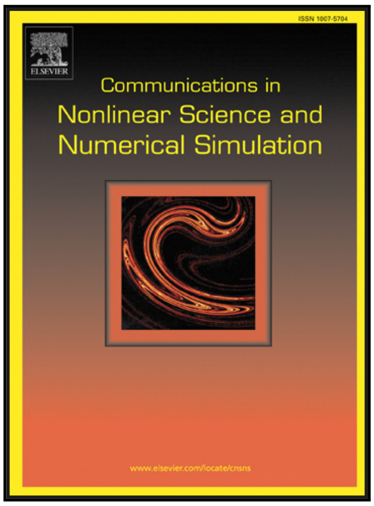

Research on price Stackelberg game model with probabilistic selling based on complex system theory

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Research on price Stackelberg game model with probabilistic selling based on complex system theory

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Highlights:

1. We considered probabilistic selling in this paper, probabilistic selling can expend the market and decrease uncertain demand. Previous literature did not study the dynamic scenario of probabilistic selling in supply chain.
2. Considering the customer has risk aversion behavior for probabilistic product, we study a scenario that two manufacturers delegate a retailer to sell two substitute brand-products and probabilistic products through dual-channel; a non-cooperative dynamic Stackelberg price game model is developed based on limited rational expectation.
3. The influences of parameters on the system stability are further analyzed; the phenomenon of flip bifurcation, chaos, and other complex phenomena are reported using bifurcation, attractor and power spectrum etc.
4. The results show that the system stability could be robust with increase in customer risk aversion, price discount and the probability of each product becoming to probability product.
5. The nonlinear feedback control method is used to control the system's chaos.

Abstract: Probabilistic selling can expand the market and decrease uncertain demand. In this paper, we study a scenario that two manufacturers delegate a retailer to sell two substitute brand-products through traditional selling channel and probabilistic products through probabilistic selling channel; the customer has risk aversion behavior for probabilistic product. A non-cooperative dynamic price Stackelberg game model is developed based on limited rational expectation. The influences of parameters on the system stability are further analyzed; the phenomenon of flip bifurcation, chaos, and other complex phenomena are reported using bifurcation, attractor and power spectrum etc. The results show that the system stability could be robust with increase in customer's risk aversion and the probability of each product becoming to probability product, and frail with increase in price discount. The manufacturer may benefit from a larger proportion of its product becoming to probabilistic product. When the chaos occurs, the market becomes abnormal, irregular and unpredictable; the two manufacturers should determine their price adjustment speed according to the parameters' values, to ensure the up and downstream enterprises keep stable. The nonlinear feedback control method is used to control the system's chaos. The derived results have very important theoretical and practical values for the two manufacturers and the common retailer.

Keywords: Probabilistic selling; Game theory; Complex analysis; Bifurcation

1. Introduction

The development of information technology (IT) and economic globalization is creating more efficient shopping environment for selling probabilistic products. In recent years, the probabilistic selling has been put forward and recently obtained great attention, which can be generated by retailers through mixing products from different suppliers [1]. Cai et al [2] studied the probabilistic selling considering Channel structure and supplier competition, and obtained that the probabilistic selling can expand the market and decrease uncertain demand. However, they neglected the effect of parameters on the participants' profits. Fay and Xie [3] developed a formal model considering buyer uncertainty which examined the purchase options, advance selling or probabilistic selling, and explored the differences via two different mechanisms homogenizing heterogeneous consumers and separating heterogeneous consumers. Anderson and Xie [4] developed a stylized model to analyze the pricing and market segmentation using opaque selling mechanisms.

Probabilistic selling can reduce the uncertainty demand which is noted by many scholars. Cheng [5] developed a quality-price game in a fully covered market where firms were uncertain about consumer tastes regarding quality. Banker et al. [6] investigated empirically and analytically the relationship between demand uncertainty and cost behavior, and obtained that the higher uncertain demand would lead the demand increasing. Chen et al. [7] investigated the long-term influences of demand uncertainty and market concentration on price instability in the hotel industry. Yang and Ng [8] established a flexible capacity strategy model with multiple market periods under investment constraints and demand uncertainty, and optimized the capacity, safety production, and the sales of each market period under different situations. Firms can benefit from creating buyer uncertainty by selling in advance [9, 10]. Fay and Xie [1] studied the strategic effect of probabilistic selling and obtained that offering probabilistic products could reduce the seller's information disadvantage, increase profit and solve the mismatch between capacity and demand. Facing risks caused by uncertainties in end product demand and component availability, a scenario in an illustrative manufacturing company was developed for modeling demand and supply uncertainties [11]. Kwag and Kim [12] constructed a reliability model of the demand resource which was generalized by a multi-state model considering customers' behaviors. Askar [13] proposed a Cournot oligopoly game where quantity-setting firm used unknown nonlinear demand function and random cost function, analyzed its complete stability and bifurcation behaviors. Moreover, this idea of uncertainty was also applied in inventory [14], network design problem [15] and bullwhip effect [16].

Above literatures complemented extant research on uncertainty demand, the probabilistic selling or opaque selling can weaken the negative effect of uncertain demand. However, they studied the probabilistic selling or opaque selling under the static environment.

Literature on the channel structure, as well as competition between the suppliers was studied by many scholars. Tetteh et al. [17] developed four view Markov chain models to investigate how to control cost of inventory by analyzing the impact of speculation in a dual-supply chain. Dan et al. [18] analyzed price and service competition in the dual-channel supply chain which consists of a manufacturer and two retailers, investigated the influence of power structure on equilibrium price and service decisions, and proposes some competitive strategies for traditional retailer under e-commerce environment. Chiarella et al. [19] used a maximum likelihood approach on data to estimate a structural model. By using differential game theory, Sayadi and Makui [20] investigated the influence of dynamic brand and channel advertising on market expansion and market share in a dual channel supply chain, and showed that a higher compatibility of a product with online marketing, a higher advertising effort for the online channel by the manufacturer.

These literatures studied the different channel structure, as well as competition between the suppliers, analyzed the optimal solution in different conditions. But they did not consider the influence of the customer's risk preference on the system's decisions.

In our knowledge, there are few literatures using dynamics methodology to study the supply chains with probability selling. Rich dynamics characteristics in the economic and society system had been found, such as chaotic, or even hyper chaotic behavior. Ma and Tu [21] Considered the macroeconomic model of money supply with time delays, discussed the effect of delay variation on system stability and Hopf bifurcation. Yang et al. [22] studied rich dynamics of a nonlinear economic model, found Chaotic and bubbling phenomena which clearly agreed with phenomena from technology bubbling. Ma and Li [23] constructed a dynamic Bertrand-Stackelberg pricing model to analyze the influence of uncertain demand on the profit and complexity. Chiarella et al. [24] incorporated the adaptive behavior of agents with heterogeneous beliefs and establishes an evolutionary capital asset pricing model (ECAPM) within the mean-variance framework. Ma and Pu [25] studied the Cournot-Bertrand duopoly model, analyzed the stability of the fixed points, and recognized the chaotic behavior of the system.

We will study the dynamics characteristics of supply chains with probabilistic selling considering the customer has risk aversion behavior for probabilistic product. A new model will be developed to dynamic analyze the pricing and the stability of the supply chain with probabilistic selling. The mutual influence between variables and parameters and the dynamic phenomena will be analyzed through numerical simulation. The research is particularly vital and urgent to the practitioners because the development in new technology is making implementation of a probabilistic selling strategy much more efficient and practical.

In this paper, our primary aim is to propose a non-cooperative dynamic price Stackelberg game model with exogenous channel structure that involves manufacturer competition and the active role of a common retailer. Toward this aim, we will construct a non-cooperative dynamic price Stackelberg game model in which two manufacturers intend to sell their products to consumers who have heterogeneous preferences and risk preferences, and a common retailer is capable of generating the probabilistic product.

