Control Design for Nonlinear Flexible Wings of a Robotic Aircraft

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***Abstract*—In this paper, the control problem for flexible wings of a robotic aircraft is addressed by using boundary control schemes. Inspired by birds and bats, the wing with flexibility and articulation is modeled as a distributed parameter system described by hybrid partial differential equations and ordinary differential equations. Boundary control for both wing twist and bending is proposed on the original coupled dynamics, and bounded stability is proved by introducing a proper Lyapunov function. The effectiveness of the proposed control is verified by simulations.**

***Index Terms*—Boundary Control, Distributed Parameter Systems, Partial Differential Equations (PDEs), Flexible Wings, Microaerial Vehicles (MAVs)**

I. Introduction

With the increasing focus on the development of aerospace engineering, the investigations for microaerial vehicles (MAVs) which belong to a class of unmanned aerial vehicles (UAVs), have gained much attention. MAVs can be used for inspection, surveillance and reconnaissance in some constrained environments, which are more suitable for the unmanned aircraft. Earlier research focuses on the fixed-rigid wings of MAVs. However, driven by military and civilian applications, designers strive to reduce weight and increase maneuverability of the system, thereby light-weight, flexible wings are widely used in modern aircraft. Different from the rigid ones [1], [2], the wings with flexibility are believed to offer major advantages such as cost-effective, fight agility, lower price, and high performance. Since the flexible property will lead to changes of the intended aerodynamics of the wings, thus potentially resulting in undesired aircraft responses [3], [4]. Consequently, there is a need to analyze the vibration control problem of the flexible wing which is described by hybrid PDEs-ODEs. In recent years, increasing attentions have been given on control design of various distributed parameter systems [5], [6], [7], [8].

Over the last decades, great progress has been witnessed in MAVs, and control is becoming an integral part. Due to the actuator and sensor limitations, boundary control is considered to be a more practical method in various flexible-structure systems [9], [10], [11]. In [4], elegant boundary control is designed for a robotic aircraft with the flexible wings, where the effectiveness of the proposed control is verified by both simulations and experiments. In [12], a novel robust controller is designed based on minimax LQG control for the control of vibrations in a flexible cantilever beam. In [13], an interdisciplinary project is discussed to set up a prototype flexible robotic link. By using a robust adaptive boundary control, an axially moving string with nonlinear behavior is discussed in [14]. In [15], by using hydraulic actuators at the top end, a three-dimensional marine riser system is stabilized by boundary controllers. By using the Laplace transform to derive the exact solution of the wave equation, boundary impedance control for a string system is investigated in [16]. A control scheme is proposed for an axially moving membrane system track a desired axial transport velocity in [17], where it is shown that transverse vibrations can also be suppressed with the proposed control. In [18], three control inputs including one control force and two control torques at boundaries are proposed for an axially moving string system to regulate both the longitudinal and transverse vibrations with velocity tracking.

In this paper, the deformation of the flexible wing has two elastic degrees of freedom: twist and bending. The structure dynamics of the flexible wing are coupled [19], [20], shown in Section 2. The system is loaded transversely with a spatiotemporally varying distributed loadings. Due to the effects of external loads, the governing equations of the system are described as two coupled nonhomogeneous PDEs with the unknown disturbance terms. The control and control-related issues are presented through theoretical analysis and simulations. Boundary control laws are proposed on the basis of the original distributed parameter system to control the deformation of the flexible wing. The exact values of external loads are not required. With the proposed control, the closed-loop system is uniformly bounded via the Lyapunov's direct method. The control performance of the system is guaranteed by suitably tuning the control parameters.

The rest of the paper is organized as follows. In Section II, the dynamic model of the flexible wing and some lemmas are given for the subsequent development. Based on the Lyapunov's direct method, boundary control schemes are proposed to control the deformation of the wing in Section III, where it is shown that the uniform boundedness of the closed-loop system can be achieved by the proposed control. Simulations are carried out to illustrate the performance of the proposed control in Section IV. The conclusion of this paper is presented in Section V.

II. Problem Formulation

Remark 1: For clarity, notations $(\cdot)' = \partial(\cdot)/\partial x$ and $\dot{(\cdot)} = \partial(\cdot)/\partial t$ are used throughout this paper.

