Mon Sept 21, 2020

Example:

Is the vector $\vec{a} = (2,5)$ a linear combination of $\vec{u} = (5,-2)$ and $\vec{v} = (-10,4)$?

$$\vec{a} = k\vec{u} + l\vec{v} \quad ?$$

$$(2,5) = k(5,-2) + l(-10,4)$$
$$(2,5) = (5k,-2k) + (-10l, 4l)$$
$$(2,5) = (5k-10l, -2k+4l)$$

$$2(5k - 10l = 2)$$
$$5(-2k + 4l = 5)$$

$$\begin{array}{r} 10k - 20l = 4 \\ + \quad -10k + 20l = 25 \\ \hline 0k + 0l \neq 29 \end{array}$$

Contradiction

Initial Point = Tail

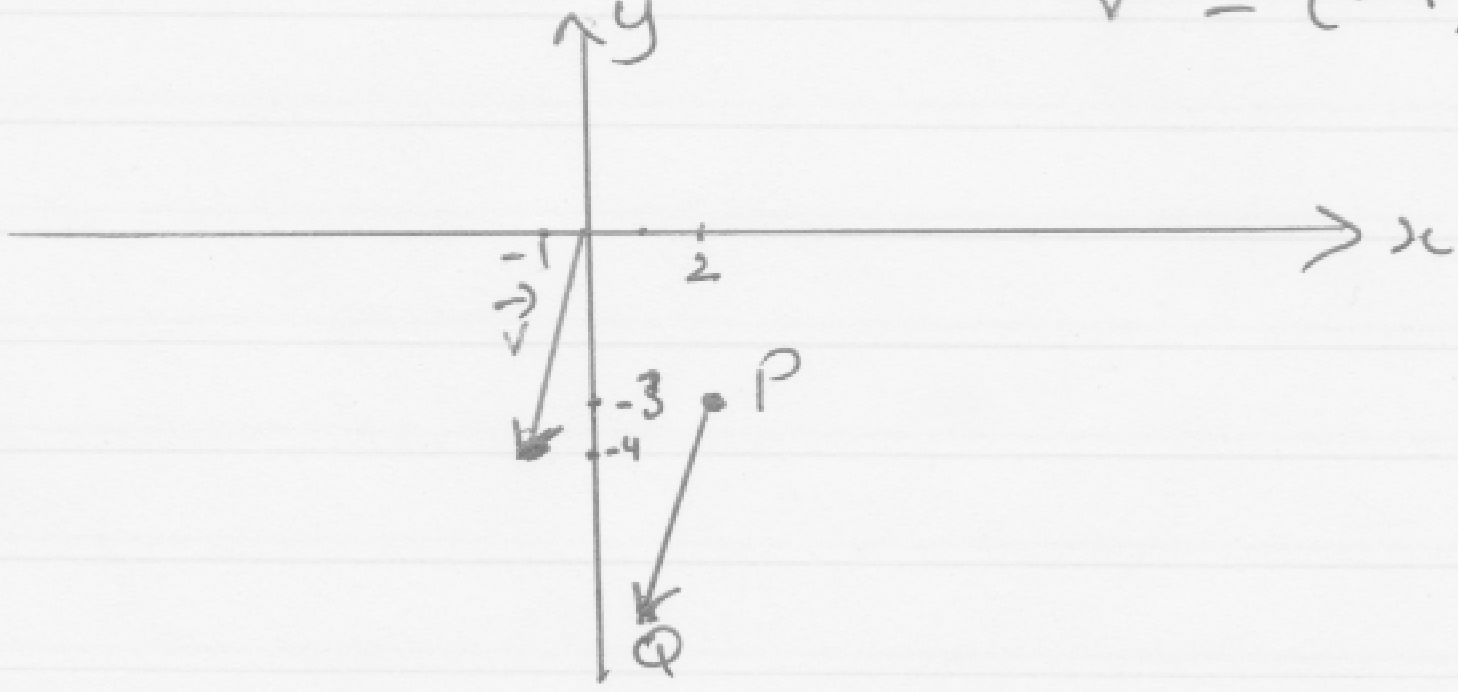
Terminal Point = Head

Definition: Let $\vec{v} = \overrightarrow{AB}$ $\quad A(x_1, y_1) \quad B(x_2, y_2)$

$$\vec{v} = (x_2 - x_1, y_2 - y_1)$$

Example Let $\vec{v}$ be the vector with initial point $P(2,-3)$ and terminal point $Q(1,-7)$. Find the components of $\vec{v}$.