The remainder of this paper is organized as follows: In section 2, we discuss the problems of consumer preferences and transaction process, and develop the dynamic price Stackelberg game model. Section 3 focuses on the stability of the dynamic price Stackelberg game model. The dynamic characteristics of the price Stackelberg game model are analyzed under different parameters change in section 4. In last section, we outline some conclusions and relevant recommendations for future research.

2. The dynamic price Stackelberg game model

2.1 Assumptions

The following assumptions are made to develop our model in this paper.

(i) Two manufacturers (A and B) provide two substitute products (a and b), a common retailer (R) can sell the probabilistic product using probabilistic selling channel and traditional product using traditional selling channel. The two manufacturers are the leaders, and the common retailer is a follower. This relationship implies the dominance of the two manufacturers over the retailer.

(ii) Facing the heterogeneous consumers, A and B sell the two substitute products and probabilistic product to customers through the common retailer, the customer has risk averse behavior for the probabilistic product.

(iii) For the simplicity of calculation, the common retailer's operating cost and the variable cost of selling the products are normalized to zero. The production cost for each manufacturer is normalized to zero.

(iv) Consumer demand follows Hotelling distribution. Each consumer buys one unit between two products at most; the heterogeneous preference of each consumer is represented by an ideal point which lies in [0, 1] .

(v) A locates at 0 and B locates at 1. They all agree to create the probabilistic product.

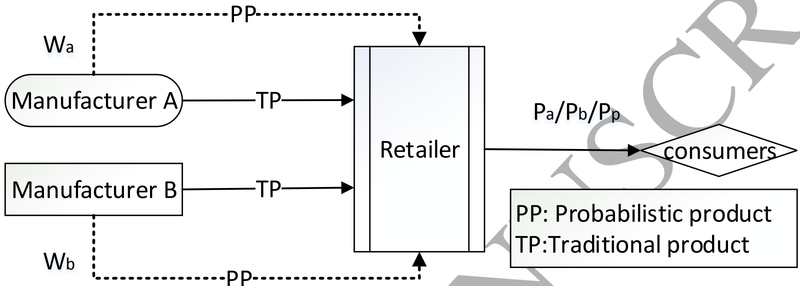


Fig.1 The supply chain system with probabilistic selling strategy

2.2 Consumer preferences

An ideal point denoted by $x \in [0,\ 1]$ is used to represent the heterogeneous preference of each consumer. The consumers will obtain the value v from buying product a or b. Because the ideal point x may differ from the manufacturers' location, the consumer incurs distance cost, denoted by c. The distance cost captures the negative utility arising from the discrepancy between the ideal point and the product position. The differentiation degree of the products is high as c increases. The net utility of the customer buying product from A is $v - cx - p_a$, where p_a is the selling price charged by product a; likewise, the net utility is $v - c(1-x) - p_b$ when purchasing from B (with p_b being the price of product b).

2.3 The definition of marginal consumers

Under the probabilistic selling, x_a is used to denote the marginal consumer who obtains the same utility between purchasing product a and the probabilistic product, and x_b to denote the marginal consumer who obtains the same utility between purchasing product b and the probabilistic product. The utility value of a consumer in $x \in (0,\ 1)$ purchasing a probabilistic product is

$$\varphi(v - cx) + (1-\varphi)[v - c(1-x)] - p_p, \tag{1}$$

where p_p is the selling price of probabilistic product. Hence, considering the consumer's risk preference of purchasing probabilistic product (β), the consumer surplus of purchasing probabilistic product is $\beta(\varphi(v - cx) + (1-\varphi)[v - c(1-x)] - p_p)$. The marginal consumers in x_a and x_b are determined by the following equations:

$$\begin{cases} v - cx_a - p_a = \beta\big(\varphi(v - cx_a) + (1-\varphi)(v - c + cx_a) - p_p\big), \\ v - c(1 - x_b) - p_b = \beta\big(\varphi(v - cx_b) + (1-\varphi)(v - c + cx_b) - p_p\big), \end{cases} \tag{2}$$

where φ is the probability of product a to the probabilistic product, $\varphi \in (0,\ 1)$ is an exogenous variable.

Solving system equation (2), the following functions can be obtained:

$$\begin{cases} x_a = \frac{v + c\beta - v\beta - c\beta\varphi - p_a + \beta p_p}{c + c\beta - 2c\beta\varphi}, \\ x_b = \frac{c - v - c\beta + v\beta + c\beta\varphi + p_b - \beta p_p}{c - c\beta + 2c\beta\varphi}. \end{cases} \tag{3}$$

We can see that x_a and x_b are related to the selling price of brand product and probabilistic product, the consumer's risk preference and the probability of product a to the probabilistic product.

2.4 Transactions

Assuming w_a and w_b represent the wholesale prices that A and B offer to the common retailer for the brand product a and b, w_a^p and w_b^p represent the wholesale prices that A and B offer to the common retailer for the probabilistic product respectively, where the superscript p depicts the probabilistic selling scenario, and they satisfy the following: $w_a^p = \rho w_a$ and $w_b^p = \rho w_b$, exogenous parameter ρ reflects the common retailer possess the relative bargaining power with A and B.

As A and B simultaneously determine the w_a, w_b, then the common retailer determines the retail price p_a and p_b for the brand product a and b, and p_p for the probabilistic product. According to the prices (p_a, p_b and p_p), customers decide which product to purchase. Therefore, the profits of the two manufacturers and the common retailer are obtained as follows.

$$\begin{cases} \pi_A = w_a x_a + \varphi\rho w_a (x_b - x_a), \\ \pi_B = p_b(1 - x_b) + \rho w_b (1-\varphi)(x_b - x_a), \\ \pi_R = x_a(p_a - w_a) + (1 - x_b)(p_b - w_b) + \varphi\big(p_p - \rho w_a\big)(x_b - x_a) \\ \qquad + (1-\varphi)\big(p_p - \rho w_b\big)(x_b - x_a). \end{cases} \tag{4}$$

2.5 The dynamic price Stackelberg game model

Note that, A and B are the leaders, and the common retailer is a follower, and this relationship implies the dominance of A and B over the retailer. This is a Stackelberg game and the game equilibrium is called Stackelberg Equilibrium. In this dynamic price Stackelberg game, A and B make decisions of wholesale prices according to the market information, then the retailer makes decision of selling prices according to the decision-making of the manufacturers. We first take the wholesale price w_a and w_b as given. Substituting x_a and x_b into (4), marginal profits of the common retailer on p_a, p_b and p_p are obtained:

$$\begin{cases} \frac{\partial \pi_R}{\partial p_a} = \frac{v + c\beta - v\beta - c\beta\varphi - 2p_a + p_p + \beta p_p + (1-\rho\varphi)w_a + \rho(\varphi - 1)w_b}{c + c\beta - 2c\beta\varphi}, \\ \frac{\partial \pi_R}{\partial p_b} = \frac{v - v\beta + c\beta\varphi - 2p_b + p_p + \beta p_p - \rho\varphi w_a + w_b - \rho w_b + \rho\varphi w_b}{c - c\beta + 2c\beta\varphi}, \\ \frac{\partial \pi_R}{\partial p_p} = \frac{-c + 2v + c\beta - 2v\beta - (1+\beta)(1+\beta(-1+2\varphi))p_a + (1+\beta)(-1-\beta+2\beta\varphi)p_b + 4\beta p_p}{c(-1+\beta(-1+2\varphi))(1+\beta(-1+2\varphi))} + \frac{(\beta - \beta^2 + 2\beta^2\varphi - 2\beta\rho\varphi)w_a}{c(-1+\beta(-1+2\varphi))(1+\beta(-1+2\varphi))} \\ \qquad + \frac{(\beta + \beta^2 - 2\beta\rho - 2\beta^2\varphi + 2\beta\rho\varphi)w_b}{c(-1+\beta(-1+2\varphi))(1+\beta(-1+2\varphi))}. \end{cases} \tag{5}$$

