Particle dynamics

Syllabus

Equation of motion of uncharged and charged particles, charged particles in constant and alternating electric field, charged particles in a magnetic field - cyclotron, magnetic focusing, charge particles in combined electric and magnetic field

Equation of motion of an uncharged particle:

According to Newton's second law of motion Force is equal to rate of change of linear momentum

i.e $\vec{F} = \frac{d\vec{p}}{dt}$

$= \frac{d(m\vec{v})}{dt}$

$= m\frac{d\vec{v}}{dt}$ $\quad [\because m = \text{constant}]$

$= m\frac{d^2\vec{r}}{dt^2}$ $\quad \left[\because \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}\right]$

$\vec{F} = m\vec{a}$ $\quad \left[\because \vec{a} = \frac{d^2\vec{r}}{dt^2}\right]$

we know,

$$\vec{a} = \frac{d^2\vec{r}}{dt^2}$$

Integrating w.r.t time(t) we get,

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$$\frac{d\vec{r}}{dt} = \vec{a}t + c_1 \quad \text{--- (1)}$$

where, c_1 = integration constant

If $t=0$, then

$$\left.\frac{d\vec{r}}{dt}\right|_{at\ t=0} = c_1 = \vec{u}.$$ which is initial velocity of the particle so ~~eqn~~ equation (1) becomes

$$\frac{d\vec{r}}{dt} = \vec{a}t + c_1 = \vec{a}t + \vec{u} \quad \text{--- (2)}$$

$$\Rightarrow \vec{v} = \vec{a}t + \vec{u} \qquad \left[\because \frac{d\vec{r}}{dt} = \vec{v} = \text{final velocity}\right]$$

$$\therefore \boxed{\vec{v} = \vec{u} + \vec{a}t} \quad \text{--- (3)}$$

Again, integrating equation (2) w.r.t time we get position of particle at instant t and given by

$$\vec{r} = \vec{u}t + \frac{\vec{a}t^2}{2} + c_2 \quad \text{--- (4)} \qquad \left[\because \int \frac{d\vec{r}}{dt} dt = \vec{r}\right]$$

Here, c_2 is constant, At $t=0$, $\vec{r}_0 = c_2$ = initial position of particle.
so equation (3) becomes

$$\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2 + \vec{r}_0$$

$$\text{or, } \vec{r} - \vec{r}_0 = \vec{u}t + \frac{1}{2}\vec{a}t^2 \quad \text{--- (5)}$$

$$\therefore \boxed{\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2} \quad \text{--- (6)} \qquad [\because \vec{s} = \vec{r} - \vec{r}_0$$ ~~= displacement~~ = displacement vector of particle at instant (t)]

Again, we have,

$$\vec{a} = \frac{d\vec{v}}{dt}$$

taking its dot product with the velocity $(\vec{v})$ of the particle at instant (t) then

$$\vec{v}\cdot\vec{a} = \vec{v}\cdot\frac{d\vec{v}}{dt}$$

$$\frac{d\vec{r}}{dt}\cdot\vec{a} = \vec{v}\cdot\frac{d\vec{v}}{dt}$$

Integrating w.r.t time (t) we get

$$\int\frac{d\vec{r}}{dt}\cdot\vec{a}\,dt = \int\vec{v}\cdot\frac{d\vec{v}}{dt}\,dt$$

$$\Rightarrow \vec{r}\cdot\vec{a} = \frac{1}{2}v^2 + c_3 \quad \text{— (7)}$$

at $t=0$, from equation (3)

$$\vec{v} = \vec{u}$$

from equation (5)

$$\vec{r} = \vec{r_0}$$

then, $\vec{a}\cdot\vec{r_0} = \frac{1}{2}u^2 + c_3$

$$c_3 = \vec{a}\cdot\vec{r_0} - \frac{1}{2}u^2$$

Substituting the value of c_3 in equation (7) we get,

$$\vec{r}\cdot\vec{a} = \frac{1}{2}v^2 + \vec{a}\cdot\vec{r_0} - \frac{1}{2}u^2$$

$$\vec{a}\cdot(\vec{r}-\vec{r_0}) = \frac{1}{2}(v^2 - u^2)$$

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or, $\vec{a}\cdot\vec{s} = \frac{1}{2}(v^2-u^2)$ $[\because \vec{r}-\vec{r_0} = \vec{s}]$

or, $2\vec{a}\cdot\vec{s} = v^2-u^2$

or, $v^2 = u^2 + 2\vec{a}\cdot\vec{s}$

$\therefore \boxed{v^2 = u^2 + 2as}$ — (8)

Here,

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$v^2 = u^2 + 2as$$

are the required equations of motion of an uncharged particle

~~Equation of motion of charged particles~~

Equations of motion for a body falling under the action of gravity

$$\vec{v} = \vec{u} + \vec{g}t$$

$$\vec{h} = \vec{u}t + \frac{1}{2}\vec{g}t^2$$

$$v^2 = u^2 + 2gh$$

Equation of motion of charged particle

A. charged particle in a uniform and constant electric field

Let us consider a charged particle of mass m carrying charge q is placed in a uniform and constant electric field $\vec{E}$. The force acting on it will be $q\vec{E}$.

