

Sec 1 Sep 22, 2020

#20 Sec 2.5

$$f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases} \quad a = 1$$

$D_f: x^2 - 1 \neq 0 \Rightarrow x \neq \pm 1$. However, the function is defined at $x = 1$ ($f(1) = 1$).

$$D_f: x \neq -1$$

At $x = 1$

✓ 1. $f(1) = 1$.

✓ 2. $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{x(x-1)}{(x+1)(x-1)} = \frac{1}{1+1} = \frac{1}{2}$

✗ $\lim_{x \rightarrow 1} f(x) \neq f(1)$

The function has a removable discontinuity at $x = 1$.
Hollow point or open point at $(1, \frac{1}{2})$

Extra I will discuss the behaviour of the function at $x = -1$.

At $x = -1$

✗ $f(-1)$ is undefined

✗ $\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} \frac{x^2 - x}{x^2 - 1} \xrightarrow{DNE}$ Infinite Limit \Rightarrow V.A. at $x = -1$

Infinite Discontinuity at $x = -1$

✗

Sec 2.5 #22.

$$f(x) = \begin{cases} \frac{2x^2 - 5x - 3}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases} \quad a = 3$$

At $x = 3$: ✓ 1. $f(3) = 6$

✓ 2. $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x - 3} = \lim_{x \rightarrow 3} \frac{(2x+1)(x-3)}{x-3} = 7$

✗ $\lim_{x \rightarrow 3} f(x) \neq f(3)$