201-114-VA (Sec1) Test 2 Dec 14, 2020

Name: ____

1. [2,2,2 marks]

The number of kilometers travelled by 25 taxi drivers, on a given day, is shown below.

257, 259, 261, 266, 268, 268, 270, 273, 273, 274, 276, 281, 281, 282, 284, 284, 285, 287, 290, 294, 295, 297, 302, 302, 304

Determine each of the following.

a) median

b) first quartile Q_1

c) third quartile Q_3

2. [2,1,1 marks]

The ages of each of the players on a school basketball team are recorded below.

15, 15, 16, 16, 17, 17, 17, 18, 18

Calculate (a) the mean, (b) the median and (c) the mode.

3. [6,2,1,1 marks]

The table below shows the amount of time, in hours, that 45 randomly selected high school students spend on their computer, per day.

Classes (hours)	f_i	$f_i m_i$	Cumulative frequency	Relative frequency
[0-2[	5			
[2-4[	12			
[4-6[	15			
[6-8[	11			
[8-10[	2			
	$\Sigma f_i =$	$\sum f_i m_i =$		

a) Complete the table

b) Calculate the mean

c) Determine the median class

d) Determine the percentage of students who spend less than 6 hours on the computer

4. [5 marks]

The marks obtained by 10 students on a math test are recorded below.

60, 70, 65, 80, 85, 70, 65, 80, 50, 95

a) Use the variance formula $s^2 = \frac{\sum x_i^2}{n} - \overline{x}^2$ to calculate the variance for the data shown above.

b) Find the standard deviation

5. [7,3 marks]

A sample of 50 students is chosen at random from a high school.

The number of absences, x_i, per student is represented in the table.

x_i	f_i	$f_i x_i$	$(x_i - \overline{x})^2$	$f_i(x_i - \overline{x})^2$
0	18			
1	12			
2	10			
3	6			
4	3			
5	1			
	$\sum f_i =$	$\sum f_i x_i =$		$\sum f_i(x_i - \overline{x})^2 =$

a) Calculate the mean, and complete the table.

b) Calculate the variance and the standard deviation

6. [4,4,4 marks]

Consider the right triangle ABC, with vertices $A(0,4)$, $B(-4,19)$, and $C(11,23)$.

a) A rotation of 90^0 counter-clockwise, followed by a translation of 10 units to the right and 9 units down is to be applied to the triangle.

The transformation matrices are, respectively $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \end{bmatrix}$

Determine a single 3×3 transformation matrix that can be used to produce the same transformation as the one described above.

b) Determine the final position of each vertex in the transformed triangle. That is, determine the coordinates of A', B' and C'.

(c) Is the resulting figure a right triangle also? Show why or why not.

7. [4 marks] Consider the triangle given in question 6.

Suppose a scale change is to be applied to the triangle, **before** applying the transformations described in 6a. Given that the transformation matrix for the scale change is $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

determine the coordinates of vertex $C(11,23)$, after all three transformations are applied.

8. [3,3 marks]

Let $A = \begin{bmatrix} -13 & -9 \\ 6 & 4 \end{bmatrix}$

a) Determine A^{-1}, the inverse of A

b) Calculate AA^{-1}

9. [2, 5 marks]

Consider the system of equations shown below.

$10x - 3y = 49$

$4x + 5y = 1$

a) Express the system in the form $A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 49 \\ 1 \end{bmatrix}$, where A is the coefficient matrix

b) Find A^{-1}, and then use it to solve the system of equations.

10. [6 marks]

Let T be a linear transformation of the form $T: R^2 \to R^2$.

The 2×2 transformation matrix transforms $\begin{bmatrix} -1 \\ 5 \end{bmatrix}$ into $\begin{bmatrix} 31 \\ -22 \end{bmatrix}$, and transforms $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ into $\begin{bmatrix} -15 \\ 14 \end{bmatrix}$.

Find the transformation matrix.

Bonus Question

Let T be a linear transformation of the form $T: R^3 \to R^2$.

Determine the transformation matrix associated with the transformation T, such that

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 8 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$$