From Newton's second law of motion

$$\vec{F} = m\vec{a}$$

$$q\vec{E} = m\frac{d^2\vec{r}}{dt^2} \qquad \left(\because \vec{a} = \frac{d^2\vec{r}}{dt^2}\right)$$

$$\frac{d^2\vec{r}}{dt^2} = \frac{q}{m}\vec{E} \qquad \text{--- (1)}$$

Integrating equation (1) w.r.t time(t), we get

$$\frac{d\vec{r}}{dt} = \frac{q}{m}\vec{E}t + c_1 \qquad \text{--- (2)}$$

At $t=0$

$$\left(\frac{d\vec{r}}{dt}\right)_{t=0} = \vec{u}_0 = c_1$$

Equation (2) becomes

$$\frac{d\vec{r}}{dt} = \frac{q\vec{E}}{m}t + \vec{u} \qquad \text{--- (3)}$$

Again integrating equation (3) w.r.t time(t) we get

$$\vec{r} = \frac{1}{2}\frac{q\vec{E}}{m}t^2 + \vec{u}t + C_2 \quad \text{--- (4)}$$

At $t=0$

$\vec{r} = \vec{r_0}$, $C_2 = \vec{r_0}$

So eqn (4) becomes

$$\vec{r} = \frac{1}{2}\frac{q\vec{E}}{m}t^2 + \vec{u}t + \vec{r_0} \quad \text{--- (5)}$$

$$\vec{r} = \frac{1}{2}\vec{a}t^2 + \vec{u}t + \vec{r_0} \quad \text{--- (5)}$$

The vector diagram shown in fig (a),

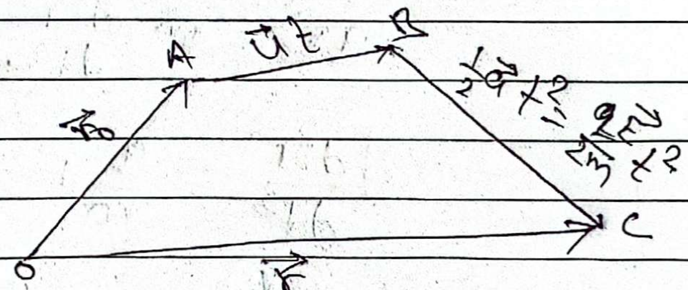

fig (a)

$\vec{OA} = \vec{r_0}$ = initial position vector at $t=0$

$\vec{AB} = \vec{u}t$ = distance covered in time t due to initial velocity $\vec{u}$

$\vec{BC} = \frac{1}{2}\vec{a}t^2 = \frac{q\vec{E}}{2m}t^2$ = the distance covered on account of the acceleration acquired due to field $\vec{E}$

$\vec{r}$ = resultant displacement

Two particular cases

a) The displacement of charged particle when the electric field acts along the direction of motion of the particle.

→ In this case both $\vec{u}$ and $\vec{r_0}$ being directed along the electric field $\vec{E}$, the acceleration produced in the particle will also be directed along $\vec{E}$. The electric field in this case called longitudinal field and the acceleration acquired by the particle is the longitudinal acceleration. Thus, whole problem becomes one dimensional.

If $\vec{r_0} = 0$ and $\vec{u} = 0$ i.e. the particle is initially at the origin and starts from rest, then from eqn (5), we get

$$\boxed{\vec{r} = \frac{q\vec{E}}{2m} t^2} \quad \text{--- (6)}$$

Eqn (6) is exactly in the manner that the distance covered by an uncharged particle, starting from rest, is given by $s = \frac{1}{2} at^2$

b) → The displacement of charged particle when electric field acts in the direction perpendicular to the motion of the charged particle.

When the electric field is perpendicular to the initial velocity of the particle then the electric field is said to be transverse and acceleration acquired by the particle is known as transverse acceleration.

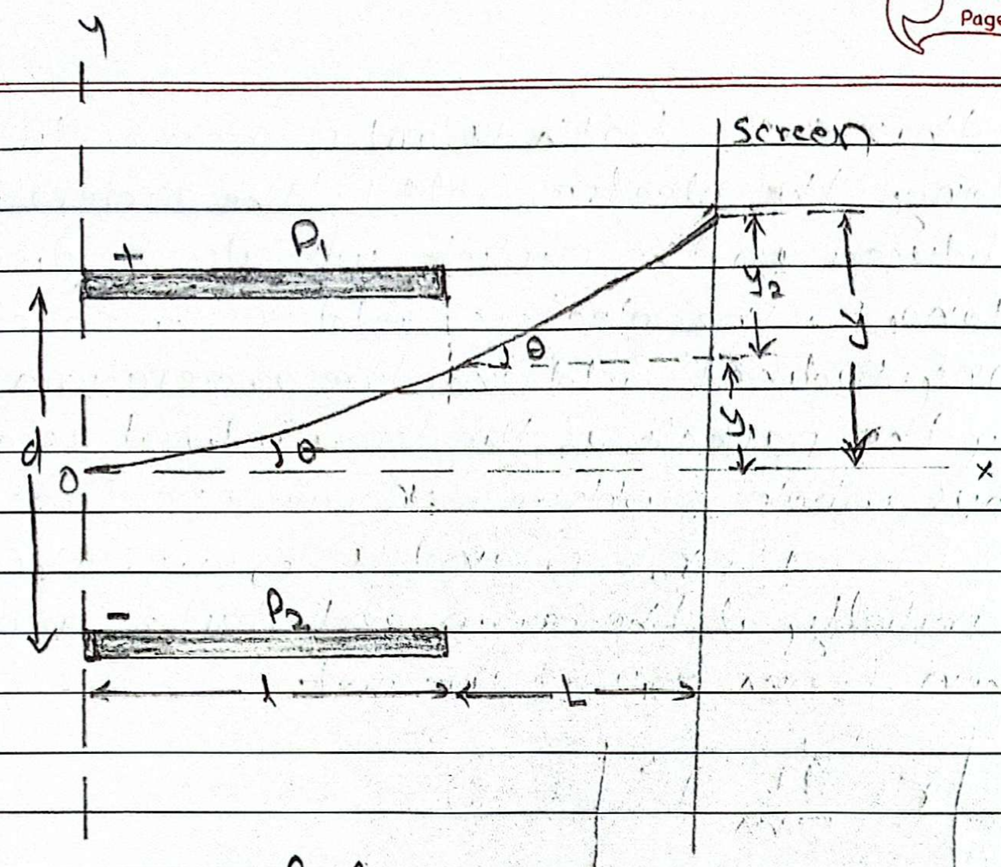

Fig. (b)

Suppose a beam of charged particles each of mass m and carrying a charge q to be travelling along the x-axis with velocity v. and passing in between two metallic plates P_1 and P_2 maintained at a constant potential difference V with upper plate positive w.r.t the lower plate.

