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Standard deviation is a measure of how spread out the numbers are. It is represented by the Greek letter sigma, σ

$$\sigma = \sqrt{\text{variance}}$$

What is variance? It is the average of the sum of the squared differences of the observations from the mean.

That is, if the data is $x_1, x_2, \ldots, x_n$, then the mean is

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum x_i}{n}$$

$$\text{Population Variance} = \sigma^2 = \frac{(\bar{x} - x_1)^2 + (\bar{x} - x_2)^2 + \cdots + (\bar{x} - x_n)^2}{n}$$

$$= \frac{\sum (\bar{x} - x_i)^2}{n}$$

And the population standard deviation is $\sigma = \sqrt{\frac{\sum (\bar{x} - x_i)^2}{n}}$

Now, the sample variance is represented by s^2 and it is calculated as follows:

$$s^2 = \frac{\sum (\bar{x} - x_i)^2}{n-1}$$

and the sample standard deviation is: $s = \sqrt{\frac{\sum (\bar{x} - x_i)^2}{n-1}}$

Example (a) What is the sample standard deviation of the following data: 75, 83, 96, 100, 121, 125?

(b) What is the sample standard deviation of the same data above?

(a) $\bar{x} = \frac{75 + 83 + 96 + 100 + 121 + 125}{6} = \frac{600}{6} = 100.$

$$s^2 = \frac{(100-75)^2 + (100-83)^2 + (100-96)^2 + (100-100)^2 + (100-121)^2 + (100-125)^2}{6-1}$$

$$= \frac{1996}{5} = 399.2 \Rightarrow \text{sample standard deviation} = \sqrt{399.2} \cong 19.98$$

(b) $\sigma^2 = \frac{1996}{6} = 332.666\ldots \Rightarrow$ population standard deviation $= \sqrt{332.66} \cong 18.24$