Centroiding (center of ~~mass~~ light (but really ADU))

1 pixel: □ No brainer

9 pixels:

	1	2	3
1	3	3	3
2	3	10	3
3	3	3	3

symmetric; center is still @ $(x,y) = (2,2)$

	1	2	3
1	4	6	3
2	7	10	3
3	3	3	3

brightness is shifted to upper left. Where is center?

to get x position: sum all columns, then get a weighted mean

1	2	3
14	19	9

$$\bar{x} = \frac{(14)1 + (19)2 + (9)3}{(14+19+9)} = 1.88$$

to get y position: sum all rows, then get a weighted mean

1	13
2	20
3	9

$$\bar{y} = \frac{(13)1 + (20)2 + (9)3}{(13+20+9)} = 1.90$$

center is @ $(x,y) = (1.88, 1.90)$

$0.88^2 \quad -.12^2 \quad -1.12^2$

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$$\sigma = \sqrt{\frac{14(.88^2) + 19(\overset{.12}{\cancel{.88}}^2) + 9(1.12)^2}{(42-1)}} = .7392$$

$\sigma_{\bar{x}} = 0.114$

$.9^2 \quad .9^2 \quad .9^2$

$.1^2 \quad .1^2 \quad .1^2$

$1.1^2 \quad 1.1^2 \quad 1.1^2$

$$\sigma = \sqrt{\frac{13(.9^2) + 20(.1^2) + 9(1.1)^2}{41}}$$

$\sigma = .726$

$\sigma_{\bar{y}} = 0.112$

$$\boxed{\begin{array}{l}\sigma_{\bar{x}} = 0.114 \\ \sigma_{\bar{y}} = 0.112\end{array}}$$

How certain are we of the center?

Make a new array w/ distance from each pixel to centroid:

e.g. (1,1) will have a distance $\sqrt{(1.88-1)^2+(1.90-1)^2} = 1.26$

1.26	0.91	1.44
0.89	0.16	1.12
1.41	1.11	1.57

Then do a ~~wei~~ mean of all these values, weighted by the brightness values in the original array.

$$S_x = \sigma \cong \frac{[(4)1.26 + 6(0.91) + 3(1.44) + \ldots]}{(4+6+3+\ldots)} = 0.91$$

This is the standard deviation of the population, effectively a measure of how spread out the distribution is. But we're interested in how precisely we know the centroid position. We can get this from the standard deviation of the mean, $\sigma_{\bar{x}}$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \frac{0.91}{\sqrt{(4+6+3+\ldots)}} = \cancel{0.02}\ 0.14$$

Assuming equally distributed errors, each axis would have $0.14/\sqrt{2} = 0.09899$

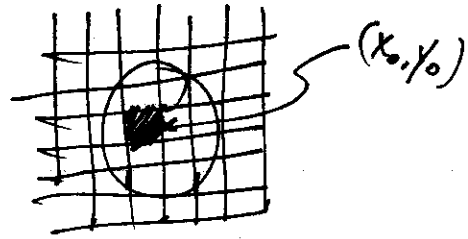
~ 0.1

$\sigma_{\bar{x}}$ & $\sigma_{\bar{y}}$ done separately on previous page

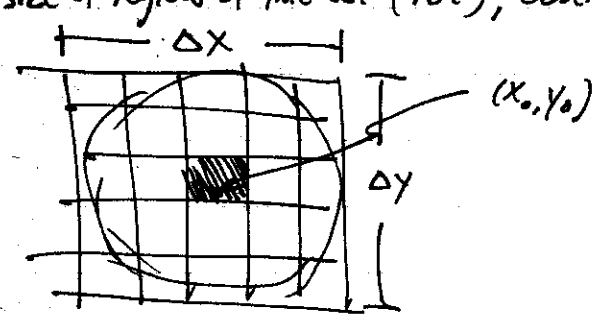
Centroid $x = 1.9 \pm .1$ $y = 1.9 \pm .1$

Centroiding Pseudo-code

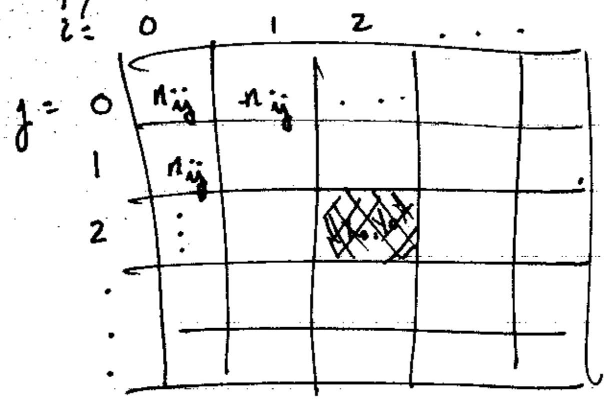
1. determine central pixel by eye (x_0, y_0)



2. determine size of region of interest (roi), centered on (x_0, y_0)



3. Use pyfits to cut out roi



(w/ x-axis denoted by i
y-axis by j
+ pixel ct by n_{ij})

4. to get x-centroid value, sum all columns $\left[x_i = \sum_j n_{ij} \right]$, then do weighted mean of all x_i-values
$\left[\bar{x} = \sum_i i \cdot x_i \Big/ \sum_i x_i \right]$
(pixel ct summed down i^{th} col.)

5. to get y-centroid value, sum all rows $\left[y_j = \sum_i n_{ij} \right]$, then do weighted mean of all y_j-values
$\left[\bar{y} = \sum_j j \cdot y_j \Big/ \sum_j y_j \right]$
(pixel ct summed across j^{th} row)

6. to get $\sigma_{\bar{x}}$, consider distance from i^{th} column to $\bar{x}$ + calculate st. dev. of the mean

$$\sigma_{\bar{x}} = \left[\frac{\sum_i (\bar{x} - i)^2 \cdot x_i}{N(N-1)} \right]^{\frac{1}{2}}$$

where $N = \sum_i x_i = \sum_j y_j$ = total pixel ct. in the roi.
$= \sum_i \sum_j n_{ij}$

7. + similarly

$$\sigma_{\bar{y}} = \left[\sum_j \frac{(\bar{y} - j)^2 \cdot y_j}{N(N-1)} \right]^{\frac{1}{2}}$$

(If you wanted to combine into 1 term, you could say that
$\sigma_{tot} = \sqrt{\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2}$)

8. Remember to add back on original centroid value (x_0, y_0) pixel, as determined in step 1. Assuming x_0, y_0 is @ center of roi, you can shift centroid by

$$\bar{x} = \bar{x} + \left(x_0 - \text{floor}\left(\frac{\Delta x}{2}\right)\right)$$

$$\cancel{\bar{y} = \bar{y} + \left(y_0 - \text{floor}\left(\frac{\Delta y}{2}\right)\right)}$$

$$\bar{y} = \bar{y} + \left(y_0 - \text{floor}\left(\Delta y/2\right)\right)$$