

Example

Solve the following system by comparison

$$\begin{aligned} 2x - 5y &= 7 \\ -5x + 2y &= -7 \end{aligned}$$

What you do is to solve for one variable in terms of the other in both equations first.

Equation ① $2x - 5y = 7 \Rightarrow 2x = 7 + 5y$

$$x = \frac{1}{2}(7 + 5y)$$

$$x = \frac{7}{2} + \frac{5}{2}y \quad \text{(*)}$$

Equation ② $-5x + 2y = -7 \Rightarrow -5x = -7 - 2y$

$$x = -\frac{1}{5}(-7 - 2y)$$

$$x = \frac{7}{5} + \frac{2}{5}y$$

Now you make the $2x$'s equal to each other.

$$\frac{7}{2} + \frac{5}{2}y = \frac{7}{5} + \frac{2}{5}y \quad \text{Solve for } y$$

$$\frac{7}{2} - \frac{7}{5} = \frac{3}{5}y - \frac{5}{2}y$$

$$\frac{7(5)}{2(5)} - \frac{7(2)}{5(2)} = \left(\frac{3}{5} - \frac{5}{2}\right)y$$

$$\frac{35}{10} - \frac{14}{10} = \left(\frac{2(2)}{5(2)} - \frac{5(5)}{2(5)}\right)y$$

$$\frac{21}{10} = -\frac{21}{10}y \Rightarrow y = -1$$

Since $x = \frac{7}{2} + \frac{5}{2}y$, then $x = \frac{7}{2} + \frac{5}{2}(-1) = \frac{7}{2} - \frac{5}{2} = 1$

Therefore, intersection point is $(1, -1)$

The system is consistent