Solutions

1. (a) $\vec{u}\cdot\vec{v} = (1,-3)\cdot(5,2) = 5-6 = -1$
(b) $\vec{u}\cdot\vec{v} = (2,1,-5)\cdot(-4,2,-10) = -8+2+50 = 44$.

2. (a) $\|\vec{v}\| = \sqrt{1^2+(-5)^2} = \sqrt{26}$ (b) $\|\vec{v}\| = \sqrt{(-3)^2+(-4)^2+6^2} = \sqrt{61}$

3. $\vec{w} = (1,2)$, $\vec{v}_1 = (5,0)$, $\vec{v}_2 = (1,3)$ and $\vec{v}_3 = (-2,5)$

We can express $\vec{w} = k_1\vec{v}_1 + k_2\vec{v}_2 + 0\vec{v}_3$

$(1,2) = k_1(5,0) + k_2(1,3) + 0(-2,5)$

$(1,2) = (5k_1+k_2,\ 3k_2)$

$5k_1 + k_2 = 1$

$3k_2 = 2 \Rightarrow k_2 = 2/3 \Rightarrow 5k_1 + \frac{2}{3} = 1 \Rightarrow 5k_1 = 1-\frac{2}{3} \Rightarrow 5k_1 = \frac{1}{3}$

$k_1 = \frac{1}{15}$

$\Rightarrow \vec{w} = (1,2) = \frac{1}{15}(5,0) + \frac{2}{3}(1,3) + 0(-2,5)$.

4. $\vec{u} = (1,-2,5)$, $\vec{v} = (3,1,-4)$ and $\vec{w} = (7,14,-41)$

$\vec{w} = x(1,-2,5) + y(3,1,-4)$

$(7,14,-41) = (x+3y,\ -2x+y,\ 5x-4y)$

$$\begin{cases} x+3y = 7 \\ -2x+y = 14 \end{cases}$$
$5x-4y = -41$.

Let's solve the system consisting of the first 2 equations by substitution. $x = 7-3y \Rightarrow -2(7-3y)+y = 14$.

$-14+6y+y = 14$.

$7y = 28 \Rightarrow y = 4$.

$x = 7-3(4) = -5$

$x = -5$ and $y = 4$. Verify the answer in all equations.

$-5+3(4) = 7$ ✓

$-2(-5)+4 = 14$ ✓

$5(-5)-4(4) = -41$ ✓

$\Rightarrow \vec{w} = -5(1,-2,5) + 4(3,1,-4)$

5. $\vec{v} = (-3,4) \Rightarrow \|\vec{v}\| = \sqrt{(-3)^2+4^2} = \sqrt{25} = 5$.

$\Rightarrow$ A unit vector with the same direction as $\vec{v}$ ~~is~~ $\frac{1}{5}(-3,4) = (-\frac{3}{5}, \frac{4}{5})$

and opp. direction of $\vec{v}$ is $(\frac{3}{5}, -\frac{4}{5})$

6. (a) $\vec{u}\cdot\vec{v} = 1(-2)+(-3)(5) = -17 \Rightarrow \boxed{\theta = \cos^{-1}\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \cos^{-1}\left(\frac{-17}{\sqrt{10}\sqrt{29}}\right)}$

(b) $\|\vec{u}+\vec{v}\| = \|(1+(-2), -3+5)\| = \|(-1,2)\| = \sqrt{(-1)^2+2^2} = \sqrt{5}$

$\|\vec{u}-\vec{v}\| = \|(1-(-2), -3-5)\| = \|(3,-8)\| = \sqrt{3^2+(-8)^2} = \sqrt{73}$

(c) The angle between $\vec{u}$ and $\vec{v}$ is obtuse since $\vec{u}\cdot\vec{v} < 0$

7. $\vec{u}\cdot\vec{v} = (-3,1,0)\cdot(-5,2,3) = 15+2+0 = 17$.

$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{17}{\sqrt{(-3)^2+1^2+0^2}\sqrt{(-5)^2+2^2+3^2}} = \frac{17}{\sqrt{10}\sqrt{38}} = \frac{17}{\sqrt{380}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{17}{\sqrt{380}}\right)$

8. (a) $-3x+2y=-13$ By substitution.

