# Transient Dynamical Indicators of Critical Transitions

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**Abstract**

Replace the text here with your abstract.
**Keywords:** tipping point, critical transition, critical slowing down, early warning signals, resilience, intensity of attraction

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# 1 Introduction

A **tipping point** or **critical transition** occurs in a dynamical system when a small perturbation to system conditions causes an abrupt overall shift in qualitative behavior. This informal concept is often understood as a local bifurcation in a dynamical system, but may also correspond to other dynamical behaviors such as a global bifurcation, a perturbation pushing a state variable across the boundary between two basins of attraction, or rate-induced tipping behavior.

Empirically, tipping points have been studied in a wide range of contexts, including Earth's climate [1,2], emerging infectious diseases [3], aquatic and land ecosystems [4,5], the onset of medical health conditions [6,7], and socio-economic systems [8], among others [9,10]. Since critical transitions often represent a shift into an undesirable or catastrophic regime, and since such transitions may not be easily or at all reversible [11–13], it is of utmost interest to try to anticipate them before they occur, in order to improve the odds of prevention. For instance, the ever-growing web of complex physical, ecological, and social pressures exerted by modern day anthropogenic practices on the Earth threaten planetary, human, and animal systems with unprecedented and potentially disastrous forms of change.

In real world systems, the conditions under which a critical transition occurs, and the underlying mechanisms driving the approach to transition can be extremely difficult to characterize; hence, there is particular interest in identifying generic mathematical signals that can warn of impending tipping in a wide variety of systems regardless of specific underlying mechanisms. Such **early warning signals** have been identified in the context of local bifurcations of ODEs, and are based on the phenomenon of **critical slowing down** [10]: roughly, as a stable rest point approaches a critical loss of stability, the resilience of the system gradually decreases. This leads to certain detectable statistical trends over time - in particular, resulting in increasing variance and autocorrelation in the system state.

# 2 Resilience Quantification

## 2.1 Asymptotic Resilience

## 2.2 Width of Basin of Attraction

## 2.3 Reactivity

## 2.4 Intensity of Attraction

# 3 Critical Slowing Down

## 3.1 Local Bifurcation

## 3.2 Critical Slowing Down

## 3.3 Early Warning Signals

# 4 Transient Dynamical Indicators of Critical Transitions

## 4.1 Indicators from Reactivity

## 4.2 Possibility for Indicators from Intensity of Attraction

# 5 Thesis Proposal

## 5.1 Continuity of Intensity of Attraction

## 5.2 Intensity through Critical Transitions

## 5.3 Further Possibilities

# 6 Conclusion

# References

[1] Timothy M. Lenton, Hermann Held, Elmar Kriegler, Jim W. Hall, Wolfgang Lucht, Stefan Rahmstorf, and Hans Joachim Schellnhuber. Tipping elements in the Earth's climate system. *Proceedings of the National Academy of Sciences*, 105(6):1786–1793, February 2008.

[2] Vasilis Dakos, Marten Scheffer, Egbert H. van Nes, Victor Brovkin, Vladimir Petoukhov, and Hermann Held. Slowing down as an early warning signal for abrupt climate change. *Proceedings of the National Academy of Sciences*, 105(38):14308–14312, September 2008.

[3] Tobias S. Brett and Pejman Rohani. Dynamical footprints enable detection of disease emergence. *PLOS Biology*, 18(5):e3000697, May 2020.

[4] Marten Scheffer, Steve Carpenter, Jonathan A. Foley, Carl Folke, and Brian Walker. Catastrophic shifts in ecosystems. *Nature*, 413(6856):591–596, October 2001.

[5] S. R. Carpenter and W. A. Brock. Rising variance: A leading indicator of ecological transition. *Ecology Letters*, 9(3):311–318, 2006.

[6] Patrick E. McSharry, Leonard A. Smith, and Lionel Tarassenko. Prediction of epileptic seizures: Are nonlinear methods relevant? *Nature Medicine*, 9(3):241–242, March 2003.

[7] Jose G. Venegas, Tilo Winkler, Guido Musch, Marcos F. Vidal Melo, Dominick Layfield, Nora Tgavalekos, Alan J. Fischman, Ronald J. Callahan, Giacomo Bellani, and R. Scott Harris. Self-organized patchiness in asthma as a prelude to catastrophic shifts. *Nature*, 434(7034):777–782, April 2005.

[8] Kees C. H. van Ginkel, W. J. Wouter Botzen, Marjolijn Haasnoot, Gabriel Bachner, Karl W. Steininger, Jochen Hinkel, Paul Watkiss, Esther Boere, Ad Jeuken, Elisa Sainz de Murieta, and Francesco Bosello. Climate change induced socio-economic tipping points: Review and stakeholder consultation for policy relevant research. *Environmental Research Letters*, 15(2):023001, January 2020.

[9] Sandip V. George, Sneha Kachhara, and G. Ambika. Early warning signals for critical transitions in complex systems. *arXiv:2107.01210 [nlin, physics:physics]*, July 2021.

[10] Marten Scheffer, Jordi Bascompte, William A. Brock, Victor Brovkin, Stephen R. Carpenter, Vasilis Dakos, Hermann Held, Egbert H. van Nes, Max Rietkerk, and George Sugihara. Early-warning signals for critical transitions. *Nature*, 461(7260):53–59, September 2009.

[11] Katharina Albrich, Werner Rammer, and Rupert Seidl. Climate change causes critical transitions and irreversible alterations of mountain forests. *Global Change Biology*, 26(7):4013–4027, 2020.

[12] Xingru Chen and Feng Fu. Imperfect vaccine and hysteresis. *Proceedings of the Royal Society B: Biological Sciences*, 286(1894):20182406, January 2019.

[13] Valerio Lucarini, Klaus Fraedrich, and Frank Lunkeit. Thermodynamic analysis of snowball Earth hysteresis experiment: Efficiency, entropy production and irreversibility. *Quarterly Journal of the Royal Meteorological Society*, 136(646):2–11, 2010.