runs of 1000 simulations for each of ten equally spaced values of S in this interval.
III. Continue the process narrowing the search for the optimal size of the pool, each time stepping the values of S a smaller amount and increasing the number of iterations for better accuracy. When you have determined the optimal value for the pool size, S, submit your choice with substantiating evidence.

Now redo the above optimization using the data for Tuesdays assuming that $p = \$36$ and $r = \$81$.

Demand for substitute teachers on Tuesdays:

Number of Teachers	Relative Percentage	Cumulative Percentage
201-275	2.5	2.5
276-350	2.5	5.0
351-425	5.0	10.0

426-500	7.5	17.5
501-575	12.5	30.0
576-650	17.5	47.5
651-725	42.5	90.0
726-800	5.0	95.0
801-875	2.5	97.5
876-950	2.5	100.0