Let $w(x,t)$ and $\theta(x,t)$ denote the bending and twist displacements, respectively. Consider the following structure dynamics of the right wing [4] with the governing equations

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as

$$m\ddot{w}(x,t) - mx_e c\ddot{\theta}(x,t) + \eta EI_b \dot{w}''''(x,t) + EI_b w''''(x,t) = F_b(x,t), \quad (1)$$
$$I_p\ddot{\theta}(x,t) - mx_e c\ddot{w}(x,t) - \eta GJ\dot{\theta}''(x,t) - GJ\theta''(x,t) = -x_a c F_b(x,t), \quad (2)$$

$\forall (x,t) \in (0,L)\times[0,\infty)$, and the boundary conditions for a tip-based actuation as

$$w(0,t) = w'(0,t) = w''(L,t) = \theta(0,t) = 0, \quad (3)$$
$$EI_b w'''(L,t) = F(t), \quad (4)$$
$$GJ\theta'(L,t) = M(t), \quad (5)$$

$\forall t \in [0,\infty)$, where L is the length of the wing, m is the mass per unit span, I_p is the polar moment of inertia of the wing cross section, EI_b is the bending stiffness, GJ is torsion stiffness, $x_e c$ denotes the distance between the center of mass and the shear center of the wing, and $x_a c$ denotes the distance between the aerodynamic center and shear center, the term η denotes the Kelvin-Voigt damping coefficient, $F(t)$ and $M(t)$ are the applied tip force and twisting moment, respectively. The wing is loaded transversely with a spatiotemporally varying distributed load $F_b(x,t)$.

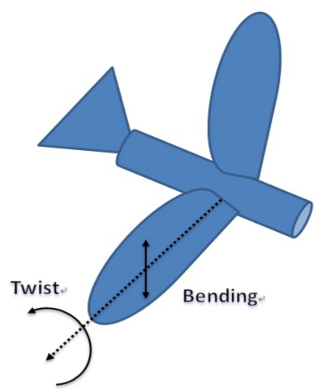

Fig. 1. Flexible wings of a robotic aircraft.

Remark 2: In order to ensure the well-posedness of the PDE, the boundary condition $EI_b\xi_{yyy} = F(t)$ in (4) is simplified from $EI_b(\xi_{yyy} + \eta\xi_{tyyy}) = F(t)$ when the Kelvin-Voigt damping coefficient $\eta \neq 0$, which essentially creates a low-pass filter for the control input F(t)[21]. As long as the dominant frequencies in F(t) are smaller than the cutoff frequency $1/\eta$, the boundary condition in (4) is sufficiently accurate for control design. What's more, the Kelvin-Voigt damping term is physically akin to distributed load [M(t) in (5)] on the wing, which does not affect the boundary conditions [22].

Assumption 1: For the unknown spatiotemporally varying distributed load $F_b(x,t)$, we assume that there exists constant $F_{b\max} \in \mathbb{R}^+$ such that $|F_b(x,t)| \le F_{b\max}$, $\forall (x,t) \in [0,L]\times[0,\infty)$. This is a reasonable assumption as the spatiotemporally varying load $F_b(x,t)$ has finite energy and thus is bounded, i.e., $F_b(x,t) \in \mathcal{L}_\infty$.

Lemma 1: [23], [24], [25] Let $\phi(x,t) \in \mathbb{R}$ be a function defined on $x \in [0,L]$ and $t \in [0,\infty)$ that satisfies the boundary condition

$$\phi(0,t) = 0, \quad \forall t \in [0,\infty), \quad (6)$$

then the following inequalities hold

$$\phi^2(x,t) \le L\int_0^L [\phi'(x,t)]^2 dx, \quad \forall x \in [0,L]. \quad (7)$$
$$\int_0^L \phi^2(x,t)dx \le L^2\int_0^L [\phi'(x,t)]^2 dx, \quad \forall x \in [0,L]. \quad (8)$$

If in addition to Eq. (6), the function $\phi(x,t)$ satisfies the boundary condition

$$\phi'(0,t) = 0, \quad \forall t \in [0,\infty), \quad (9)$$

then the following inequality also holds

$$\int_0^L [\phi'(x,t)]^2 dx \le L^2\int_0^L [\phi'']^2 dx, \quad \forall x \in [0,L]. \quad (10)$$

III. CONTROL DESIGN

In this section, boundary control $F(t)$ and $M(t)$ are designed to suppress the bending and twist displacements $w(x,t)$ and $\theta(x,t)$ of the flexible wing system, i.e. , to ensure both w(x,t), $\theta(x,t)$ are bounded by a constant, respectively. Firstly, we give the following definition [26].