$$\vec{v} = \overrightarrow{PQ} = (1,-7) - (2,-3) = (1-2, -7-(-3))$$
$$\vec{v} = (-1, -4).$$



Example

In each case, find $\overrightarrow{PQ}$ and $\|\overrightarrow{PQ}\|$

(a) $P(1,-1)$ and $Q(3,1)$ (b) $P(0,1)$ and $Q(0,-3)$.

(a) $\overrightarrow{PQ} = (3-1, 1-(-1)) = (2,2)$ $\|\overrightarrow{PQ}\| = \sqrt{2^2+2^2} = \sqrt{8}$

(b) $\overrightarrow{PQ} = (0-0, -3-1) = (0,-4)$ $\|\overrightarrow{PQ}\| = \sqrt{0^2+(-4)^2} = 4$.

Distance between 2 points $A(x_1,y_1)$ and $B(x_2,y_2)$.

The distance $d = \|\overrightarrow{AB}\| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Ex Find the distance between points $P(-2,4)$ and $Q(-5,2)$.

$$d = \sqrt{(-5-(-2))^2+(2-4)^2} = \sqrt{9+4} = \sqrt{13}$$

Ex Let $P(x,2)$ and $Q(3,x)$ be points in the xy-coordinate system. Find x such that $\|\overrightarrow{PQ}\| = 5$.

$$\overrightarrow{PQ} = (3-x, x-2)$$

$$\|\overrightarrow{PQ}\| = \sqrt{(3-x)^2+(x-2)^2} = 5$$ Square both sides

$$(3-x)^2+(x-2)^2 = 25$$

$$(3-x)(3-x)+(x-2)(x-2) = 25$$

$$9-3x-3x+x^2+x^2-2x-2x+4 = 25$$

$$2x^2-10x+13-25 = 0$$

$$2x^2-10x-12 = 0.$$

$$2(x^2-5x-6) = 0$$

$$2(x-6)(x+1) = 0$$

-6

-1 6

-6 $+1$

$ax^2+bx+c=0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x-6=0$ or $(x+1)=0$

$x=6$ or $x=-1$.

Either (i) $P(6,2)$ and $Q(3,6)$.
or (ii) $P(-1,2)$ and $Q(3,-1)$.

Verify (i) $\|\overrightarrow{PQ}\| = \sqrt{(3-6)^2+(6-2)^2} = \sqrt{9+16} = 5$ ✓

(ii) $\|\overrightarrow{PQ}\| = \sqrt{(3-(-1))^2+(-1-2)^2} = \sqrt{16+9} = 5$ ✓.

<u>Example</u> In each case, find a point Q such that $\overrightarrow{PQ}$ has (i) the same direction as $\vec{v}$
(ii) the opposite direction of $\vec{v}$.

(a)(i) $P(-1,2)$ $\vec{v}=(1,3)$. Let $Q(x,y)$.

$\overrightarrow{PQ} = (x-(-1), y-2) = (1,3)$
$(x+1, y-2) = (1,3)$

$x+1=1$
$y-2=3$

$Q(0,5)$

(ii) $P(-1,2)$ $\vec{v}=(1,3)$.

$\overrightarrow{PQ} = (x+1, y-2) = -(1,3)$
$(x+1, y-2) = (-1,-3)$

$x+1=-1 \Rightarrow x=-2$
$y-2=-3 \Rightarrow y=-1$.

$Q(-2,-1)$.

(b) $P(3,0)$ $\vec{v}=(2,-1)$.

(i) $Q(5,-1)$ (ii) $(1,1)$

Two Vectors $\vec{u}$ and $\vec{v}$ are parallel if and only if $\vec{u}=k\vec{v}$.