114 Nov 16, 2020.

A function is a rule that accepts inputs and produces outputs.

$$\underset{\text{input}}{f(x)} = \underset{\text{output}}{x^2}$$

$f(2) = 2^2 = 4$ $\qquad$ $f(-4) = (-4)^2 = 16$.

$$\underset{3\times 2}{\begin{pmatrix} 1 & -2 \\ 3 & 1 \\ 5 & 2 \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} -3 \\ 2 \end{pmatrix}} = \underset{3\times 1}{\begin{pmatrix} -7 \\ -7 \\ -11 \end{pmatrix}}.$$

$$(-3, 2) \longrightarrow (-7, -7, 11).$$

$Ax = b$. is a function that transforms the vector x into the vector b.

independent variable vector

(Both x and b are column matrices).

dependent variable vector.

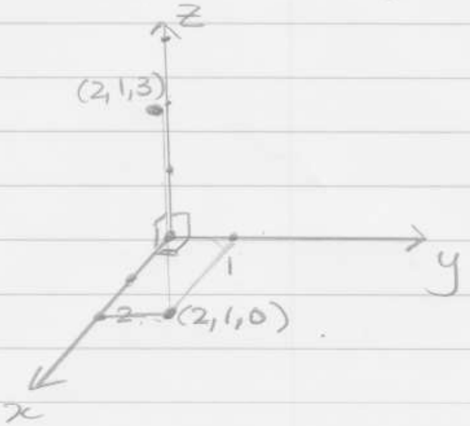
A is called the associated matrix transformation.

<u>Projection Transformation</u>

Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ describe the function $b = Ax$

$Ax = b$. geometrically

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$



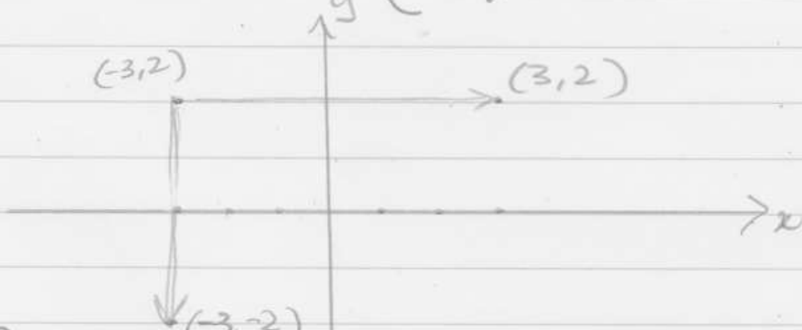
Multiplication by the matrix A projects a vector from xyz plane onto the xy-coordinate system.

Reflection Transformation.

Let $A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ describe the function $Ax = b$. geometrically

$$Ax = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_1 \\ x_2 \end{pmatrix}.$$

If the input is $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$, then the output is $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$



Reflection w.r.t. y-axis has the associate transformation matrix $A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

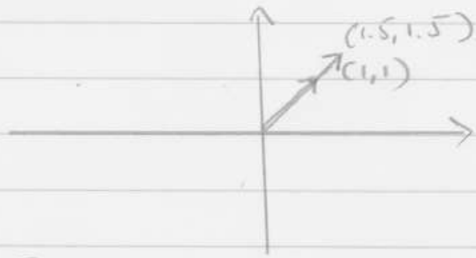
Reflection w.r.t. x-axis has the associate transformation matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

Dilation Transformation.

Let $A = \begin{pmatrix} 1.5 & 0 \\ 0 & 1.5 \end{pmatrix}$ describe the function $Ax = b$ geometrically.

$$Ax = \begin{pmatrix} 1.5 & 0 \\ 0 & 1.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1.5x_1 \\ 1.5x_2 \end{pmatrix}$$

Matrix A dilates the vector (x_1, x_2) into $(1.5x_1, 1.5x_2)$
" " scales " " " " "



Rotation Transformation.

Let $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ describe the function $Ax = b$ geometrically.