Therefore, best reply functions of the common retailer are as follows:

$$
\begin{cases}
p_a = \frac{-2c+4v+c\beta-8v\beta-3c\beta^2+4v\beta^2+3c\beta^3+c\beta^4-2c\beta\varphi-10c\beta^3\varphi-4c\beta^4\varphi+4c\beta^2\varphi^2+8c\beta^3\varphi^2+4c\beta^4\varphi^2}{4(-1+\beta)^2} \\
\qquad + \frac{(-1+\beta)[-1+\beta^2(-1+2\varphi)+\beta(2+2\varphi-4\rho\varphi)]w_a-(-1+\beta)[-1-2\beta(-1+2\rho)(-1+\varphi)+\beta^2(-1+2\varphi)]w_b}{4(-1+\beta)^2}, \\
p_p = \frac{-2c+2v+c\beta-4v\beta+2v\beta^2+c\beta^3-4c\beta^2\varphi-4c\beta^3\varphi+4c\beta^2\varphi^2+4c\beta^3\varphi^2}{2(-1+\beta)^2} \\
\qquad + \frac{(-1+\beta)[1-2\rho\varphi+\beta(-1+2\varphi)]w_a-(-1+\beta)(-1-\beta+2\rho+2\beta\varphi-2\rho\varphi)w_b}{2(-1+\beta)^2}, \\
p_b = \frac{-2c+4v-c\beta-8v\beta+c\beta^2+4v\beta^2+c\beta^3+c\beta^4+2c\beta\varphi-8c\beta^2\varphi-6c\beta^3\varphi-4c\beta^4\varphi+4c\beta^2\varphi^2+8c\beta^3\varphi^2+4c\beta^4\varphi^2}{4(-1+\beta)^2} \\
\qquad + \frac{(-1+\beta)[1+\beta(2-4\rho)\varphi+\beta^2(-1+2\varphi)]w_a-(-1+\beta)[1+\beta^2(-1+2\varphi)+\beta(-4+4\rho+2\varphi-4\rho\varphi)]w_b}{4(-1+\beta)^2}.
\end{cases} \tag{6}
$$

Formula (6) is the optimal decision prices of the common retailer on the premise of w_a and w_b, the common retailer can obtain the decision after it observe the manufacturer's behaviors. Similarly, substitute (6) into (4), the two manufacturers' marginal profit is obtained as follows:

$$
\begin{cases}
\frac{\partial\pi_A}{\partial w_a} = \frac{c(-2+\beta+\beta^2-2\beta\varphi-2\beta^2\varphi+4\beta\rho\varphi)-2(-1+\beta)w_a+(-1+\beta)w_b}{4c(-1+\beta)} = 0, \\
\frac{\partial\pi_B}{\partial w_b} = \frac{c(-2+\beta^2(-1+2\varphi)+\beta(-1+4\rho+2\varphi-4\rho\varphi))+(-1+\beta)w_a-2(-1+\beta)w_b}{4c(-1+\beta)} = 0.
\end{cases} \tag{7}
$$

Let the equation (7) equal to zero, we can obtain the following equations:

$$
\begin{cases}
w_a = \frac{c[6+\beta^2(-1+2\varphi)+\beta(-1+2\varphi)-4\beta\rho(\varphi+1)]}{3(\beta-1)}, \\
w_b = \frac{c[-6+\beta^2(-1+2\varphi)+\beta(-1+2\varphi)-4\rho\beta(-2+\varphi)]}{3(-1+\beta)}.
\end{cases} \tag{8}
$$

We can see that manufacturers' wholesale price is related to distance cost, the relative bargaining power that common retailer possess, the probability of product a to the probabilistic product, consumer's risk preference of purchasing probabilistic product.

It is important for A and B to specify the market information set to determine the price behavior over time. A and B always hope to gain more profits through the active management behaviors. In this paper, the two manufacturers make decisions of the price in next period based on the limited rationality [26]. That is, the two manufacturers adjust the transfer wholesale price $w_a(t+1), w_b(t+1)$ on basis of the one of previous period $w_a(t)$, $w_b(t)$. Then, the two-dimensional system that characterizes the price adjustment mechanism of the two manufacturers and the common retailer is the following:

$$
\begin{cases}
w_a(t+1) = w_a(t) + k_1 w_a(t)\frac{\partial\pi_A}{\partial w_a}, \\
w_b(t+1) = w_b(t) + k_2 w_b(t)\frac{\partial\pi_B}{\partial w_b},
\end{cases} \tag{9}
$$

where k_1 and k_2 are coefficients that capture the speed at which the two manufacturers adjust its price according to the consequent marginal change in its profit, respectively.

3 The stability of the system (9)

In system (9), let $w_a(t+1) = w_a(t), w_b(t+1) = w_b(t)$. Then,

$$
\begin{cases}
\frac{c(-2+\beta+\beta^2-2\beta\varphi-2\beta^2\varphi+4\beta\rho\varphi)-2(-1+\beta)w_a+(-1+\beta)w_b}{4c(-1+\beta)} = 0, \\
\frac{c(-2+\beta^2(-1+2\varphi)+\beta(-1+4\rho+2\varphi-4\rho\varphi))+(-1+\beta)w_a-2(-1+\beta)w_b}{4c(-1+\beta)} = 0.
\end{cases} \tag{10}
$$

We can get the four fixed points and only the Nash equilibrium point $E^*(w_a,\ w_a)$ has the economic meaning.

According to the actual situation, first we assign some values to parameters: $v = 110, c = 4, \varphi = 0.4, \beta = 0.8, \rho = 0.95$. The Nash equilibrium value is $E^*(w_a,\ w_a)$= (9.653, 9.547). To study the stability of the fixed points, the Jacobian matrix of the E^* is needed:

$$J = \begin{vmatrix} J_{11} & \frac{k_1 w_a}{4c} \\ \frac{k_2 w_b}{4c} & J_{22} \end{vmatrix} \tag{11}$$

where

$$J_{11} = 1 - \frac{k_1 w_a}{2c} + \frac{k_1[c(-2+\beta-2\varphi\beta+4\varphi\beta\rho+\beta^2-2\varphi\beta^2)-2w_a(-1+\beta)+w_b(-1+\beta)]}{4c(-1+\beta)},$$

$$J_{22} = 1 - \frac{k_2 w_b}{2c} + \frac{k_2[c(-2-\beta+4\rho\beta+2\varphi\beta-4\rho\varphi\beta-\beta^2+2\varphi\beta^2)+w_a(-1+\beta)-2w_b(-1+\beta)]}{4c(-1+p)}.$$

We only consider the stability of the Nash equilibrium point. The characteristic polynomial of system (9) at Nash Equilibrium point is.:

$$f(\lambda) = \lambda^2 + A_1\lambda + A_0 \tag{12}$$

where $A_1 = -J_{11} - J_{22}$, $A_0 = J_{11}J_{22} - J_{12}J_{21}$.

