

A line given in this form $L(t) = (-5t+1, 3-t)$ can be written in this form:

$$\begin{aligned}(x, y) &= (-5t+1, -t+3) \\ &= (-5t, -t) + (1, 3) \\ &= t(-5, -1) + (1, 3) \\ &= -5t\left(1, \frac{1}{5}\right) + (1, 3) \Rightarrow \text{the slope of the straight line is } \frac{1}{5} \text{ and passes through the point } (1, 3)\end{aligned}$$

$$y = \frac{1}{5}x + b$$

$$3 = \frac{1}{5}(1) + b \Rightarrow 3 - \frac{1}{5} = b \Rightarrow b = \frac{14}{5}$$

$$y = \frac{1}{5}x + \frac{14}{5}$$

Perpendicular lines have their direction vectors to be perpendicular to each other.

So the line $(x, y) = r\left(1, \frac{1}{5}\right) + (1, 3)$

is perpendicular to the line $(x, y) = t(1, -5) + (5, 1)$ for example.

because $\left(1, \frac{1}{5}\right) \cdot (1, -5) = 1 + \left(\frac{1}{5}\right)(-5) = 1 - 1 = 0$.

Determine whether the following lines are parallel, intersecting or identical.

(a) $r(t) = (-2+t, 5-3t)$ and $s(t) = (-3+2t, 3-6t)$

$$r(t) = (-2, 5) + t(1, -3) \quad s(t) = (-3, 3) + t(2, -6)$$

Since the direction vectors are $(1, -3)$ and $(2, -6)$, then the 2 lines are either // or identical.

If they have at least one common point, then they are identical.

$$(-2+t_1, 5-3t_1) = (-3+2t_2, 3-6t_2)$$

$$-2+t_1 = -3+2t_2 \Rightarrow t_1 - 2t_2 = -1$$

$$5-3t_1 = 3-6t_2 \Rightarrow -3t_1 + 6t_2 = -2$$

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$$\begin{array}{rcl} 3t_1 - 6t_2 & = & -3 \\ -3t_1 + 6t_2 & = & -2 \\ \hline 0 + 0 & = & -5 \end{array} \quad \text{Contradiction} \Rightarrow \text{The 2 lines are } \parallel.$$

(b) $r(t) = (1+5t, -3+2t)$ and $s(t) = (2+3t, 4+5t)$
 $\vec{d}_1 = (5, 2)$ $\vec{d}_2 = (3, 5)$
 $\vec{d}_1 \not\parallel \vec{d}_2$ \vec{d}_1 and \vec{d}_2 are not parallel nor identical

\Rightarrow They are intersecting

(c) $r(t) = (1+3t, -3+5t)$ $s(t) = (4-6t, 2-10t)$
 $\vec{d}_1 = (3, 5)$ $\vec{d}_2 = (-6, -10)$
 $-2\vec{d}_1 = \vec{d}_2 \Rightarrow \vec{d}_1 \parallel \vec{d}_2$

The lines are either parallel or identical

Let's see if there are any intersection points

$$\begin{array}{lcl} 1+3t_1 & = & 4-6t_2 \Rightarrow 3t_1+6t_2=3 \Rightarrow 5t_1+30t_2=15 \\ -3+5t_1 & = & 2-10t_2 \Rightarrow 5t_1+10t_2=5 \Rightarrow 15t_1+30t_2=15 \end{array}$$

Same line \Rightarrow Identical