

Answers to Homework

1(a) $P_1(4, 8)$ $P_2(3, 7)$

$$\vec{P_1P_2} = (3-4, 7-8) = (-1, -1)$$

(b) $P_1(-1, 0, 2)$, $P_2(0, -1, 0)$

$$\vec{P_1P_2} = (0-(-1), -1-0, 0-2) = (1, -1, -2)$$

2. (i) Let the terminal point of \vec{u} be $Q(x, y, z)$.
Then $\vec{PQ} = (x-(-1), y-3, z-5) = (x+1, y-3, z-5)$

$$\vec{PQ} = \vec{v} \Rightarrow (x+1, y-3, z-5) = (6, 7, -3)$$

$$x+1=6 \Rightarrow x=5$$

$$y-3=7 \Rightarrow y=10$$

$$z-5=-3 \Rightarrow z=2$$

So terminal point of \vec{u} is $Q(5, 10, 2)$.

(ii) Let the terminal point of \vec{u} be $Q(x, y, z)$.
 $\vec{PQ} = -\vec{v} \Rightarrow (x+1, y-3, z-5) = (-6, -7, 3)$

$$x+1=-6 \Rightarrow x=-7$$

$$y-3=-7 \Rightarrow y=-4$$

$$z-5=3 \Rightarrow z=8$$

So terminal point of \vec{u} is $Q(-7, -4, 8)$

3. Let the initial point of \vec{u} be $P(x, y, z)$

(i) $\vec{PQ} = (3-x, -y, -5-z)$

$$\vec{PQ} = \vec{v} \Rightarrow (3-x, -y, -5-z) = (4, -2, -1)$$

$$3-x=4 \Rightarrow x=-1$$

$$-y=-2 \Rightarrow y=2$$

$$-5-z=-1 \Rightarrow z=-4$$

So initial point of \vec{u} is $P(-1, 2, -4)$

$$(ii) (3-x, -y, -5-z) = (-4, 2, 1)$$

$$3-x = -4 \Rightarrow x = 7$$

$$-y = 2 \Rightarrow y = -2$$

$$-5-z = 1 \Rightarrow z = -6$$

So initial point of $\vec{u} = (7, -2, -6)$

$$4. \vec{u} = (-3, 1, 2), \vec{v} = (4, 0, -8) \text{ and } \vec{w} = (6, -1, -4)$$

$$\begin{aligned} (2\vec{u} - 7\vec{w}) - (8\vec{v} + \vec{u}) &= 2\vec{u} - 7\vec{w} - 8\vec{v} - \vec{u} \\ &= \vec{u} - 7\vec{w} - 8\vec{v} \\ &= (-3, 1, 2) - 7(6, -1, -4) - 8(4, 0, -8) \\ &= (-3 - 42 - 32, 1 + 7 - 0, 2 + 28 + 64) \\ &= (-77, 8, 94) \end{aligned}$$

$$5. 2\vec{u} - \vec{v} + \vec{x} = 7\vec{x} + \vec{w}$$

Solve for \vec{x} first

$$\vec{x} - 7\vec{x} = \vec{w} - 2\vec{u} + \vec{v}$$

$$-6\vec{x} = (6, -1, -4) - 2(-3, 1, 2) + (4, 0, -8)$$

$$-6\vec{x} = (16, -3, -16)$$

$$\vec{x} = -\frac{1}{6}(16, -3, -16)$$

$$= \left(-\frac{8}{3}, \frac{1}{2}, \frac{8}{3}\right)$$

$$6. 3a - 2b = -13$$

$$a + b = -1 \Rightarrow a = -1 - b$$

$$3(-1-b) - 2b = -13$$

$$-3 - 3b - 2b = -13$$

$$-5b = -13 + 3 \Rightarrow -5b = -10 \Rightarrow b = 2$$

$$\Rightarrow a = -1 - 2 = -3$$

$$\boxed{a = -3, b = 2}$$

$$\Rightarrow -3(3, 1) + 2(-2, 1) = (-13, -1)$$

$$7. \|\vec{v}\| = \sqrt{2^2 + 3^2} = \sqrt{13}.$$

$$\|\vec{w}\| = \sqrt{(-7)^2 + (-2)^2 + 1^2} = \sqrt{54} = \sqrt{9(6)} = 3\sqrt{6}$$

$$8(a) \text{ Distance} = \sqrt{(-1+3)^2 + (-4-6)^2} = \sqrt{4+100} = \sqrt{104} = 2\sqrt{26}.$$

$$(b) \text{ Distance} = \sqrt{(-7-7)^2 + (-2+5)^2 + (-1-1)^2} = \sqrt{196+9+4} = \sqrt{209}$$

$$9. \vec{v} = (-1, 2, 5) \quad \|k\vec{v}\| = 4.$$

$$k\vec{v} = (-k, 2k, 5k)$$

$$\|k\vec{v}\| = \sqrt{(-k)^2 + (2k)^2 + (5k)^2} = \sqrt{k^2 + 4k^2 + 25k^2} = \sqrt{30k^2}$$

$$\sqrt{30k^2} = 4$$

Square both sides

$$30k^2 = 16$$

$$k^2 = \frac{16}{30}$$

Square root both sides

$$\Rightarrow k = \pm \sqrt{\frac{16}{30}} = \pm \frac{4}{\sqrt{30}}$$