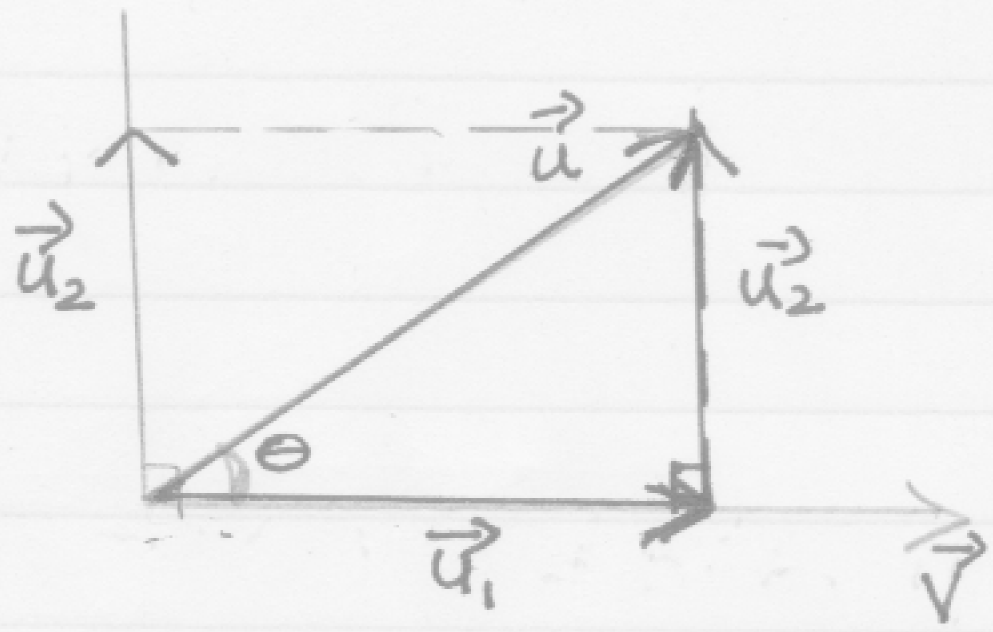
201-114. Oct 19, 2020.

Projections.