According to the Jury conditions [27], the necessary and sufficient condition of the local stability of Nash equilibrium point should satisfy the following conditions:

$$\begin{cases} 1 + A_1 + A_0 > 0, \\ 1 - A_1 + A_0 > 0, \\ |A_0| < 1. \end{cases} \tag{13}$$

Solving the conditions (13), the local stable region of the E^* about the price adjustment speed $k_i, i = 1, 2$ can be obtained under the different parameter values. Figure 2 shows the stable region of system (9) with different β. Figure 3 shows the stable region of system (9) with different ρ. Figure 4 shows the stable region of system (9) with different φ. The increase of β shows customer's risk attitude tends to be neutral, with the increase of β the stable region reduces. We can make a conclusion that the stability of the system could be robust with increase in customer risk aversion, and weaken with increase in price discount; the stability of k_1 could be robust and the stability of k_2 could be weaken with increase in the probability of product a to the probabilistic product. From figures (2), (3), (4), the values of the Nash Equilibrium is unrelated to price adjustment speed, but the system (9) will become unstable and even falls into chaos if one manufacturer adjusts price speed too fast and pushes the price adjustment speed out of the stable region. So A and B should control the price adjustment speed according to the value of parameters so as to make the market stable.

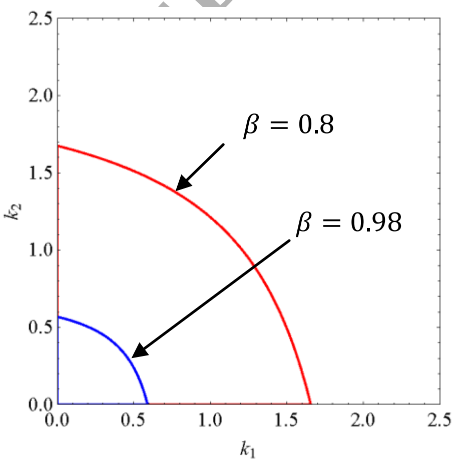


Fig.2 The stable region of Nash Equilibrium in (k_1, k_2) plan with different β

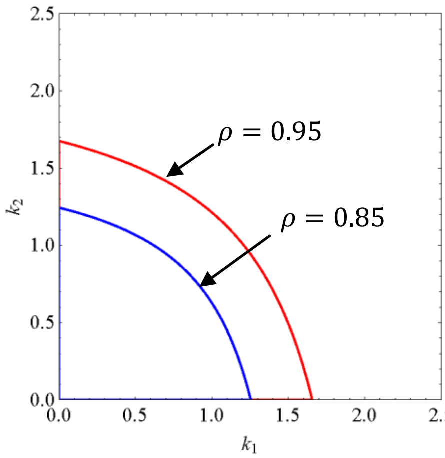


Fig.3 The stable region of Nash Equilibrium in (k_1, k_2) plan with different ρ

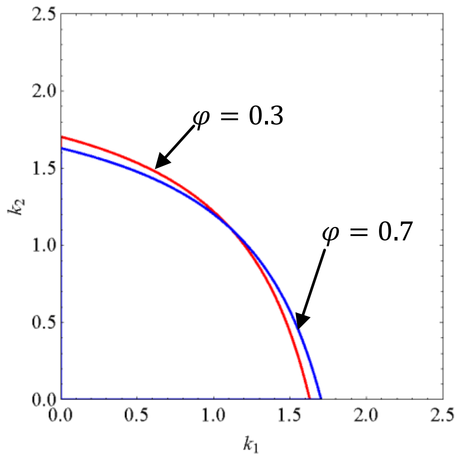


Fig.4 The stable region of Nash Equilibrium in (k_1, k_2) plan with different φ

The economic meaning of the stable region is that A and B will eventually maintain at Nash equilibrium price after a finite game whatever initial price is chosen in the local stable region. Numerical simulation method is used to analyze the dynamic characteristics of system (9) with change of $k_i, i = 1, 2$, such as the bifurcation diagram, the strange attractor, the Lyapunov exponent and the sensitive dependence on initial conditions will be considered in the following section.

4 Dynamic characteristics of the system (9)

In this game, the two manufacturers make the optimal wholesale price decisions to get the maximum profits and adjust wholesale prices based on the marginal profits of last period. The two manufacturers have the driving force to adjust their price in the hope of achieving more profits. However, the price adjustment speed and parameters affect the game results seriously. The influences of the price adjustment speed and the other parameters on the behaviors of system (9) will be analyzed in the following subsections.

4.1 The influences of price adjustment speed on the stability of the system (9)

The stability of Nash equilibrium point will be changed if A or B accelerate the price adjustment speed and push $k_i, i = 1, 2$, out of the stable region. As the similarity of channel structure, we only discuss the influence of k_1 on the dynamic characteristic of the system (9). From figures 5 and 6, the price bifurcation diagram agrees with the corresponding Lyapunov exponent diagram and attractor diagram with $k_2 = 0.2$ when $k_1 \in (0,\ 2.4)$. We can find that with the change of the price adjustment speed $k_i, i = 1, 2$, the system (9) presents the complex dynamic behaviors. In addition, the larger the positive Lyapunov exponent is, the more obvious the chaos of the system is. For $k_1 = 1.644$, the system (9) becomes 2-period bifurcation, after that the system (9) enters 2-period state which can be depicted by its attractor in figure 6(a) and its wave plot in figure 7(a). When $k_1 > 2.2$, the system (9) enters into chaotic state which can be depicted by its attractor in figure 6(c) and (d) and its wave plot in figure 7(b). We can find $w_a > w_b$, this is because the probability of product a to the probabilistic product is less than the one of product b.

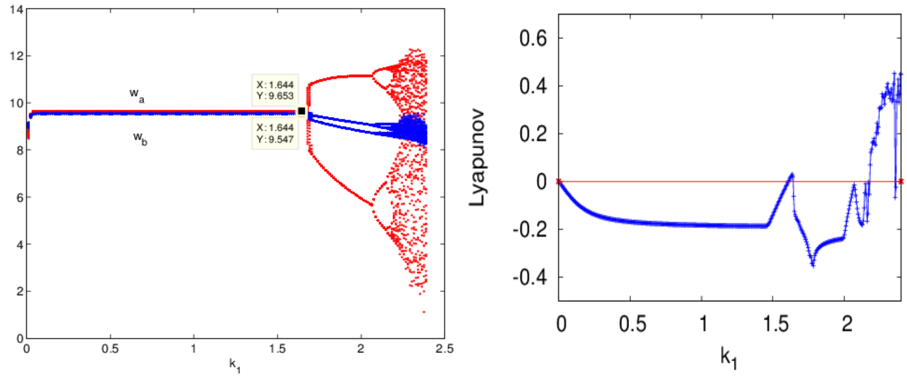
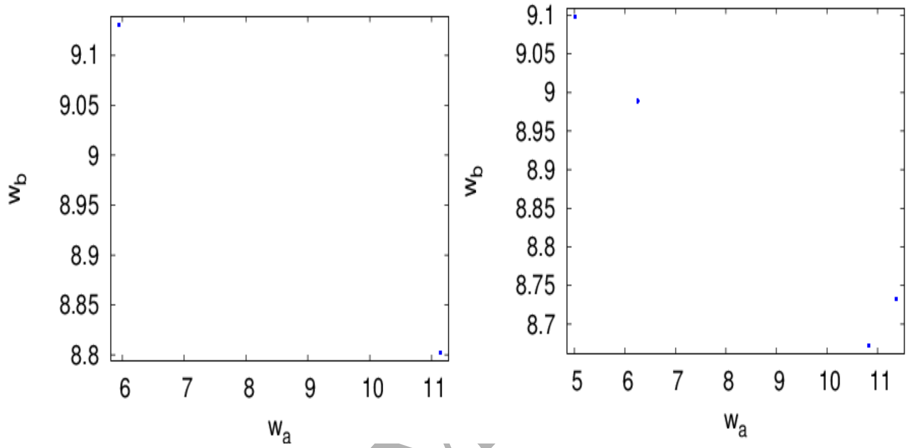
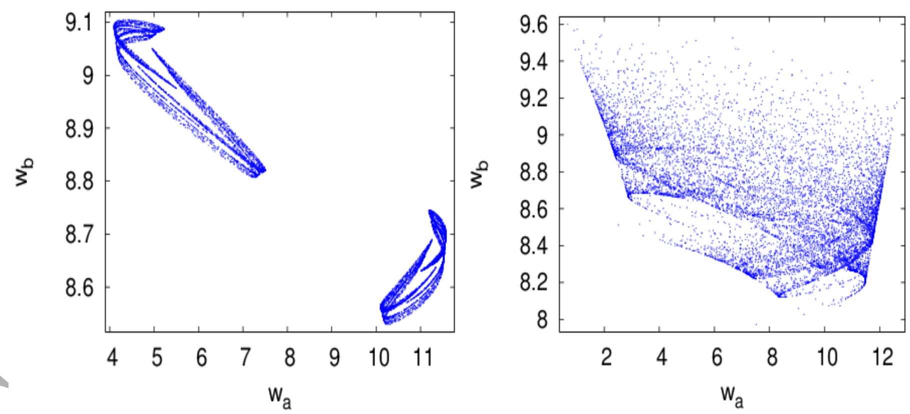