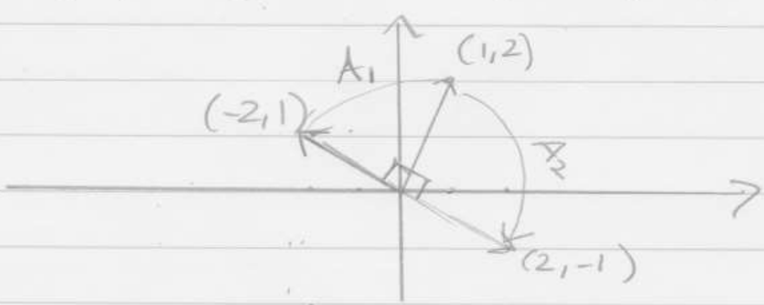
A rotates the vector (x_1, x_2) counter clockwise through the angle θ.

Let $\theta = 90°$.

$$A_1 = \begin{pmatrix} \cos 90° & -\sin 90° \\ \sin 90° & \cos 90° \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$A_1 x = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix}$$

$$A_1\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$



Let $A_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ describe the function $Ax = b$ where A rotates the vector (x_1, x_2) clockwise through an angle of 90°.

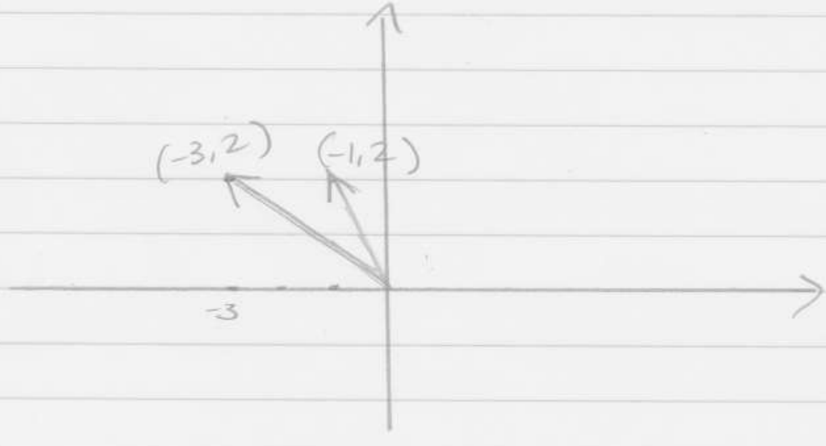
$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Shear Transformation.

Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ describe the function $Ax = b$ geometrically

$$A\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 \end{pmatrix}.$$

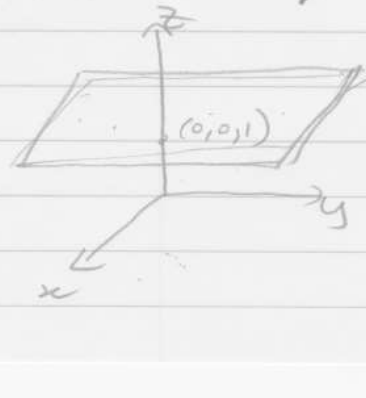
$$A\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$



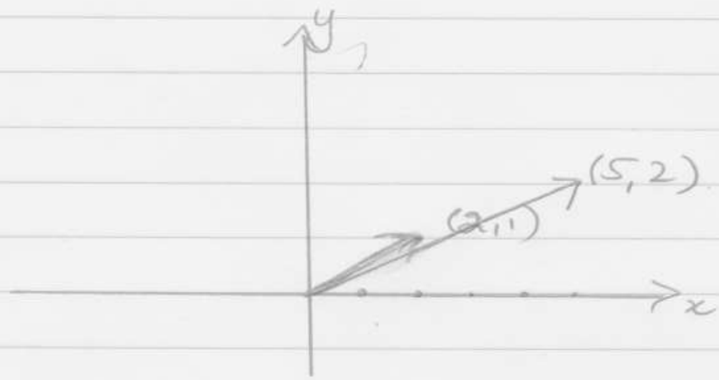
Translation Transformation.

Let $A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ Let all our inputs in our transformation be vectors of the form $(x_1, x_2, 1)$.

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} x_1 + 3 \\ x_2 + 1 \\ 1 \end{pmatrix}$$



$$(x_1, x_2) \longrightarrow (x_1 + 3, x_2 + 1).$$



$(x_1, x_2) = (2, 1)$
$\downarrow$
$(5, 2)$

The website is:
textbooks.math.gatech.edu/ila/matrix-transformations.html