

Removable Discontinuity at  $x=3$ .  
(Open point is  $(3, 7)$ )

Sec 2.5 #18

$$f(x) = \begin{cases} \frac{1}{x+2} & \text{if } x \neq -2 \\ 1 & \text{if } x = -2 \end{cases}$$

$$a = -2. \quad D_f: \mathbb{R}.$$

At  $x = -2$ .

✓ 1.  $f(-2) = 1$ .

✗  $\lim_{x \rightarrow -2} f(x) = \lim_{x \rightarrow -2} \frac{1}{x+2} = \frac{1}{0} = \text{Infinite limit} \quad \text{DNE} \quad \text{V.A asymptote at } x = -2.$

✗  
Infinite Discontinuity at  $x = -2$ .

Example: Find the numbers at which  $f$  is discontinuous.

$$f(x) = \begin{cases} 2^x & \text{if } x \leq 1 \\ 3-x & \text{if } 1 < x \leq 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$$

$$D_f: \mathbb{R}.$$

At  $x = 1$ .

✓ 1.  $f(1) = 2^1 = 2$

✓ 2.  $\lim_{x \rightarrow 1} f(x) \text{ exists?}$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 2^x = 2^1 = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 3-x = 3-1 = 2$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = 2$$

✓ 3.  $\lim_{x \rightarrow 1} f(x) = f(1)$

$f$  is continuous at  $x = 1$

At  $x = 4$ .

✓ 1.  $f(4) = 3-4 = -1$