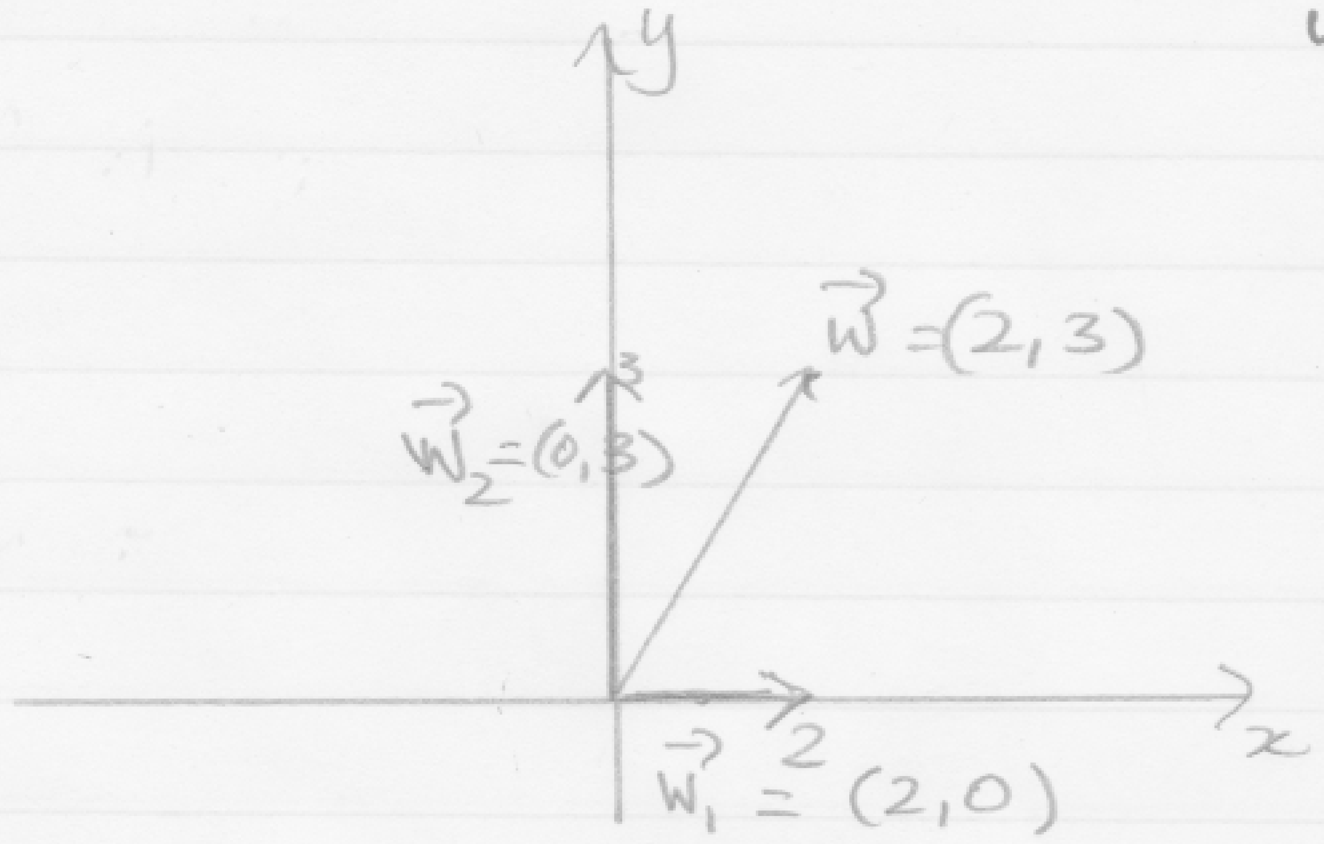


$\vec{u}_1$ is the projection of $\vec{u}$ on $\vec{v}$ (or along $\vec{v}$)

$$\vec{u}_1 = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\,\vec{v}$$

$$\vec{u}_1 + \vec{u}_2 = \vec{u}$$

$$\vec{u}_2 = \vec{u} - \vec{u}_1$$



$\vec{w} = (2,3)$

$\vec{w} = \vec{w}_1 + \vec{w}_2$

$(2,3) = (2,0) + (0,3)$

$\vec{w}_1 \cdot \vec{w}_2 = 0$. $\vec{w}_1 \perp \vec{w}_2$

Example. Let $\vec{u} = (1,-3)$ and $\vec{v} = (5,2)$.

(a) Find the component of $\vec{u}$ along $\vec{v}$ $(\vec{u}_1)$

(b) Find the vector component of $\vec{u}$ orthogonal to $\vec{v}$. $(\vec{u}_2)$

(The vector component of $\vec{u}$ along $\vec{v}$ is also called projection of $\vec{u}$ on $\vec{v}$ $\vec{u}_1 = \text{proj}_{\vec{v}}\vec{u}$).

$$\vec{u}_1 = \text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\,\vec{v}$$

$$\vec{u}\cdot\vec{v} = (1,-3)\cdot(5,2) = (1)(5) + (-3)(2) = -1$$

$$\vec{u}_1 = \frac{-1}{\left(\sqrt{5^2+2^2}\right)^2}(5,2) = \frac{-1}{\left(\sqrt{29}\right)^2}(5,2) = -\frac{1}{29}(5,2).$$

$$= \left(\frac{-5}{29}, \frac{-2}{29}\right)$$

(a) $\boxed{\vec{u}_1 = \text{proj}_{\vec{v}}\vec{u} = \left(\frac{-5}{29}, \frac{-2}{29}\right)}$

(b) $\vec{u} = \vec{u}_1 + \vec{u}_2 \Rightarrow \vec{u}_2 = \vec{u} - \vec{u}_1$.

$\vec{u}(1,-3)$, $\vec{u}_1 = \left(\frac{-5}{29}, \frac{-2}{29}\right)$

$$= (1,-3) - \left(\frac{-5}{29}, \frac{-2}{29}\right).$$

$$= \left(1 + \frac{5}{29}, -3 + \frac{2}{29}\right)$$

$$= \left(\frac{29}{29} + \frac{5}{29}, \frac{-3(29)}{29} + \frac{2}{29}\right)$$

(b) $\boxed{\vec{u}_2 = \left(\frac{34}{29}, \frac{-85}{29}\right)}$