Definition 1: Let $f(t) : R^n \to R^m$ be a continuous function or piecewise continuous function. The p-norm of f is defined by

$$\|f\|_p = \left(\int_0^\infty |f(t)|^p dt\right)^{1/p}, \quad p \in [0,\infty) \quad (11)$$
$$\|f\|_\infty = \sup_{t\in[0,\infty)} |f(t)|, \quad p = \infty \quad (12)$$

By letting $p = \infty$, the corresponding spaces are called L_∞. More precisely, let $f(t)$ be a function on $[0,\infty)$ of the signal spaces, they are defined as

$$\mathcal{L}_\infty \triangleq \left\{ f : R_+ \to R, \|f\|_\infty = \sup_{t\in[0,\infty)} |f| < \infty \right\} \quad (13)$$

From a signal point of view, the ∞-norm is its absolute maximum amplitude or peak value. The definition of the norms for vector functions are not unique [27].

Consider the Lyapunov function candidate as

$$V(t) = V_1(t) + \Delta(t), \quad (14)$$

where $V_1(t)$ and $\Delta(t)$ are defined as

$$V_1(t) = \frac{\beta}{2}m\int_0^L [\dot{w}(x,t)]^2 dx + \frac{\beta}{2}EI_b\int_0^L [w''(x,t)]^2 dx + \frac{\beta}{2}I_p\int_0^L [\dot{\theta}(x,t)]^2 dx + \frac{\beta}{2}GJ\int_0^L [\theta'(x,t)]^2 dx, \quad (15)$$
$$\Delta(t) = \alpha m\int_0^L \dot{w}(x,t)w(x,t)dx + \alpha I_p\int_0^L \dot{\theta}(x,t)\theta(x,t)dx - \alpha m x_e c\int_0^L \left[\dot{w}(x,t)\theta(x,t) + w(x,t)\dot{\theta}(x,t)\right]dx - \beta m x_e c\int_0^L \dot{w}(x,t)\dot{\theta}(x,t)dx, \quad (16)$$

where α and β are two small positive weighting constants.

Define a new function as

$$\kappa(t) = \int_0^L \{[\dot{w}(x,t)]^2 + [\dot{\theta}(x,t)]^2 + [w''(x,t)]^2 + [\theta'(x,t)]^2\}dx, \quad (17)$$

then we obtain the upper and lower bound of $V_1(t)$ as

$$\gamma_2\kappa(t) \le V_1(t) \le \gamma_1\kappa(t), \quad (18)$$

where $\gamma_1 = \frac{\beta}{2}\max(m, I_p, EI_b, GJ)$,
and $\gamma_2 = \frac{\beta}{2}\min(m, I_p, EI_b, GJ)$.

Applying inequalities (8) and (10), we obtain

$$\begin{aligned}|\Delta(t)| \le\ & (\alpha m + \alpha m x_e c + \beta m x_e c)\int_0^L [\dot{w}(x,t)]^2 dx \\ & + (\alpha I_p + \alpha m x_e c + \beta m x_e c)\int_0^L [\dot{\theta}(x,t)]^2 dx \\ & + (\alpha m + \alpha m x_e c) L^4 \int_0^L [w''(x,t)]^2 dx \\ & + (\alpha I_p + \alpha m x_e c) L^2 \int_0^L [\theta'(x,t)]^2 dx \\ \le\ & \gamma_3\kappa(t) \end{aligned} \quad (19)$$

where

$$\begin{aligned}\gamma_3 = \max\{& \alpha m + \alpha m x_e c + \beta m x_e c, \alpha I_p + \alpha m x_e c + \beta m x_e c \\ & (\alpha m + \alpha m x_e c) L^4, (\alpha I_p + \alpha m x_e c) L^2\}\end{aligned} \quad (20)$$

Consider a positive β satisfying that $\beta > \frac{2\gamma_3}{\min(m, I_p, EI_b, GJ)}$, we have

$$0 \le \lambda_2\kappa(t) \le V(t) \le \lambda_1\kappa(t), \quad (21)$$

where $\lambda_2 = \gamma_2 - \gamma_3$ and $\lambda_1 = \gamma_1 + \gamma_3$.

Differentiating $V(t)$ leads to

$$\dot{V}(t) = \dot{V}_1(t) + \dot{\Delta}(t), \quad (22)$$

where $\dot{V}_1(t)$ is given as

$$\begin{aligned}\dot{V}_1(t) =\ & \beta m \int_0^L \dot{w}(x,t)\ddot{w}(x,t)dx \\ & + \beta I_p \int_0^L \dot{\theta}(x,t)\ddot{\theta}(x,t)dx \\ & + \beta GJ \int_0^L \theta'(x,t)\dot{\theta}'(x,t)dx \\ & + \beta EI_b \int_0^L w''(x,t)\dot{w}''(x,t)dx. \end{aligned} \quad (23)$$

