Transient Dynamical Indicators of Critical Transitions

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Abstract

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Keywords: tipping point, critical transition, critical slowing down, early warning signals, resilience, intensity of attraction

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1 Introduction

A **tipping point** or **critical transition** occurs in a dynamical system when a small perturbation to system conditions causes an abrupt overall shift in qualitative behavior. Empirically, tipping points have been studied in contexts as diverse as Earth's climate [1, 2], emerging infectious diseases [3], aquatic and land ecosystems [4, 5], the onset of medical health states [6, 7], socio-economic systems [8], and more [9, 10]. Since critical transitions often represent a shift into an undesirable or catastrophic regime, and since such transitions may not be easily or at all reversible [11–13], it is of pressing interest to anticipate them before they occur, in order to inform management strategies and improve the odds of prevention. Unfortunately, in complex real world systems, the conditions under which a critical transition occurs, and the underlying mechanisms driving the approach to transition are usually extremely difficult to characterize.

As a result, there is particular interest in generic mathematical signals that can warn of impending tipping in a wide variety of systems without reference to specific underlying mechanisms. Such **early warning signals** have been most commonly studied as precursors of local codimension-1 bifurcations of ODEs, where they are based on the phenomenon of **critical slowing down** [10]. Roughly speaking, as the bifurcation parameter gradually nears its critical value, the resilience of the system drops (becoming slower to recover from perturbations), and this produces certain detectable statistical trends over time. In the context of critical slowing down, the term resilience refers specifically to what is known in the ecology literature as **asymptotic resilience**. In section 2, we review asymptotic resilience, and also other quantitative definitions of resilience. In section 3, we summarize the theory of critical slowing down.

Early warning signals derived from critical slowing down are a powerful tool for anticipating critical transitions, , and their usefulness has already been demonstrated in numerous empirical contexts, including . citations But a major limitation is their assumption that the system experiences only small, infrequent perturbations that do not drive the system state very far from equilibrium and that leave sufficient time for recovery in between disturbances. In particular, there is a neglect of transient behavior within the domain of attraction but potentially far from the equilibrium. Such transient states can arise from large, closely repeated, or continual disturbances, as are common in real world ecological systems.

In section 4

2 Resilience Quantification

In this section, we review some different approaches to quantifying resilience. First, we discuss the classic and most commonly used definition of resilience in theoretical ecology, also known as asymptotic resilience. Asymptotic resilience is defined to be the dominant eigenvalue of the Jacobian matrix at a stable equilibrium, and represents the long-term rate of return to that equilibrium after a small isolated perturbation away from it. Second,

Finally, we discuss intensity of attraction, a quantification of resilience defined by Katherine Meyer in her PhD thesis [14].

2.1 Asymptotic Resilience

the dominant eigenvalue of the Jacobian at the stable rest point. The real part of this eigenvalue approximately governs the rate of return to equilibrium after a small perturbation to the system. Because local bifurcation is characterized by the real part of the dominant eigenvalue passing through zero, the system recovers slower when nearer to bifurcation. This definition of resilience (dominant eigenvalue of the Jacobian) is also

2.2 Width of Basin of Attraction

2.3 Reactivity

2.4 Intensity of Attraction

3 Critical Slowing Down

this results in certain detectable statistical trends over time – in particular, gradually increasing variance and auto-correlation in the system state

3.1 Local Bifurcation

3.2 Critical Slowing Down

3.3 Early Warning Signals

3.4 Limitations

Early warning signals derived from critical slowing down are a powerful tool for anticipating critical transitions, and their usefulness has already been demonstrated in numerous empirical contexts, including . citations However, they have at least a few significant limitations. First, being based on a linear approximation at a stable equilibrium, they are relevant only to small perturbations that do not move the system state very far from equilibrium. Second, being a measure of long term rates of return to equilibrium, they (1) may overlook short term behavior that occurs immediately after the perturbation and (2) are relevant only to infrequent perturbations, so that the system has enough time to recover in between disturbances. In particular, they are not reliable in cases of closely repeating or continual disturbances, as are common in real world ecological systems. Third, they specifically precede local bifurcations, while the informal tipping point concept may correspond to other dynamical behaviors such as global bifurcations, perturbations pushing a state variable across the boundary between two basins of attraction, or rate-induced tipping behavior.

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4.1 Indicators from Reactivity

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Machine learning based early warning signals.

Possible connection between machine-learning based and analytical theory based early warning signals? i.e. using theory to inform ML design.

Mention papers where critical transitions occur with no lead warning

As pressures exerted by modern day anthropogenic practices on the Earth grow in magnitude and complexity, threatening physical, ecological, and social systems on all scales with unprecedented forms of change, this goal becomes even more pressing.

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