The electric field is setup between the plates is given by

$E_y = \frac{V}{d}$ is directed from the upper to the lower plate.

Suppose the charged particle enters with velocity v_x at the origin O within transverse electric field E_y, the velocity along x-axis remains constant and only changes along y-axis

Now, time taken by the charged particle to cover the length of plate (l) is

$$t = \frac{l}{v_x}$$

The transverse acceleration gained by particle is

$$a_y = \frac{qE_y}{m}$$

So, the transverse velocity at the end of the plate leaving is

$$v_y = u_y + a_y t$$

$$v_y = 0 + \frac{qE_y}{m} \cdot \frac{l}{v_x}$$

$$v_y = \frac{qE_y l}{m v_x} \quad \text{--- (1)}$$

The transverse displacement of the beam during the time $(t) = \frac{l}{v_x}$ that while it leaves the plate is

$$y_1 = u_y + \frac{1}{2} a_y t^2$$

$$= 0 + \frac{qE_y}{2m} \left(\frac{l}{v_x}\right)^2$$

$$y_1 = \frac{qE_y l^2}{2m v_x^2} \quad \text{--- (2)}$$

This is clearly the equation to a parabola ($\frac{qE_y}{2mv_x^2}$ being a constant quantity). So that the beam of charged particles takes a parabolic path in between the two plates strikes the screen at point Q making an angle 'θ' with the x-axis as shown in figure.

Total vertical displacement

$$y = y_1 + y_2 \quad \text{--- (3)}$$

from geometry,

$$y_2 = L \tan\theta$$

where,

$$\tan\theta = \frac{v_y}{v_x} = \frac{qE_y l / m v_x}{v_x}$$

$$\tan\theta = \frac{qE_y l}{m v_x^2}$$

So,

$$y_2 = L \frac{qE_y l}{m v_x^2} \quad \text{--- (4)}$$

From equation (2), (3) and (4)

$$y = y_1 + y_2$$

$$= \frac{qE_y l^2}{2 m v_x^2} + \frac{L qE_y l}{m v_x^2}$$

$$y = \frac{qE_y l}{m v_x^2}\left[L + \frac{l}{2}\right] \quad \text{--- (5)}$$

Since, $\frac{qE_y l}{m v_x^2} = \tan\theta$ and $\left(L + \frac{l}{2}\right)$ = distance D of the screen from the centre of the electric field between the plates, we have

$$y = D \tan\theta$$

This is the principle underlying the cathode ray oscillograph, television picture tubes etc

Charged Particle in an Alternating Electric Field

Suppose we have a particle of mass m and carrying a charge q placed in an alternating electric field whose intensity E at any instant t is given by $E = E_0 \sin\omega t$

where,

E_0 = maximum or the peak value of E

Time period $= \frac{2\pi}{\omega}$

If $\frac{d^2r}{dt^2}$ is the acceleration produced in the particle due to this field, then we have.

$$m\frac{d^2r}{dt^2} = qE$$

$$m\frac{d^2r}{dt^2} = qE_0 \sin\omega t$$

$$\frac{d^2r}{dt^2} = \frac{qE_0}{m} \sin\omega t \quad \text{--- (1)}$$

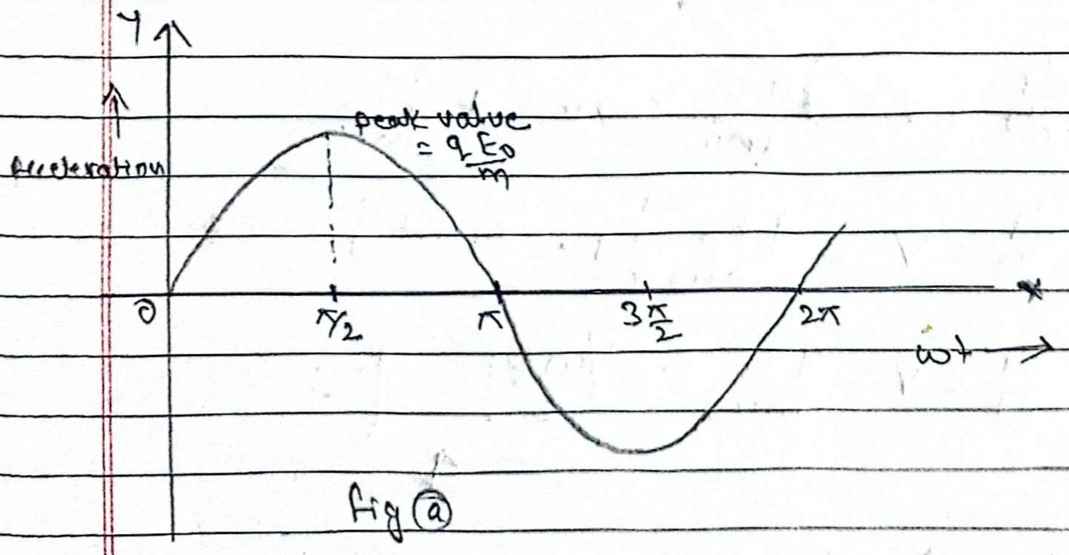

Fig (a)

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eqⁿ ① indicates that the acceleration of the particle, like the applied electric field varies sinusoidally with time as shown in figure.

Integrating eqⁿ ① w.r.t time, we have

$$\frac{dr}{dt} = v = -\frac{qE_0}{m\omega}\cos\omega t + C_1 \quad \text{— ②}$$

where C_1 is integration constant to be determined from the initial or boundary conditions.