Fig.5 Bifurcation diagram and the corresponding Lyapunov exponents of system (9) with $k_1 \in (0,\ 2.4),\ k_2 = 0.2$

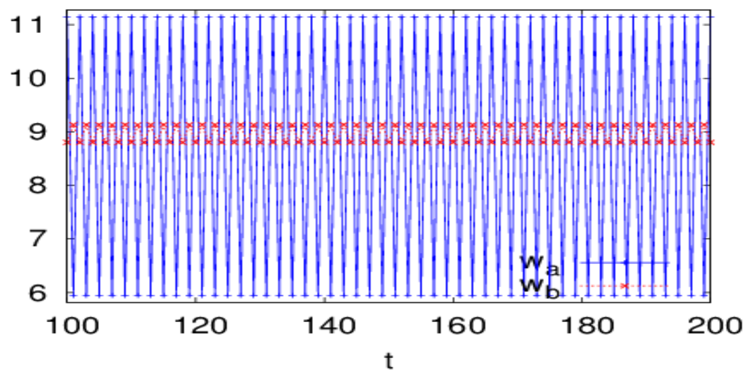


(a) $k_1 = 2$ (b) $k_1 = 2.1$

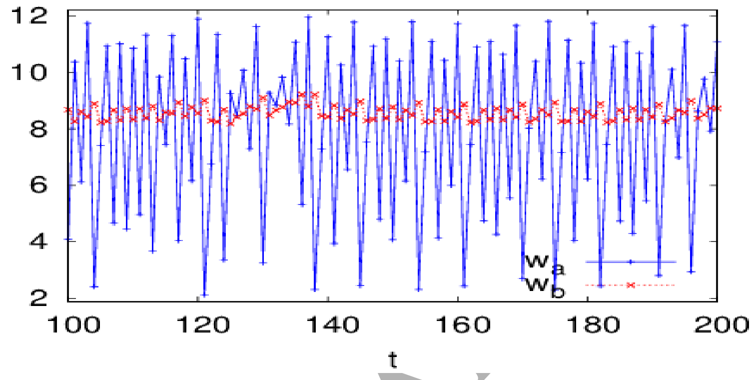


(c) $k_1 = 2.2$ (d) $k_1 = 2.4$

Fig.6 Phase plot of system (9) with $k_2 = 0.2$



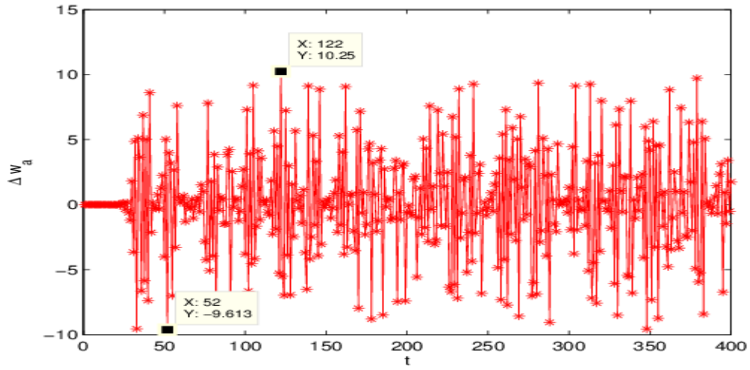
(a) $k_1 = 2$



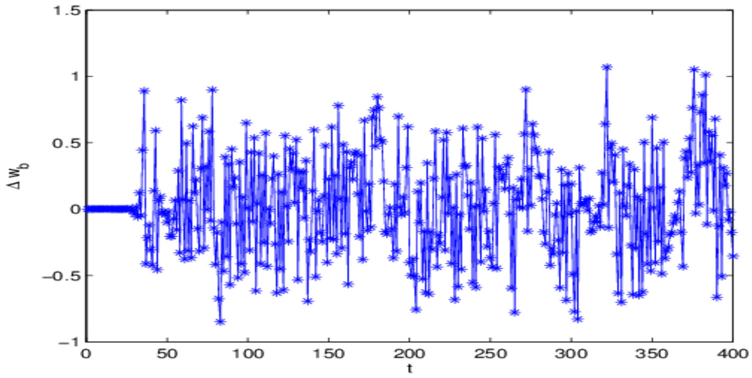
(b) $k_1 = 2.4$

Fig.7 The wave plots of system (9) with change of time with $k_2 = 0.2$

One of the most important characteristics of the chaos is extremely sensitive dependence on initial conditions. Figure 8 shows the changes in wholesale price with change of time when system (9) has different initial conditions. We can see that there is almost no distinction between them in the beginning, but the changes between them become larger with the number of games increasing. It indicates that a slight difference between initial values can lead to a great effect on the game results. Figure 8 affirms the system (9) is in a chaotic state. When the system (9) is in a chaotic state, the market will be destroyed and it is difficult for the two manufactures to make long term plan. Therefore, every action of the two manufacturers can result in a great loss.



(a)



(b)

Fig.8 The difference in w_a and w_b for $k_1 = 2.4, k_2 = 0.2$ and the initial conditions are $(w_a, w_b) =$ (6, 2) and (6.001, 2)

Figure 9 shows the profits of the two manufacturers and the common retailer with change of k_1 when $k_2 = 0.2$. We can see that the profit of B is larger than the one of A. In other words, the larger the probability of product becoming to the probabilistic product is, the larger the profit of product is. Hence, the manufacturer may benefit from a larger proportion of its product becoming to probabilistic product. This suggests a tight connection between the retailer decision on probabilistic product and the manufacturers' profitability [2].

