201-114-VA Test 1 Oct 21, 2020

1. [3,1,1,2 marks] Let $\vec{u} = \langle 11,2 \rangle$ and $\vec{v} = \langle 12,9 \rangle$.
   (a) Find $\frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2}$.
   (b) Use the result from part (a) to calculate $\overrightarrow{w_1}$, where $\overrightarrow{w_1} = \frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2} \vec{v}$.
   (c) Calculate $\overrightarrow{w_2}$, where $\overrightarrow{w_2} = \vec{u} - \overrightarrow{w_1}$.
   (d) Are $\overrightarrow{w_1}$ and $\overrightarrow{w_2}$, parallel, perpendicular or neither? State why.

2. [6 marks] Let $\vec{u} = \langle 2, -4 \rangle$, $\vec{v} = \langle 9,8 \rangle$ and $\vec{w} = \langle 46,12 \rangle$.
   Find scalars c_1 and c_2 such that $c_1\vec{u} + c_2\vec{v} = \vec{w}$.

3. [5 marks] Let $\vec{u} = \langle 2, -5,1 \rangle$, $\vec{v} = \langle 5,3, -1 \rangle$, and $\vec{w} = \langle 11,19, -5 \rangle$ be vectors in R^3.
   Express $\vec{w}$ as a linear combination of $\vec{u}$ and $\vec{v}$, if possible.

4. [3 marks] Find two non-parallel vectors $\vec{v}$ and $\vec{w}$ in R^3perpendicular to $\vec{u} = \langle 8, -3,7 \rangle$.

5. [3 marks] Find a unit vector (having magnitude 1) parallel to the vector $\vec{v} = \langle 4, ,3, -12 \rangle$

6. [3,3 marks] Let $\vec{u} = \langle 8, -15 \rangle$ and $\vec{v} = \langle 12,16 \rangle$ be two sides of a parallelogram.
   (a) If the vectors are placed such that their initial points coincide, find the magnitude of the diagonal $\vec{u} - \vec{v}$.
   (b) Determine the angle Θ between $\vec{u}$ and $\vec{v}$.

7. [3 marks] Determine the equation of the line passing through the points P(12,35) and Q(18,-7).

8. [2,3,2 marks] Given the line $8x + 17y = 46$, determine:
   (a) The equation of the line parallel to the given line, and passing through the point P(9,-4).
   (b) The equation of the line perpendicular to the given line and passing through P(64,10).
   (c) Determine the x-intercept of your line in part (a).

9. [4 marks] Solve the system:
$$\begin{aligned} 8x &- 3y &&= 30 \\ 5x &- 2y &&= 19 \end{aligned}$$

10. [2,4 marks] A line contains the points A(-6,11) and B(9,-19).
    (a) Determine a vector which is parallel to the line.
    (b) Determine another point, P, on the line, such that $\left\|\overrightarrow{BP}\right\| = 2\left\|\overrightarrow{AB}\right\|$.