Initially i.e. $t=0$, electric field $(E)=0$ the initially velocity of the particle too is zero and eqⁿ ② becomes

$$0 = -\frac{qE_0}{m\omega} + C_1$$

$$\Rightarrow C_1 = \frac{qE_0}{m\omega}$$

So eqⁿ ② becomes

$$\frac{dr}{dt} = v = -\frac{qE_0}{m\omega}\cos\omega t + \frac{qE_0}{m\omega}$$

$$\text{or} \quad \frac{dr}{dt} = v = \frac{qE_0}{m\omega}\left[1-\cos\omega t\right] \quad \text{— ③}$$

$$\text{or} \quad \frac{dr}{dt} = v = \frac{qE_0}{m\omega}\sin^2\left[\frac{\omega t}{2}\right] \quad \left[\because \cos 2\theta = 1 - 2\sin^2\theta\right]$$

— ④

eqⁿ ④ indicates that velocity of the particle at any given instant is always positive, in the direction of E_0 and hence unidirectional though it varies after the manner of $\sin^2$-curve as shown in figure (b) and has an amplitude or peak value $\frac{2qE_0}{m\omega}$

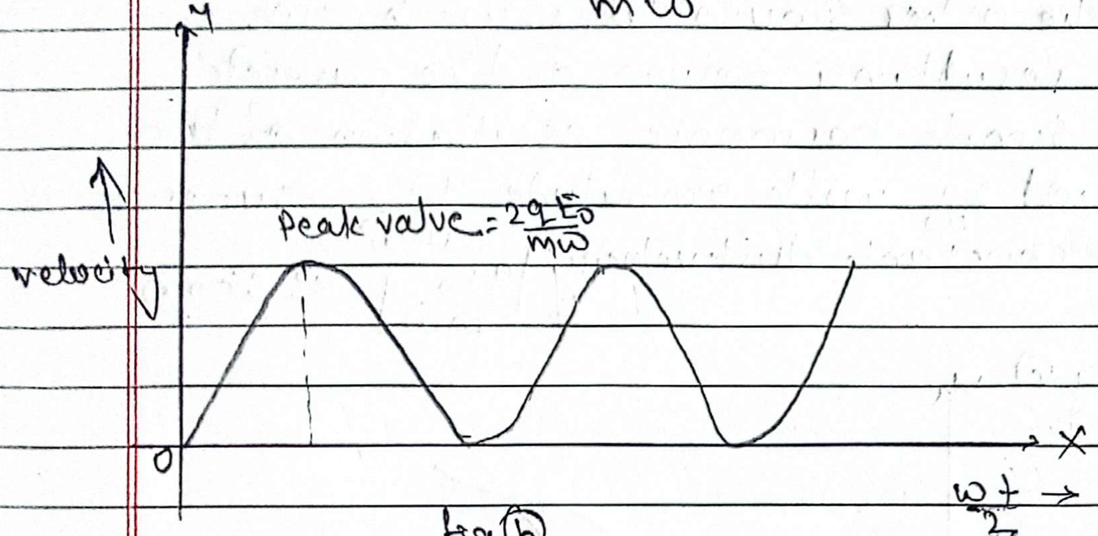

fig. (b)

Again, integrating eqn (3) w.r.t time, we get

$$r = \frac{qE_0}{m\omega}t - \left(\frac{qE_0}{m\omega^2}\right)\sin\omega t + C_2$$

where C_2 is another constant of integration

At $t = 0$, $r = r_0$ = initial position vector

$\therefore$ $C_2 = r_0$

So,

$$r = \left(\frac{qE_0}{m\omega}\right)t - \left(\frac{qE_0}{m\omega^2}\right)\sin\omega t + r_0$$

where, r_0 may be zero or have a finite value, depending upon the choice of the origin.

if $r_0 = 0$ then,

$$r = \left(\frac{qE_0}{m\omega}\right)t - \left(\frac{qE_0}{m\omega^2}\right)\sin\omega t \quad \text{--- (5)}$$

eqⁿ (5) represents displacement vector (r) which is the sum of two vectors, with the value of one varying linearly, and that of the other sinusoidally, with time so the resulting motion of the particle is a simple harmonic oscillation of time period $\frac{2\pi}{\omega}$ with amplitude $\frac{qE_0}{m\omega^2}$, superimposed on a constant drift velocity $\left(\frac{qE_0}{m\omega}\right)$ as shown in fig. (C)

