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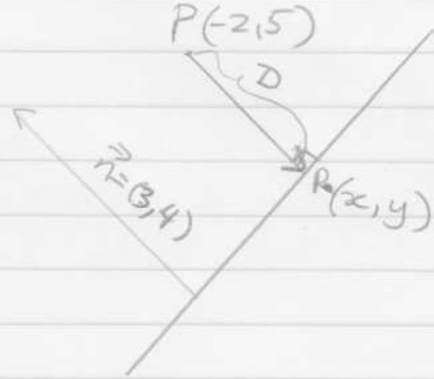
Ex Find the point $R(x,y)$ on the line $3x+4y-8=0$ which is closest to the point $P(-2,5)$

$3x+4y=8$

By Method 1 ~~by~~ using Slopes, we found that $R\left(\frac{-68}{25}, \frac{101}{25}\right)$

By Method 2 (Using Vectors)



The normal vector is $\vec{n}=(3,4)$

$$\vec{PR} = k\vec{n}$$

$$D = \frac{|ax_0+by_0-8|}{\sqrt{a^2+b^2}}$$

$$= \frac{|3(-2)+4(5)-8|}{\sqrt{3^2+4^2}} = \frac{6}{\sqrt{25}} = \frac{6}{5}$$

$$\|\vec{PR}\| = \|k\vec{n}\| = \frac{6}{5} \qquad \|\vec{n}\| = \sqrt{3^2+4^2} = \sqrt{25} = 5$$

$$\|k\vec{n}\| = |k|\,\|\vec{n}\| = \boxed{|k|(5) = \frac{6}{5}} \Rightarrow |k| = \frac{6}{25}$$

$$\Rightarrow k = \pm\frac{6}{25} \qquad \frac{\frac{6}{5}}{5}$$

If $k=\frac{6}{25}$, then $\vec{PR} = \frac{6}{25}(3,4) = \left(\frac{18}{25}, \frac{24}{25}\right)$

$$\vec{PR} = (x,y)-(-2,5) = (x+2, y-5)$$

$$x+2 = \frac{18}{25} \Rightarrow x = \frac{18}{25} - 2 = \frac{18}{25} - \frac{50}{25} = \frac{-32}{25}$$

$$y-5 = \frac{24}{25} \Rightarrow y = \frac{24}{25} + 5 = \frac{24}{25} + \frac{125}{25} = \frac{149}{25}$$

The possible coordinates of R are $\left(-\frac{32}{25}, \frac{149}{25}\right)$

$3x+4y-8 \stackrel{?}{=} 0$

$3\left(-\frac{32}{25}\right)+4\left(\frac{149}{25}\right)-8 \neq 0$

Not satisfied

If $k = -\frac{6}{25}$, then $\overrightarrow{PR} = \frac{-6}{25}(3,4) = \left(\frac{-18}{25}, \frac{-24}{25}\right)$.

$\overrightarrow{PR} = (x+2, y-5)$ //

$x + 2 = -\frac{18}{25} \Rightarrow x = -\frac{18}{25} - 2 = -\frac{18}{25} - \frac{50}{25} = -\frac{68}{25}$

$y - 5 = -\frac{24}{25} \Rightarrow y = -\frac{24}{25} + 5 = -\frac{24}{25} + \frac{125}{25} = \frac{101}{25}$

$R\left(-\frac{68}{25}, \frac{101}{25}\right)$ is the point on the line that is closest to the point $P(-2,5)$ because $3\left(-\frac{68}{25}\right) + 4\left(\frac{101}{25}\right) - 8 \stackrel{?}{=} 0$

$-\frac{204}{25} + \frac{404}{25} - 8 = 0$ ✓.

Matrices.

$$\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$$

	M	T	W	T	F	S	S
Math	1	1	0	2	0	1	0
Java Script	1	0	1	0	2	1	0
English	0	1	1	0	0	0	0
French	0	1	0	1	0	0	0
Humanities	1	0	0	0	1	0	0

$$\begin{array}{c} \\ M \\ JS \\ E \\ F \\ H \end{array} \begin{array}{c} \begin{array}{ccccccc} M & T & W & T & F & S & S \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 2 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \rightarrow 5 \times 7$$

5 rows 7 columns

The size of a matrix is obtained from the number of rows by the number of columns. Each number of the matrix is called an entry (or an element).

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is 2×2 $B = \begin{bmatrix} 1 \\ 0 \\ -5 \\ 2 \end{bmatrix}$ is 4×1

$C = \begin{bmatrix} 2 & 13/5 & 1 & \pi \\ -5 & 3.2 & 5 & 7 \end{bmatrix}$ is 2×4.

In general, a matrix A can be written as follows if it has the size $m \times n$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

a_{13}: 1^{st} row, 3^{rd} column

a_{52}: 5^{th} row, 2^{nd} column.

