

A line given in this form $L(t) = (-5t+1, 3-t)$
can be written in this form:

$$\begin{aligned}(x, y) &= (-5t+1, -t+3) \\ &= (-5t, -t) + (1, 3) \\ &= t(-5, -1) + (1, 3) \\ &= -5t(1, \frac{1}{5}) + (1, 3)\end{aligned}$$

\Rightarrow the slope of the straight line is $\frac{1}{5}$ and passes through the point $(1, 3)$

$$\begin{aligned}y &= \frac{1}{5}x + b \\ 3 &= \frac{1}{5}(1) + b \Rightarrow 3 - \frac{1}{5} = b \Rightarrow b = \frac{14}{5}\end{aligned}$$

$$y = \frac{1}{5}x + \frac{14}{5}$$

Perpendicular lines have their direction vectors to be perpendicular to each other.

$$\text{So the line } (x, y) = r(1, \frac{1}{5}) + (1, 3)$$

is perpendicular to the line
 $(x, y) = t(1, -5) + (5, 1)$ for example.

$$\text{because } (1, \frac{1}{5}) \cdot (1, -5) = 1 + (\frac{1}{5})(-5) = 1 - 1 = 0$$

Determine whether the following lines are parallel, intersecting or identical.

$$(a) r(t) = (-2+t, 5-3t) \text{ and } s(t) = (-3+2t, 3-6t)$$

$$r(t) = (-2, 5) + t(1, -3) \quad s(t) = (-3, 3) + t(2, -6)$$

since the direction vectors are $(1, -3)$ and $(2, -6)$,
then the 2 lines are either \parallel or identical.

If they have at least one common point, then they are identical. $(-2+t_1, 5-3t_1) = (-3+2t_2, 3-6t_2)$.

$$-2+t_1 = -3+2t_2 \Rightarrow t_1 - 2t_2 = -1$$

$$5-3t_1 = 3-6t_2 \Rightarrow -3t_1 + 6t_2 = -2$$

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$$3t_1 - 6t_2 = -3$$

$$\begin{array}{r} -3t_1 + 6t_2 = -2 \\ \hline 0 + 0 = -5 \end{array}$$

Contradiction \Rightarrow The 2 lines are \parallel .

(b) $r(t) = (1+5t, -3+2t)$ and $s(t) = (2+3t, 4+5t)$

$$\vec{d}_1 = (5, 2)$$

$$\vec{d}_2 = (3, 5)$$

$\vec{d}_1 \neq \vec{d}_2$ \vec{d}_1 and \vec{d}_2 are not parallel nor identical

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\Rightarrow They are intersecting

(c) $r(t) = (1+3t, -3+5t)$ $s(t) = (4-6t, 2-10t)$

$$\vec{d}_1 = (3, 5)$$

$$\vec{d}_2 = (-6, -10)$$

$$-2\vec{d}_1 = \vec{d}_2 \Rightarrow \vec{d}_1 \parallel \vec{d}_2$$

The lines are either parallel or identical

Let's see if there are any intersection points

$$1+3t_1 = 4-6t_2 \Rightarrow 3t_1 + 6t_2 = 3 \Rightarrow 15t_1 + 30t_2 = 15$$

$$-3+5t_1 = 2-10t_2 \Rightarrow 5t_1 + 10t_2 = 5 \Rightarrow 15t_1 + 30t_2 = 15$$

Same line \Rightarrow Identical