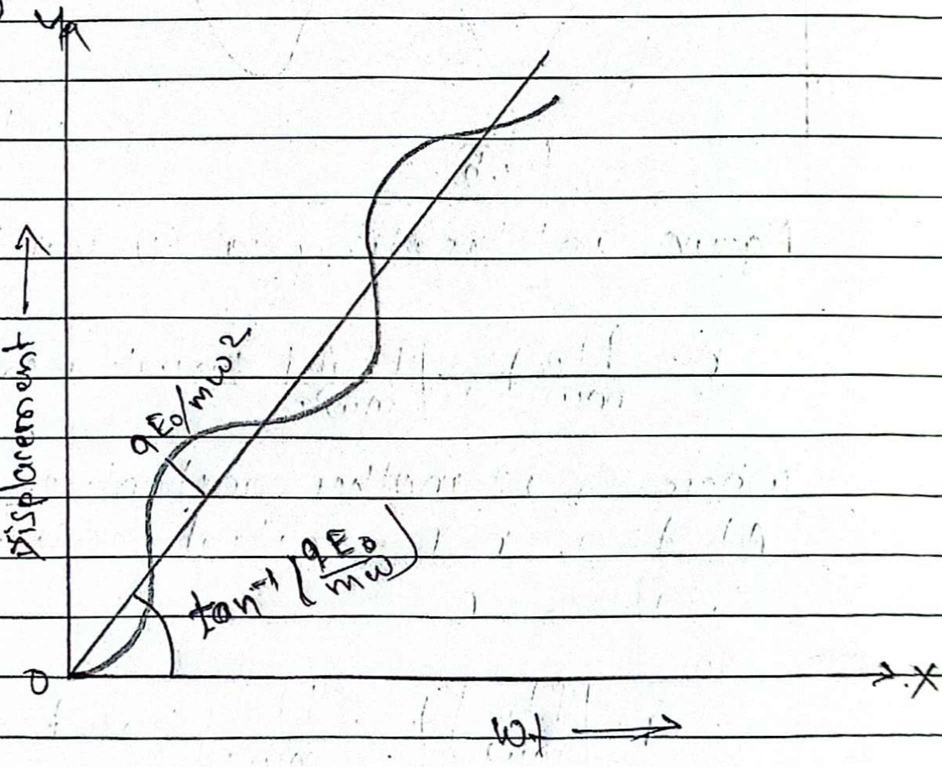

fig. (C)

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Charged particle in a uniform and constant magnetic field

Suppose a charged ~~particle~~ particle carrying a charge q moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$. Then the Lorentz force acted upon the particle due to magnetic field is

Lorentz force $(\vec{F}) = q(\vec{v} \times \vec{B})$

If m be the mass of the particle and $\frac{d^2\vec{r}}{dt^2}$ be its acceleration then,

$$\vec{F} = m\vec{a} = q(\vec{v} \times \vec{B})$$

$$m\frac{d^2\vec{r}}{dt^2} = q(\vec{v} \times \vec{B})$$

If velocity is perpendicular to magnetic field $(\vec{B})$ then Lorentz force is perpendicular to the velocity. Magnitude of Lorentz force remains same in this case and direction of force changes continuously so resulting that the particle is moving in a circular path and force is directed towards the centre.

This Lorentz force is balanced by centrifugal force acting on the particle outwards so,

$$\frac{mv^2}{r} = Bqv\sin 90°$$

$$\boxed{r = \frac{mv}{Bq}}$$

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If the velocity ($\vec{v}$) of the particle is not normal to the magnetic field. Then the velocity can be resolved into two components. One is parallel (v_1) to the magnetic field and another is v_2 perpendicular to the magnetic field.

v_1 makes the particle move in the direction of the field and v_2 makes the particle to move in the circular path of radius

$$r = \frac{m v_2}{qB}$$

The combined effect of the circular and the linear horizontal motion is said to be a helical motion. as shown in figure (b)

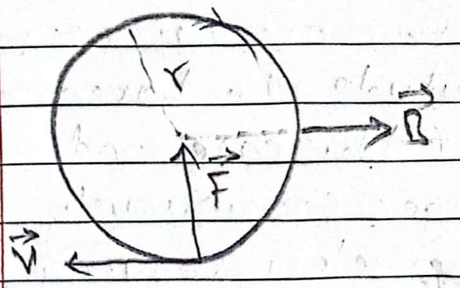


fig. (a) for $\vec{v}$ perpendicular to $\vec{B}$ (circular path)

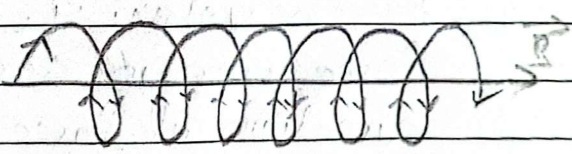


fig. (b) for $\vec{v}$ not perpendicular to $\vec{B}$ (Helical motion)

The radius r of the circular or the helical path, described by a charged particle in a uniform magnetic field is sometimes called gyro radius or cyclotron radius.

If ω be the angular velocity of the particle in circular or helical motion in the magnetic field, we have.

$$\omega = \frac{v}{r} = \frac{Bqr}{mr}$$

$$\therefore \omega = \frac{Bq}{m} \quad \left[\because r = \frac{mv}{Bq}\right]$$

$$\text{Time period } (T) = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{Bq}{m}}$$

$$\therefore T = \frac{2\pi m}{Bq}$$

~~Again,~~

The distance through which the particle advances forward as it completes one full rotation (in case of helical motion) or the linear distance it covers in one full time-period T is called the pitch of the helix.

$$\text{Pitch of the helix} = v_1 T$$

$$= v_1 \cdot \frac{2\pi m}{Bq}$$

$$= \frac{2\pi m}{Bq} v_1$$

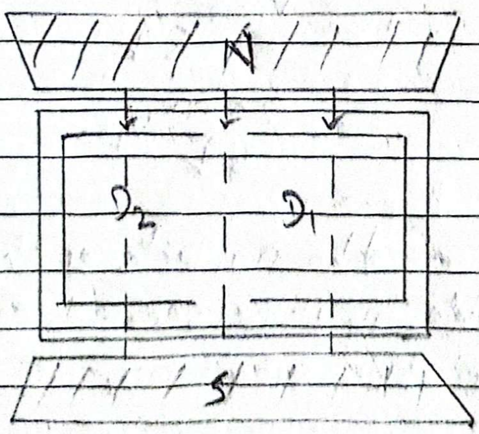
The frequency of the particle or the number of rotations made by it per second, called gyro frequency or cyclotron frequency.

$$\text{frequency } (f) = \frac{1}{T} = \frac{Bq}{2\pi m}$$

The gyro frequency or cyclotron frequency of a particle is quite independent of the velocity.

