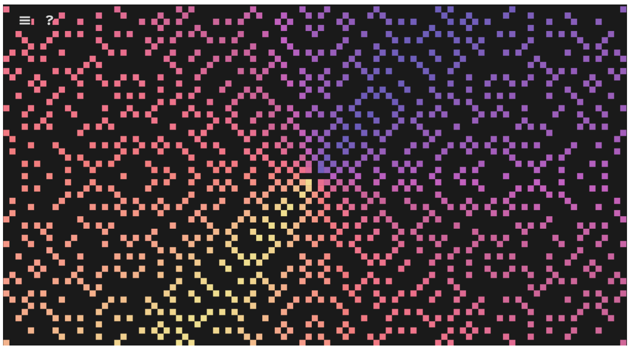
Gaussian Primes

10 seconds, 256 megabytes

Gaussian Primes (Complex Primes) are Gaussian integers $z = a+bi$ (a and b are integers, $i = \sqrt{-1}$) satisfying one of the following properties.
1. If both a and b are nonzero then, $a + bi$ is a Gaussian prime if and only if $a^2 + b^2$ is an ordinary prime.
2. If $a = 0$, then bi is a Gaussian prime if and only if $|b|$ is an ordinary prime and $|b| \equiv 3 \pmod 4$.
3. If $b = 0$, then a is a Gaussian prime if and only if $|a|$ is an ordinary prime and $|a| \equiv 3 \pmod 4$.
Such examples of Gaussian Primes are $1 + i$ and $2 + i$. (**Credit:** Gaussian Prime)



Quadratic Primes Visualizer

Or a simpler definition from Gaussian Primes Visually: firstly, a **prime** is usually defined as an positive integer greater than 1 only divisible by 1 and itself.
For the reason of, if 1 was a prime, "Unique Factorization" would be broken.
If we extend the idea of primes across all integers, we can get that a prime number (extended to the integers) is only divisible by units (1 and -1 which divides every number) and its associates (Numbers separated by a unit such as $2 \times -1 = -2$.)
Following the same concepts, we can extend that to complex integers.
Getting that Gaussian Primes are a subset of Gaussian Integers $z = a + bi$ which can be found using an altered version of **Sieve of Eratosthenes**: starting with all of the Gaussian Integers other than 0 and the units.
Find the smallest circle around 0 (in a 2D complex plane) getting $1 + i$ and its associates.
Remove their multiples; repeating that once again, we get $2 + i$, $2 - i$ and their associates.
Repeat until we get all Gaussian primes.
Getting that each has a radius which is prime (Property 2 or 3) or a square root (Property 1) of a prime. And every prime appears in in a radius.
Now we can get properties 2 and 3 using some number theory covered in Eisenstein Primes Visually.

Following the definition:
Find the **Gaussian Primes** that is closer or equal in length to the origin $(0, 0)$ (again on a 2D plane) than N which is a non-negative integer.
In order of length to the origin from least to most, but if the length to the origin is equal, order them buy the real part first from least to most and then order the imaginary part from least to most.

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Input

1 Line:
First Line: An integer N: the max distance to the origin that is allowed. $(0 \leq N \leq 1000)$

Output

For M (number of answers) lines:
In order of length to the origin from least to most, but if the length to the origin is equal, order them buy the real part first from least to most and then order the imaginary part from least to most.
With a new line every new answer and a blank line sectioning every new group (that has equal length to the origin).
Ith $(1 \leq I \leq M)$ Line: A Gaussian integer Z_I: a Gaussian Prime that is closer or equal in length to the origin than N.

Input and Output Examples

Input Examples	Output Examples
2	-1 - i -1 + i 1 - i 1 + i
5	-1 - i -1 + i 1 - i 1 + i -2 - i -2 + i -1 + -2i -1 + 2i 1 + -2i 1 + 2i 2 - i 2 + i -3 -3i 3i 3 -3 + -2i -3 + 2i -2 + -3i -2 + 3i 2 + -3i 2 + 3i 3 + -2i 3 + 2i -4 - i -4 + i -1 + -4i -1 + 4i 1 + -4i 1 + 4i 4 - i 4 + i