$2x+3y=0 \Rightarrow 3y=-2x \Rightarrow y = -\frac{2}{3}x$.

$-3x+2\left(-\frac{2}{3}x\right) = -13$.

$-3x - \frac{4}{3}x = -13 \Rightarrow -9x-4x=-39$.

$-13x=-39$

$x=3 \Rightarrow y = -\frac{2}{3}(3)$

$y=-2$

$x=3$ and $y=-2$ Verifying in the system:

$-3(3)+2(-2) = -13$ ✓

$2(3)+3(-2) = 0$. ✓

(b) $-5x+2y=23$ By elimination.

$3x-5y=-29$

$3(-5x+2y) = 3(23)$ $-15x+6y=69$

$5(3x-5y) = 5(-29)$ $15x-25y=-145$

$-19y=-76 \Rightarrow y=+4$

$-5x+2(+4)=23$

$-5x=15 \Rightarrow x=-3$

$\boxed{\begin{matrix} x=-3 \\ y=4 \end{matrix}}$ Verify in the 2 equations

$-5(-3)+2(4)=23$ ✓

$3(-3)-5(4)=-29$ ✓

9. $3x + 2y = 5$ $\quad 2(3x + 2y) = 2(5) \Rightarrow 6x + 4y = 10$
$-6x - 4y = 1$ $\quad -6x - 4y = 1 \Rightarrow -6x - 4y = 1$

$$0 = 11$$

Contradiction $\Rightarrow$ No Solution.

10. The line through A and B has slope $m_1 = \frac{7-(-1)}{3-(-1)} = \frac{8}{4} = 2$.

$y = 2x + b \qquad A(-1,-1)$
$-1 = 2(-1) + b \Rightarrow b = 1 \Rightarrow$ The equation of the line through A and B is $y = 2x + 1$.

The midpoint of the line segment from A to B is $M\left(\frac{-1+3}{2}, \frac{-1+7}{2}\right)$ or $M(1,3)$.
The slope of the perpendicular line to the line through A and B is $-\frac{1}{2} \Rightarrow$ The perpendicular line is

$\Rightarrow y = -\frac{1}{2}x + b$. If M is on the perpendicular line
then $3 = -\frac{1}{2}(1) + b \Rightarrow b = 3 + \frac{1}{2} = \frac{7}{2}$.

$y = -\frac{1}{2}x + \frac{7}{2}$ describes the set of points in $\mathbb{R}^2$ that are equidistant from the points $A(-1,-1)$ and $B(3,7)$.

11. $\overrightarrow{P_1P_2} = (1-2, -2-1, 0-(-2)) = (-1,-3,2)$

(a) $\overrightarrow{P_1P} = \frac{1}{5}\overrightarrow{P_1P_2}$ $\qquad$ Let $P(x,y,z)$

$\Rightarrow \overrightarrow{P_1P} = (x-2, y-1, z+2)$
$(x-2, y-1, z+2) = \frac{1}{5}(-1,-3,2)$
$x - 2 = -\frac{1}{5} \Rightarrow x = \frac{9}{5}$
$y - 1 = -\frac{3}{5} \Rightarrow y = \frac{2}{5}$
$z + 2 = \frac{2}{5} \Rightarrow z = -\frac{8}{5} \Rightarrow P\left(\frac{9}{5}, \frac{2}{5}, -\frac{8}{5}\right)$

(b) $\overrightarrow{PP_2} = \frac{1}{4}\overrightarrow{P_1P_2} \Rightarrow \overrightarrow{PP_2} = (1-x, -2-y, 0-z) = \frac{1}{4}(-1,-3,2)$

$1 - x = -\frac{1}{4} \Rightarrow x = \frac{5}{4}$.
$-2 - y = -\frac{3}{4} \Rightarrow y = -2 + \frac{3}{4} = -\frac{5}{4}$
$-z = \frac{2}{4} \Rightarrow z = -\frac{1}{2}$.

$P(5/4, -5/4, -\frac{1}{2})$