The Cyclotron

This is to obtain high energy charged particles like protons, deutrons, α-particles. The cyclotron consists of 2 hollow semi circular metal boxes D_1 and D_2 called dees. A source of ions is located near the midpoint of the gap between the dees. Dees are insulated from each other and are enclosed in vacuum chamber. Dees are connected to a powerful radiofrequency oscillator. The whole apparatus is placed between the pole-pieces of a strong electromagnet. The magnetic field is perpendicular to the plane of the dees



fig(3)

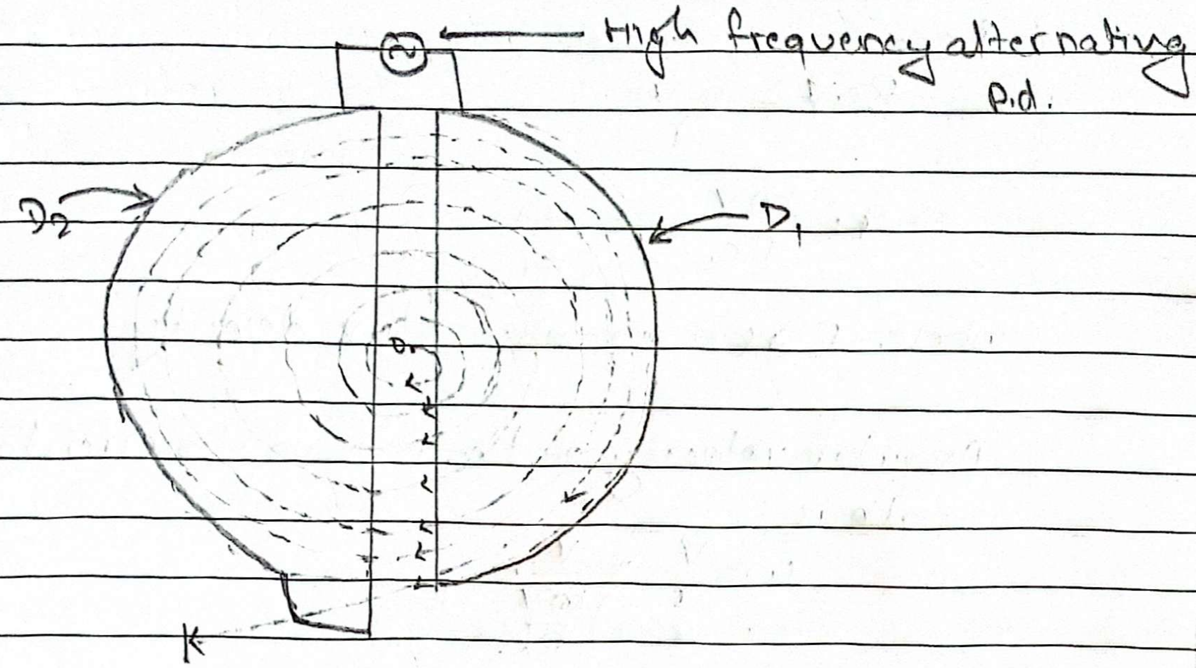


fig (b)

Theory

Suppose a +ve ion leaves the ion source at the centre of the chamber at the instant when D_1 and D_2 dees are at the maximum -ve and +ve A.C. potentials respectively. The +ve ion will be accelerated towards the -ve dee D_1 before entering it. The ion enter the space inside the dee with a velocity v given by

$$eV = \frac{1}{2}mv^2$$

where,

V is the applied voltage, e and m are charge and mass of the ion respectively. When the ion is inside the dee it is not accelerated since the space is field free. Inside the dee under the action of applied magnetic field, the ions travel in a circular path of radius r given by

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$$Bev = \frac{mv^2}{r}$$

$$\Rightarrow r = \frac{mv}{Be} \quad \text{---} (1)$$

where, B is magnetic flux density

Angular velocity of the ion in its circular path is

$$\omega = \frac{v}{r} = \frac{v}{\frac{mv}{Be}}$$

$$\therefore \omega = \frac{Be}{m} \quad \text{---} (2)$$

Again, the time taken by the ion to travel the semi circular path is

$$t = \frac{\pi}{\omega} = \frac{\pi}{\frac{Be}{m}}$$

$$\therefore t = \frac{\pi m}{Be} \quad \text{---} (3)$$

The strength of the field (B) or the frequency of the oscillator (f) is so adjusted that by the time the ion has described a semicircular path and just enters the space between D_1 and D_2 then D_2 becomes -ve. w.r.t D_1. The ion is then accelerated towards D_2 and enters the space inside it with a greater velocity. Since the ion is now moving with greater velocity, it will describe a semicircle of greater radius in the 2^{nd} dee.

Here,

$$t = \frac{\pi m}{Be}$$

It is clear that the time taken by the ion to describe a semicircle is independent of both the radius of the path (r) and the velocity of the ion (v). Hence the ion describes all semi-circles in exactly the same time. This process continues until the ion reaches the periphery of the dees. The ion thus spirals round in circles of increasing radius and acquires high energy. The ion will finally come out of the dees in the direction indicated through the window. The particles are ejected out of the cylotron not continuously but as pulsed streams.

Energy of an ion

Let r_{max} be the radius of the outermost orbit described by the ion and V_{max} be the maximum velocity gained by the ion in its final orbit.

Then,

The equation of motion for the motion of the ion in a magnetic field

$$BeV_{max} = \frac{mV_{max}^2}{r_{max}}$$

$$\Rightarrow V_{max} = \frac{Ber_{max}}{m} \quad (4)$$

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Energy of the ion

$$E = \frac{1}{2} m v_{max}^2$$

$$= \frac{1}{2} m \left(\frac{B e r_{max}}{m} \right)^2$$

$$\therefore E = \frac{(B e r_{max})^2}{2m} \quad \text{——— (5)}$$

but,

$$\text{frequency } (f) = \frac{Be}{2\pi m}$$

$$\Rightarrow B = \frac{2\pi m f}{e} \quad \text{——— (6)}$$

from (5) and (6)

$$E = 2\pi^2 f^2 m r_{max}^2 \quad \text{——— (7)}$$

Magnetic Focussing

Particles carrying the same charge (q) and having the same momentum (p), even if moving in different directions, can ~~be~~ all be brought to focus at very nearly the same point on a screen by means of a suitable applied magnetic field. This is called magnetic focussing, and since the particles come to a common focus after describing an angle of 180° from their point of entry into the magnetic field, we qualify it as 180° magnetic focussing.

