$\checkmark$ 1. $f(4) = -1$.

$\times$ $\lim_{x\to 4} f(x)$ exists? $\times$

$$\left.\begin{array}{l}\lim_{x\to 4^-} 3-x = -1 \\ \lim_{x\to 4^+} \sqrt{x} = 2\end{array}\right\} \neq \quad \lim_{x\to 4} f(x) \text{ DNE}$$

Jump Discontinuity at $x=4$

Definition: A function f is continuous from the right at $x=a$ if $\lim_{x\to a^+} f(x) = f(a)$.

Likewise, f is continuous from the left at $x=a$ if $\lim_{x\to a^-} f(x) = f(a)$

Note: In the previous example $\lim_{x\to 4^-} f(x) = -1 = f(4)$

$\Rightarrow f$ is continuous from the left at $x=4$.

However, $\lim_{x\to 4^+} f(x) = 2 \neq f(4) \Rightarrow f$ is not continuous from the right at $x=4$.

<u>Example</u> Consider the function $H(x) = \begin{cases} 0 & \text{when } x<0 \\ 1 & \text{when } x \geq 0 \end{cases}$

Is the function continuous from the right at $x=0$?
from the left at $x=0$?

$\checkmark$ 1. $H(0) = 1$

$\times$ $\lim_{x\to 0} H(x)$ $\left\langle\begin{array}{l}\lim_{x\to 0^+} 1 = 1 \\ \lim_{x\to 0^-} 0 = 0\end{array}\right\rangle \neq$

limit does not exist

H is discontinuous at $x=0$
(H is cont. from the right at $x=0$ and H is discont. from the left at $x=0$)