Substituting the governing equations (1) and (2), we obtain

$$\dot{V}_1(t) = A_1 + A_2 + ... + A_6, \quad (24)$$

where

$$\begin{aligned}A_1 =\ & -\beta EI_b \int_0^L \dot{w}(x,t)w''''(x,t)dx \\ & + \beta EI_b \int_0^L w''(x,t)\dot{w}''(x,t)dx, \end{aligned} \quad (25)$$

$$A_2 = -\beta\eta EI_b \int_0^L \dot{w}(x,t)\dot{w}''''(x,t)dx, \quad (26)$$

$$A_3 = \beta m x_e c \int_0^L [\dot{w}(x,t)\ddot{\theta}(x,t)l + \ddot{w}(x,t)\dot{\theta}(x,t)dx]dx, \quad (27)$$

$$\begin{aligned}A_4 =\ & \beta \int_0^L \dot{w}(x,t)F_b(x,t)dx \\ & - \beta x_a c \int_0^L \dot{\theta}(x,t)F_b(x,t)dx, \end{aligned} \quad (28)$$

$$\begin{aligned}A_5 =\ & \beta GJ \int_0^L \dot{\theta}(x,t)\theta''(x,t)dx, \quad (29) \\ & + \beta GJ \int_0^L \theta'(x,t)\dot{\theta}'(x,t)dx, \end{aligned}$$

$$A_6 = \beta\eta GJ \int_0^L \dot{\theta}(x,t)\dot{\theta}''(x,t)dx, \quad (30)$$

Using integration by parts and boundary conditions (3) and (4), we have

$$A_1 = -\beta EI_b\dot{w}(L,t)w'''(L,t) = -\beta\dot{w}(L,t)F(t), \quad (31)$$

From the boundary condition (3), we can obtain that $\dot{w}(0,t) = \dot{w}'(0,t) = 0$, then applying inequalities (8) and (10), we obtain

$$\begin{aligned}A_2 \le\ & -\beta\eta\dot{w}(L,t)\dot{F}(t) - \frac{\beta\eta EI_b}{2L^4}\int_0^L [\dot{w}(x,t)]^2 dx \\ & - \frac{\beta\eta EI_b}{2}\int_0^L [\dot{w}''(x,t)]^2 dx, \end{aligned} \quad (32)$$

We can further obtain

$$\begin{aligned}A_4 \le\ & \sigma_1\beta\int_0^L [\dot{w}(x,t)]^2 dx + \sigma_2\beta x_a c\int_0^L [\dot{\theta}(x,t)]^2 dx \\ & + \left(\frac{\beta}{\sigma_1} + \frac{\beta x_a c}{\sigma_2}\right) L F_{b\max}^2, \end{aligned} \quad (33)$$

where σ_1 and σ_2 are positive constants. Similarly, we have

$$A_5 = \beta GJ\dot{\theta}(L,t)\theta'(L,t) = \beta\dot{\theta}(L,t)M(t), \quad (34)$$

$$\begin{aligned}A_6 \le\ & \beta\eta\dot{\theta}(L,t)\dot{M}(t) - \frac{\beta\eta GJ}{2L^2}\int_0^L [\dot{\theta}(x,t)]^2 dx \\ & - \frac{\beta\eta GJ}{2}\int_0^L [\dot{\theta}'(x,t)]^2 dx. \end{aligned} \quad (35)$$

Combining A_1 - A_6, we obtain the derivative of $V_1(t)$ as

$$\begin{aligned}\dot{V}_1(t) \le\ & -\left(\frac{\beta\eta EI_b}{2L^4} - \sigma_1\beta\right)\int_0^L [\dot{w}(x,t)]^2 dx \\ & - \left(\frac{\beta\eta GJ}{2L^2} - \sigma_2\beta x_a c\right)\int_0^L [\dot{\theta}(x,t)]^2 dx \\ & - \frac{\beta\eta EI_b}{2}\int_0^L [\dot{w}''(x,t)]^2 dx \\ & - \frac{\beta\eta GJ}{2}\int_0^L [\dot{\theta}'(x,t)]^2 dx \\ & + \beta m x_e c\int_0^L \left[\dot{w}(x,t)\ddot{\theta}(x,t) + \ddot{w}(x,t)\dot{\theta}(x,t)\right] dx \\ & - \beta\dot{w}(L,t)\left[F(t) + \eta\dot{F}(t)\right] \\ & + \beta\dot{\theta}(L,t)\left[M(t) + \eta\dot{M}(t)\right] \end{aligned}$$

$$
\begin{aligned}
&+\beta\dot{\theta}(L,t)\left[M(t)+\eta\dot{M}(t)\right]\\
&+\left(\frac{\beta}{\sigma_1}+\frac{\beta x_a c}{\sigma_2}\right)LF_{b\max}^2. \qquad (36)
\end{aligned}
$$