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If, on the other hand, the particles carry the same charge but have different momenta, they are bought to focus at different points on the screen.

To take the second case, if a beam of particles, each carrying a charge q, enters normally through a slit S in a screen, [fig ①], into a magnetic field of flux density B, perpendicular to the beam [i.e. perpendicular to the plane of paper in the case shown]

The force experienced by particle is

$$F = Bqv \sin 90°$$

$$= Bqv \quad \text{where, } v = \text{velocity of the particle}$$

Under the action of this force, the particle takes a circular path.

If m be the mass of the particle and r be the radius of its circular path then, we have

$$\frac{mv^2}{r} = Bqv$$

$$r = \frac{mv}{Bq}$$

B and q being constants i.e. the same for each particle r varies with the momentum $p = mv$ of the particle. So that, particles having different momenta come to focus at different points a, b, c on the screen. The ~~def~~ device thus acts as a momentum ~~detector~~ selector.

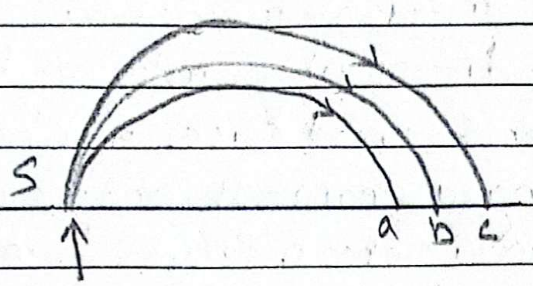

fig (a)

Charged particle in a combined electric and magnetic field

When the magnetic field and electric field are parallel – Thomson parabolas

Let a magnetic field B and an electric field E be both along the axis of y [fig (a)] and let a particle of mass m and carrying a charge q be travelling along the negative direction of the z-axis with velocity v i.e.

$\vec{B} = B\hat{j}$, $\vec{E} = E\hat{j}$ and $\vec{v} = -v\hat{k}$

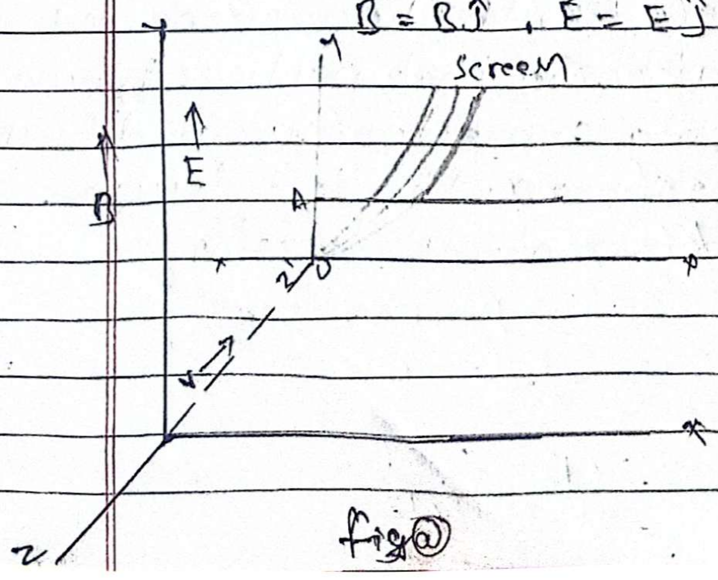

fig (a)

Action of magnetic field

Force acting on charge particle due to the magnetic field is

$$F = Bqv \sin 90°$$

$= Bqv$ is along the axis of x perpendicular to both B and v in accordance with Fleming's left hand rule.

$$F = Bqv$$

$$ma = Bqv$$

$$m \frac{d^2r}{dt^2} = Bqv$$

$$\frac{d^2r}{dt^2} = \frac{Bqv}{m}$$

If the length of the region occupied by the magnetic field be d_1, the time taken by the particle to cross the field is

$$t = \frac{d_1}{v}$$

During this time t, the particle will, therefore move or get displaced along the axis of x through a distance

$$x_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\frac{Bqv}{m}\left[\frac{d_1}{v}\right]^2$$

since its initial velocity along the axis of x is zero i.e.,

$$x_1 = \frac{1}{2}\frac{qBd_1^2}{mv}$$

The velocity acquired by the particle along the axis of x in time t when it just emerges from the magnetic field is

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$$v_x = u + at$$
$$= 0 + \frac{Bqv}{m}\left[\frac{d_1}{v}\right]$$
$$= \frac{Bqd_1}{m}$$

If the distance of the screen or the photographic plate from the point where the magnetic field ends be d_2 along the axis of z, the particle travelling along this axis will take time $\frac{d_2}{v}$ to cover it. During this time the particle will get further displaced along the axis x through a distance

$$x_2 = v_x \cdot \frac{d_2}{v} = \frac{Bqd_1}{m} \cdot \frac{d_2}{v} = \frac{Bqd_1d_2}{mv}$$

Thus the total displacement of the particle along the x-axis

$$x = x_1 + x_2$$
$$= \frac{1}{2}\frac{qBd_1^2}{mv} + \frac{qBd_1d_2}{mv}$$
$$x = \frac{qB}{mv}\left[\frac{d_1^2}{2} + d_1d_2\right] \quad \text{—— (1)}$$

b) Action of the electric field

The force experienced by the charge particle is

$F = qE$ along the direction of field i.e. along y-axis

$$ma = qE$$
$$a = \frac{qE}{m}$$
$$\frac{d^2r}{dt^2} = \frac{qE}{m}$$

Proceeding exactly as in the case of the magnetic field above, we have

Total displacement of the particle along the y-axis

$$y = y_1 + y_2$$

$$= \frac{1}{2}\frac{qEd_1'^2}{mv^2} + \frac{qEd_1'd_2}{mv^2}$$

$$y = \frac{qE}{mv^2}\left(\frac{d_1'^2}{2} + d_1'd_2'\right) \quad \text{(ii)}$$

where, d_1' is the length of the region occupied by the electric field and d_2' the distance of the screen or the photographic plate from the point where the electric field ends.