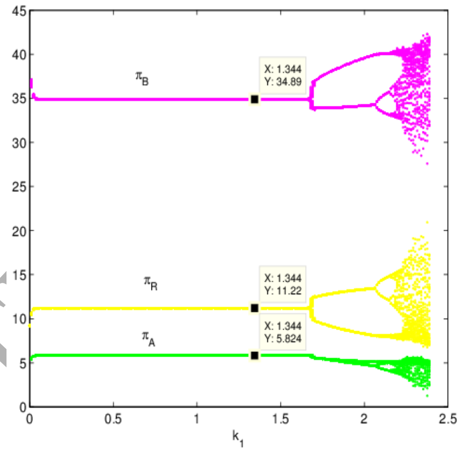


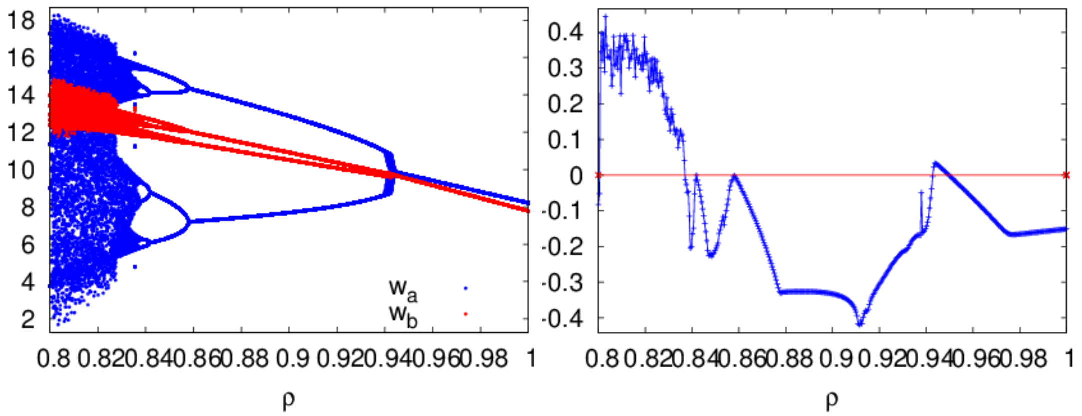
Fig.9 The profits of the two manufacturers and the common retailer with change of k_1 when $k_2 = 0.2$

4.2 The dynamic characteristics of system (9) with change of ρ and φ

For any given values of the price adjustment speed, the movement direction of the wholesale prices will be analyzed considering the role played by the discount parameter ρ and parameter φ.

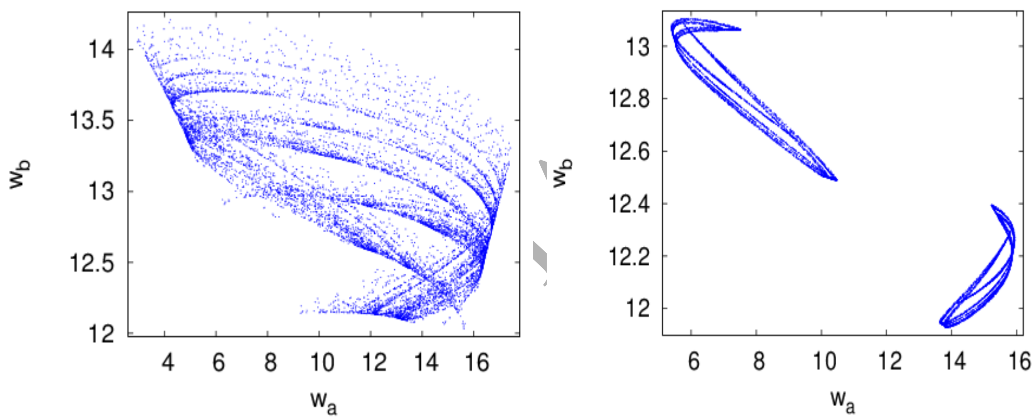
First, we consider the case of ρ increasing. When $k_1 = 1.4$, $k_2 = 0.2$, figure 10 shows that the price bifurcation diagrams well coincide with the corresponding the biggest Lyapunov exponent diagram; Figure 11 is the attractors of the system (9) in different state. We can see when $k_1 = 1.4$, $k_2 = 0.2$, with the increasing of ρ the system (9) experiences the complex characteristics from chaos state to stable state through the slip bifurcation, and w_a and w_b decrease gradually. When $\rho = 0.82$ and $\rho = 0.83$, the system (9) is in chaotic state; when $\rho = 0.84$, the system (9) is in 8-period; when $\rho = 0.85$, the system (9) is in 4-period; when

$\rho = 0.94$, the system (9) is in stable state. Figure 12 is the wave plots of system (9) in different state.

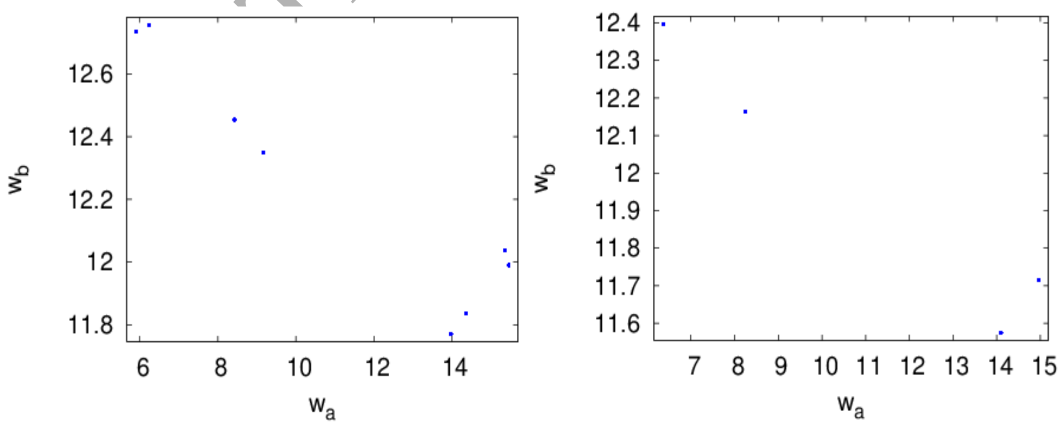


(a) wholesale price bifurcation (b) Lyapunov exponent

Fig.10 Bifurcation diagram and corresponding Lyapunov exponents of system (9) with $\rho \in [0.8,\ 1],\ k_1 = 1.4,\ k_2 = 0.2$