In general a_{ij}: i^{th} row, j^{th} column.

$1 \leq i \leq m$.
$1 \leq j \leq n$.

Equal Matrices.

2 Matrices are equal to each other if and only if they have the same size and they have the same corresponding entries.

Ex $A = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}_{2 \times 2}$ $B = \begin{bmatrix} 1 & x+2 \\ y & 5 \end{bmatrix}_{2 \times 2}$

$2 = x+2 \Rightarrow x = 0$
$y = -3$

Find the values of x and y so that $A = B$.

$x = 0$ and $y = -3$ are the values that make $A = B$.

Addition. Let A and B be 2 matrices of the same size.

$A = \begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 & -8 \\ 2 & 15 & -3 \end{bmatrix}$

$$\underset{\text{The sum.}}{A+B} = \begin{bmatrix} -2+3 & 1+4 & 4+(-8) \\ 5+2 & 0+15 & -3+(-3) \end{bmatrix} = \begin{bmatrix} 1 & 5 & -4 \\ 7 & 15 & -6 \end{bmatrix}$$

Note: Each vector is a matrix. But not all matrices are vectors.

If A and B are 2 matrices of different sizes, then A+B is not possible.

Ex let $A = \begin{bmatrix} 1 & 2 \\ -5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 7 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 2 & 0 \end{bmatrix}$.

Find $A+B$, $B+C$ and $A+C$ if possible.

$B+C$ and $A+C$ are not possible since the sizes are different. A and B are 2×2 and C is 2×3.

$$A+B = \begin{bmatrix} 1+3 & 2+(-2) \\ -5+7 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2 & 1 \end{bmatrix}$$

In general, $A+B = B+A$ (Commutativity of addition).

Zero Matrix

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2\times1}, \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}_{1\times3}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times3}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2}$$

$O_{m\times n}$ is the notation for the zero matrix.

In general, if A is an $m\times n$ matrix, then $A + O_{m\times n} = A$

The zero matrix is called the additive identity.

Matrix Subtraction.

Let A and B be 2 matrices of the same size. Then A-B is possible

Ex Let $A = \begin{bmatrix} 4 & -2 & 5 \\ 3 & 0 & 1 \\ -6 & 9 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 7 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -3 & 2 \\ 1 & 5 & 8 \\ -3 & 10 & -5 \end{bmatrix}$

Perform A-B, A-C and B-C if possible.

A-B and B-C are not possible since they are of different sizes A is 3x3, B is 2x3, and C is 3x3.

$$A-C = \begin{bmatrix} 4-0 & -2-(-3) & 5-2 \\ 3-1 & 0-5 & 1-8 \\ -6-(-3) & 9-10 & 7-(-5) \end{bmatrix} = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -5 & -7 \\ -3 & -1 & 12 \end{bmatrix}$$

Scalar Multiplication

$A = \begin{bmatrix} -3 & 5 \\ 25 & -1/2 \end{bmatrix}$ $\quad -2A = \begin{bmatrix} -2(-3) & -2(5) \\ -2(25) & -2(-1/2) \end{bmatrix} = \begin{bmatrix} 6 & -10 \\ -50 & 1 \end{bmatrix}$

The negative of a matrix A.

If $A = \begin{bmatrix} -3 & 5 \\ 25 & -1/2 \end{bmatrix}$, then $-A = \begin{bmatrix} 3 & -5 \\ -25 & 1/2 \end{bmatrix}$

Note: $A + (-A) = O_{2\times 2}$

Linear Combinations. Let $A_1, A_2, \dots, A_k$ be all $m \times n$ matrices and let $c_1, c_2, \dots, c_k$ be scalars. Then the expression:

$$c_1 A_1 + c_2 A_2 + \cdots + c_k A_k$$

is a linear combination of $A_1, A_2, \ldots, A_k$ with coefficients $c_1, c_2, \ldots, c_k$ respectively.

Ex Determine whether $C = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$ is a linear combination of $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$.

$$C \stackrel{?}{=} c_1 A + c_2 B.$$

$$c_1 A + c_2 B = c_1 \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & 2c_1 \\ 0 & 3c_1 \end{bmatrix} + \begin{bmatrix} 2c_2 & 0 \\ c_2 & c_2 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 + 2c_2 & 2c_1 + 0 \\ 0 + c_2 & 3c_1 + c_2 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$$

$$\begin{aligned} c_1 + 2c_2 &= 2 \ (*) \\ 2c_1 &= -1 \Rightarrow c_1 = -1/2 \\ c_2 &= 3 \\ 3c_1 + c_2 &= 1 \end{aligned}$$

$$c_1 + 2c_2 = -\frac{1}{2} + 2(3) = \frac{11}{2} \neq 2. \ (*)$$

The system is inconsistent.

$\therefore$ C is not a linear combination of A and B.