Eliminating v between relations (i) and (ii)

$$\frac{x^2}{y} = \frac{q}{m}\frac{B^2}{E}\left(\frac{d_1^2/2 + d_1d_2}{d_1^2/2 + d_1'd_2'}\right)$$

$$= \frac{q}{m}\frac{B^2}{E}\cdot k \quad \text{where } k = \frac{d_1^2/2 + d_1d_2}{d_1^2/2 + d_1'd_2'} = \text{constant}$$

— (iii)

eqn (iii) represents a parabola about y-axis.

b) When the magnetic field and electric fields are crossed or mutually perpendicular

Let the electric field $\vec{E}$ be along the y-axis and the magnetic field $\vec{B}$ along z-axis [fig (a)]. If at any given instant the velocity of the particle, subjected to the two fields simultaneously, be $\vec{v}$, we have

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

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The force experienced by a charged particle in a combined magnetic and electric field.

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

$$m\frac{d\vec{v}}{dt} = q\left[\vec{E} + (\vec{v} \times \vec{B})\right]$$

$$m\frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) = q\left[E\hat{j} + \{(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) \times B\hat{k}\}\right]$$

$$= qE\hat{j} + qB(v_y\hat{i} - v_x\hat{j})\}$$

$$\frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = \frac{1}{m}\left[qE\hat{j} + qB(v_y\hat{i} - v_x\hat{j})\right]$$

$$= \left(\frac{qE}{m} - \frac{qBv_x}{m}\right)\hat{j} + \frac{qBv_y}{m}\hat{i}$$

Thus, the component acceleration along the three coordinate axes are.

$$\frac{dv_x}{dt} = \frac{qB}{m}v_y \quad \text{--- (i)}$$

$$\frac{dv_y}{dt} = \frac{qE}{m} - \frac{qB}{m}v_x \quad \text{--- (ii)}$$

and $$\frac{dv_z}{dt} = 0 \quad \text{--- (iii)}$$

From eqn (iii) we have v_z = a constant indicating that the velocity component of the particle along the direction of the magnetic field remains unaltered in the cross field.

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Differentiating eqn (ii) w.r.t t we have

$$\frac{d^2 \cancel{v_x}}{dt^2} =$$

$$\frac{d^2 v_y}{dt^2} = -\frac{qB}{m} \cdot \frac{dv_x}{dt}$$

$$\frac{d^2 v_y}{dt^2} = -\frac{q^2 B^2}{m^2} v_y \qquad \left[\because \frac{dv_x}{dt} = \frac{qB}{m} v_y\right]$$

putting $\frac{qB}{m} = \omega$ = angular frequency of the particle, we have

$$\cancel{\frac{d^2 v_y}{dt^2} = -\omega^2 v_y}$$

$$\frac{d^2 v_y}{dt^2} = -\omega^2 v_y \quad -(iv)$$

which is the equation of a simple harmonic motion

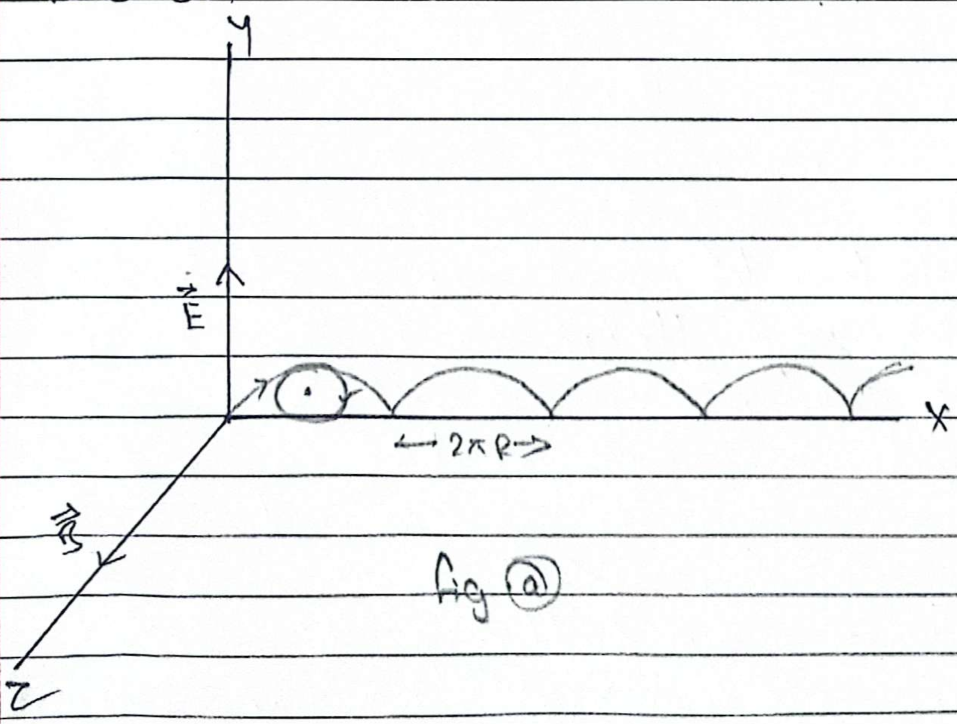

fig (a)

The solution of equation (iv) is

$$v_y = A\sin(\omega t + \phi)$$

Substituting the value of v_y in eqn (i) we have

$$v_x = \frac{E}{B} - A\cos(\omega t + \phi)$$