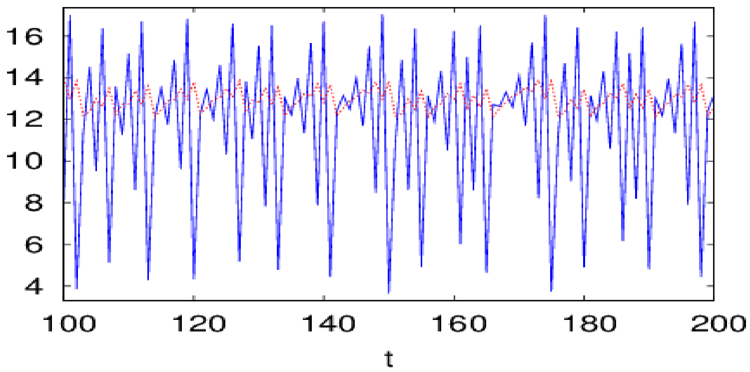


(a) $\rho = 0.82$ (b) $\rho = 0.83$

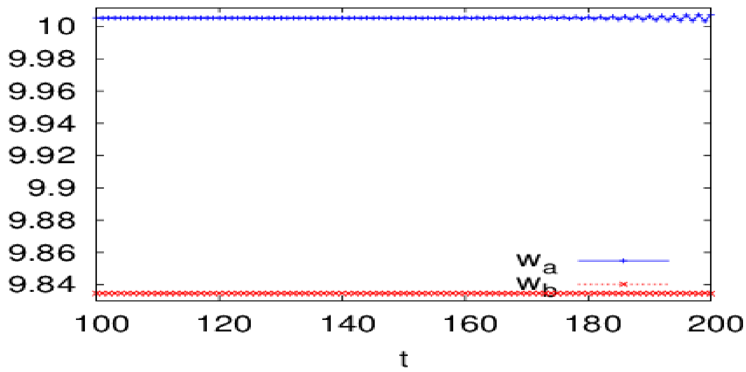


(c) $\rho = 0.84$ (d) $\rho = 0.85$

Fig.11 Chaos attractor of system (9) when $k_1 = 1.4,\ k_2 = 0.2$



(a) $\rho = 0.82$

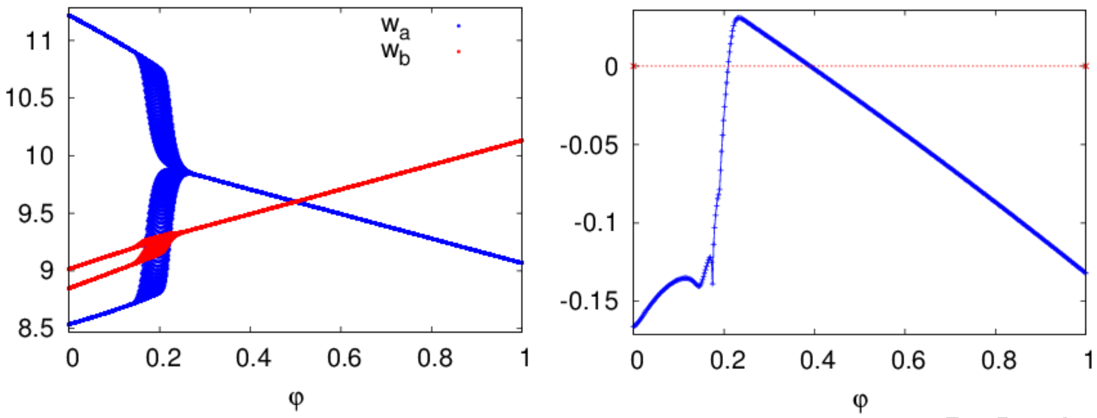


(b) $\rho = 0.95$

Fig.12 The wave plots of system (9) with change of time with $k_2 = 0.2$

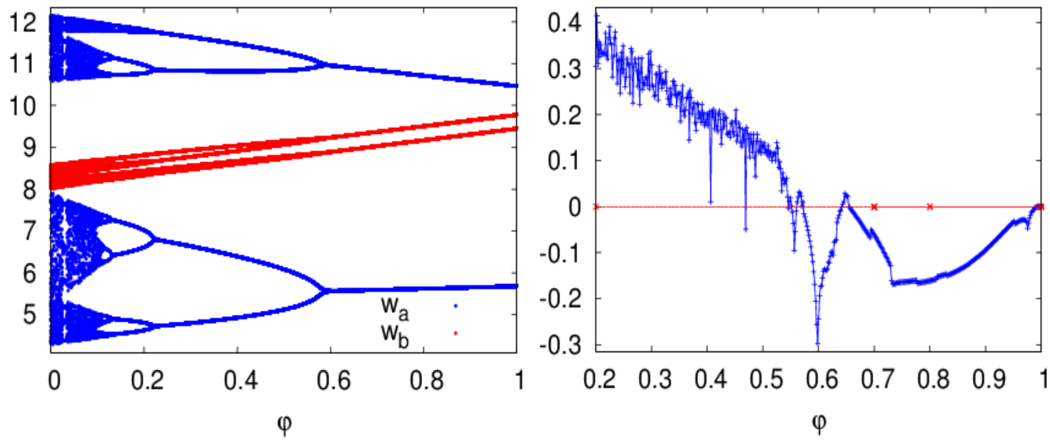
Next, we will consider the case in which the φ varies. Figure 13 is the price bifurcation diagram and the Lyapunov exponent diagram of the system (9) when $k_1 = 1.6$, $k_2 = 0.2$, the system (9) gradually changes from 2-period bifurcation state to the stable state; figure 14 is the price bifurcation diagram and the Lyapunov exponent diagram of the system (9) when $k_1 = 2.2$, $k_2 = 0.2$, the system (9) gradually changes from chaos state to the 2-period bifurcation state. When $k_1 = 2.2$, $k_2 = 0.2$, the system (9) is in chaotic state. What is more, it can be found that the attractor diagram in figure 15 well coincides with evolution process of the price bifurcation diagram in figure 14 respectively. Figure 16 is the wave plots of system (9) with change of time.

Cai et al. [2] found that it was generally optimal to assign an equal probability to each component product as the probabilistic product in a static game model, this is because deviating from such an equal probability will diminish the two positive effects of probabilistic selling: market expansion and price discrimination. In the view of dynamic game, when k_1 makes the system (9) in 2-period bifurcation state, adjusting the value of φ can make the system (9) back to stable state; once k_1 makes the system (9) in 4-period bifurcations state, or even in a chaotic state, no matter how to adjust the value of φ, can only make the system (9) back to the 2-period bifurcation state. Managers can supervise and control the market competition according to the relation of price adjustment speed and the probability of product becoming to the probabilistic product.



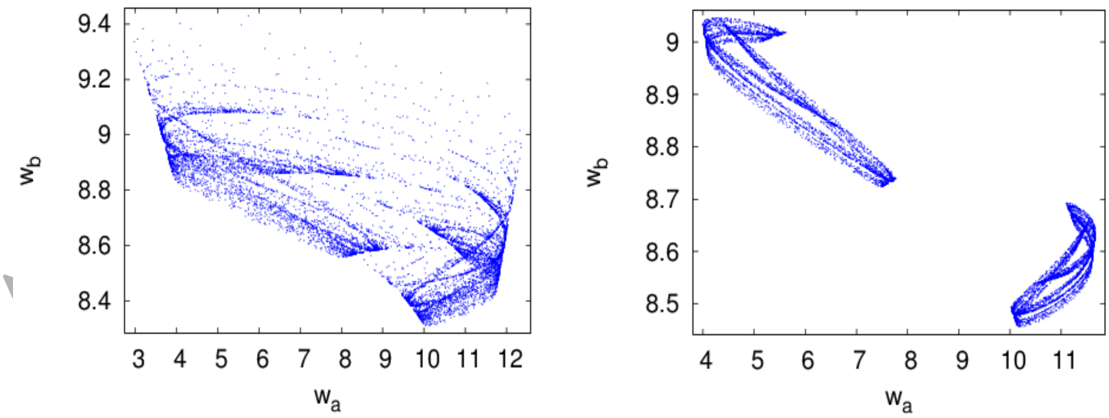
(a) Wholesale price bifurcation (b) The largest Lyapunov exponent

Fig.13 Bifurcation diagram and corresponding Lyapunov exponents of system (9) with $\varphi \in [0,\ 1],\ k_1 = 1.6,\ k_2 = 0.2$

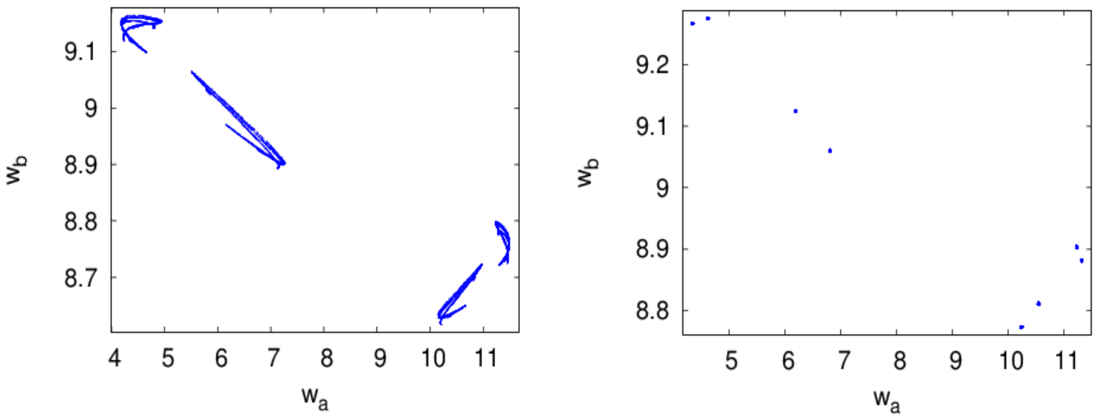


(a) Wholesale price bifurcation (b) The largest Lyapunov exponent

Fig.14 Bifurcation diagram and corresponding Lyapunov exponents of system (9) with $\varphi \in [0,\ 1],\ k_1 = 2.2,\ k_2 = 0.2$



