Definition: A function f is continuous on an interval if it is continuous at every number in that interval.

For example, the function H in the previous example is continuous on $(-\infty, 0) \cup (0, \infty)$.

Example: Let $f(x) = 2x^2 + 3x - 4$. f is continuous everywhere on $\mathbb{R}$ because if a is any real number, then

✓ 1. $f(a)$ is defined.

✓ 2. $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} 2x^2 + 3x - 4 = 2a^2 + 3a - 4$

✓ 3. $\lim\limits_{x \to a} f(x) = f(a)$.

Theorem: If f and g are continuous at a and c is a constant, then the following functions are also continuous at $x = a$.

1. $f + g$ 2. $f - g$ 3. cf
4. fg 5. f/g if $g(a) \neq 0$.

Theorem: The following types of functions are continuous at every number in their domain.

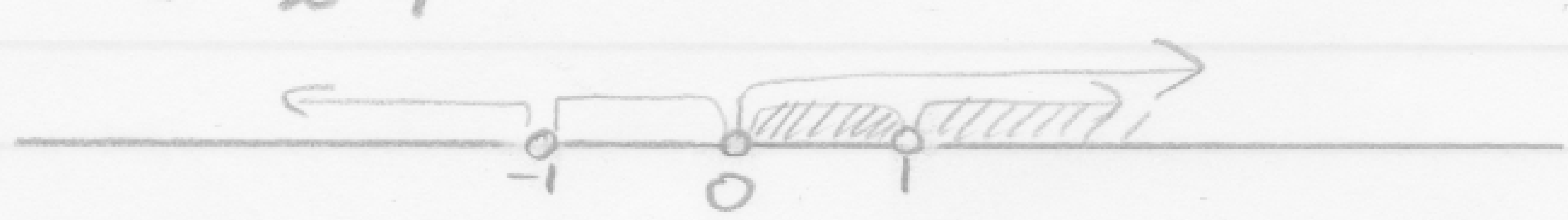
polynomial, rational, root, trigonometric, inverse trigonometric, exponential, logarithmic, algebraic

Example: Where is $f(x) = \dfrac{\ln x + \tan^{-1} x}{x^2 - 1}$ continuous?

The domain of $y = \ln x$ is $x > 0$ $(0, \infty)$

The domain of $y = \tan^{-1} x$ is $\mathbb{R}$ $(-\infty, \infty)$

The domain of $y = \dfrac{1}{x^2 - 1}$ is $x \neq \pm 1$. $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$



$D_f : (0, 1) \cup (1, \infty) \Rightarrow f$ is continuous everywhere on $(0, 1) \cup (1, \infty)$