(a) $\varphi = 0.3$ (b) $\varphi = 0.4$



(c) $\varphi = 0.5$ (d) $\varphi = 0.6$

Fig.15 Chaos attractor of system (9) with $k_1 = 2.2,\ k_2 = 0.2$

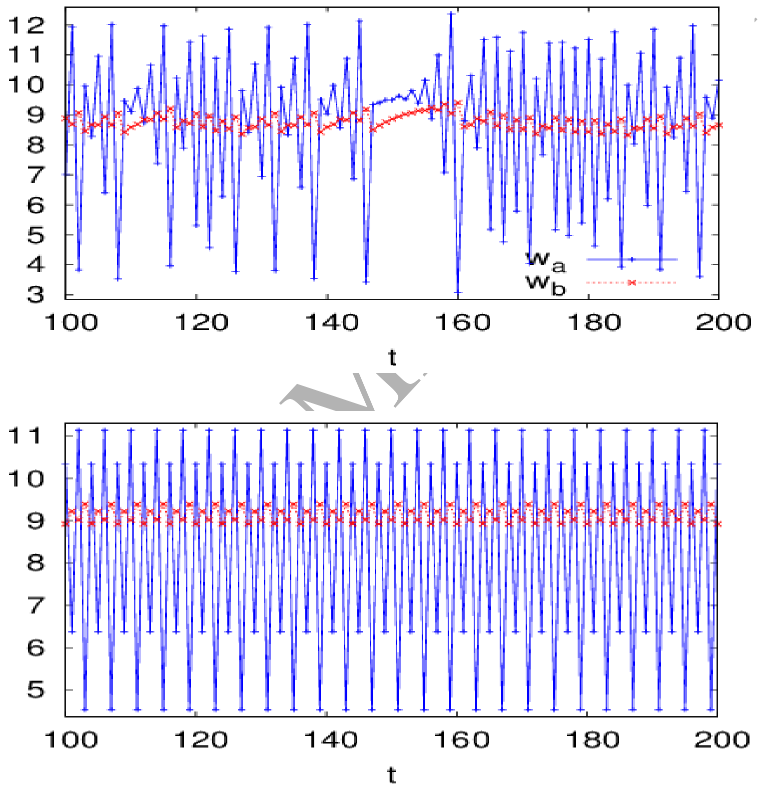


Fig.16 The wave plots of system (9) with change of time when $k_1 = 2.2,\ k_2 = 0.2$

5 Chaos control

In a complex and dynamic supply chain, the market equilibrium state is very short. Many factors in the system (such as the manufacturer's price adjustment speed, the customer's risk attitude to the probability product, the discount factor, etc.) will make the market deviate the equilibrium state, and become unstable and even chaos. Once the market is in a chaos state, the market will sensitively depend on the parameter's values, and parameter changes will lead the market's long-term trajectory unpredictable.

Because the chaos behaviors in market are harmful for the participant' profits, based on the system operation mechanism, certain control methods must be adopted to suppress or eliminate market chaotic behavior that apt to happen in the system.

In this section, we will use nonlinear feedback control method to establish a control model of the system (9) [28]. Assume the original system (9) is $w_a(t+1) = f_1(w_a(t),\ w_b(t))$, $w_b(t+1) = f_2(w_a(t),\ w_b(t))$, the control part is: $\mu(\alpha,\ x_n) = \alpha\sum_{i=1}^{p}(x_n - x_i)$, where α is feedback control coefficient, p is the number of unstable periodic orbit, $\{x_i\}$ is the instability of the fixed point or unstable periodic orbit. The controlled system can be represented as follows:

$$\begin{cases} w_a(t+1) = f_1(w_a(t),\ w_b(t)) + \alpha\sum_{i=1}^{p}(x_n - x_i) \\ w_b(t+1) = f_2(w_a(t),\ w_b(t)) \end{cases} \tag{14}$$

α can make the system stable at any periodic orbit. According to the values: $v = 110, c = 4, \varphi = 0.4, \beta = 0.8, \rho = 0.95$, the system (9) is in chaotic state when $k_1 = 2.4$, $k_2 = 2$. Here, we will make the system (14) stable in the 2-period point, namely $w_a = 6.673$ and 11.09. So the controlled system (14) can be represented as follows:

$$\begin{cases} w_a(t+1) = f_a(w_a(t),\ w_b(t)) + \alpha(w_a(\mathrm{t}) - 6.673)(w_a(\mathrm{t}) - 11.09) \\ w_b(t+1) = f_b(w_a(t),\ w_b(t)) \end{cases} \tag{15}$$

Figure 17 is the bifurcation diagram and the largest Lyapunov index of the controlled system (15), we can obtained that the controlled system (15) is stable in 2-period points when $\alpha \in [0.0913,\ 0.1782]$, the system (15) has a very good control effect under the nonlinear feedback control.

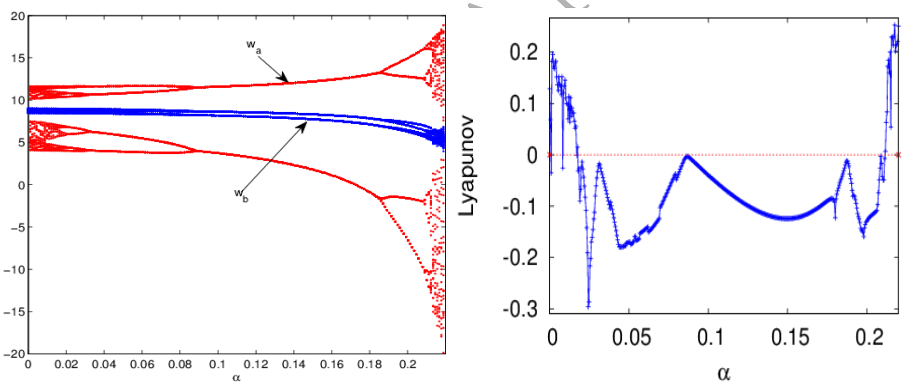


Fig.17 Bifurcation diagram and corresponding Lyapunov exponents of system (15) with the change of α when $k_1 = 2.4,\ k_2 = 0.2$

6 Conclusions

In this paper, we consider that two manufacturers sell two substitute products through the common retailer using the traditional selling channel and probabilistic selling channel. Based on the limited rational expectations, a dynamic price Stackelberg game model considering the customer has risk aversion behavior for probabilistic product is developed. The phenomenon of flip bifurcation, chaos, and other complex phenomena is reported using bifurcation, attractor and power spectrum etc. The influences of parameters on the stability of the system are further analyzed. The results show that the stability of the system could be robust with increase in customer risk aversion, and weaken with increase in price discount. When the chaos occurs, the market becomes abnormal, irregular and unpredictable. So each manufacturer should consider the market reaction of competitors and retailers in order to ensure the up and downstream enterprises keep stable and the stability of the supply chain. The nonlinear feedback control method is used to control the system. The derived results have very important theoretical and practical values for two manufacturers and the common retailer.

The research of complexity of the dynamic price Stackelberg system has theoretical and practical significance. According to the conclusion of this manuscript, the supply chain managers can do the correct and efficient decision. Nonetheless, we have made several assumptions in this paper. Loosing these assumptions may allow us to understand the interactive dynamics of the model better. For instance, the risk sensitivity of decision makers will be considered, and the model will close to the actual situation. Second, other channel structure between the two manufacturers and the common retailer should be taken into account, as it may shed lights on whether the current results will hold. We believe that the ideas and the model presented in this paper will lay the motivational ground for future research in these directions.

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Conflicts of interest

The author(s) declare(s) that there is no conflict of interests regarding the publication of this article.

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