The derivative of $\Delta(t)$ is given as

$$
\begin{aligned}
\dot{\Delta}(t) \;=\; & \alpha m\int_0^L \ddot{w}(x,t)w(x,t)dx+\alpha m\int_0^L[\dot{w}(x,t)]^2dx\\
&+\alpha I_p\int_0^L\ddot{\theta}(x,t)\theta(x,t)dx+\alpha I_p\int_0^L[\dot{\theta}(x,t)]^2dx\\
&-\beta m x_e c\int_0^L\left[\dot{w}(x,t)\ddot{\theta}(x,t)+\ddot{w}(x,t)\dot{\theta}(x,t)\right]dx\\
&-\alpha m x_e c\int_0^L[\ddot{w}(x,t)\theta(x,t)+2\dot{w}(x,t)\dot{\theta}(x,t)\\
&+w(x,t)\ddot{\theta}(x,t)]dx. \qquad (37)
\end{aligned}
$$

Substituting the governing equations (1) and (2), we obtain

$$\dot{\Delta}(t) \;=\; B_1+B_2+...+B_8, \qquad (38)$$

where

$$
\begin{aligned}
B_1 \;&=\; -\alpha EI_b\int_0^L w(x,t)w''''(x,t)dx, && (39)\\
B_2 \;&=\; -\alpha\eta EI_b\int_0^L w(x,t)\dot{w}''''(x,t)dx, && (40)\\
B_3 \;&=\; \alpha GJ\int_0^L\theta(x,t)\theta''(x,t)dx, && (41)\\
B_4 \;&=\; \alpha\eta GJ\int_0^L\theta(x,t)\dot{\theta}''(x,t)dx, && (42)\\
B_5 \;&=\; \alpha m\int_0^L[\dot{w}(x,t)]^2dx+\alpha I_p\int_0^L[\dot{\theta}(x,t)]^2dx, && (43)\\
B_6 \;&=\; -\beta m x_e c\int_0^L\left[\dot{w}(x,t)\ddot{\theta}(x,t)+\ddot{w}(x,t)\dot{\theta}(x,t)\right]dx, && (44)\\
B_7 \;&=\; -2\alpha m x_e c\int_0^L\dot{w}(x,t)\dot{\theta}(x,t)dx, && (45)\\
B_8 \;&=\; \alpha\int_0^L w(x,t)F_b(x,t)dx && (46)\\
&\quad -\alpha x_a c\int_0^L\theta(x,t)F_b(x,t)dx. && (47)
\end{aligned}
$$

Using boundary conditions (3) and (4), we obtain

$$
\begin{aligned}
B_1 \;&=\; -\alpha w(L,t)F(t)-\alpha EI_b\int_0^L[w''(x,t)]^2dx, && (48)\\
B_2 \;&\le\; -\alpha\eta w(L,t)\dot{F}(t)+\frac{\alpha\eta EI_b}{\sigma_3}\int_0^L[w''(x,t)]^2dx\\
&\quad +\sigma_3\alpha\eta EI_b\int_0^L[\dot{w}''(x,t)]^2dx, && (49)\\
B_3 \;&=\; \alpha\theta(L,t)M(t)-\alpha GJ\int_0^L[\theta'(x,t)]^2dx && (50)\\
B_4 \;&\le\; \alpha\eta\theta(L,t)\dot{M}(t)+\frac{\alpha\eta GJ}{\sigma_4}\int_0^L[\theta'(x,t)]^2dx,\\
&\quad +\sigma_4\alpha\eta GJ\int_0^L[\dot{\theta}'(x,t)]^2dx && (51)\\
B_7 \;&\le\; 2\alpha m x_e c\sigma_5\int_0^L[\dot{w}(x,t)]^2dx\\
&\quad +\frac{2\alpha m x_e c}{\sigma_5}\int_0^L[\dot{\theta}(x,t)]^2dx && (52)
\end{aligned}
$$

Furthermore, we have

$$
\begin{aligned}
B_8 \;\le\; & \sigma_6\alpha L^4\int_0^L[w''(x,t)]^2dx+\sigma_7\alpha L^2x_ac\int_0^L[\theta'(x,t)]^2dx\\
&+\left(\frac{\alpha}{\sigma_6}+\frac{\alpha x_a c}{\sigma_7}\right)LF_{b\max}^2, \qquad (53)
\end{aligned}
$$

where σ_3 - σ_7 are positive constants.

Combining B_1 - B_8, we obtain

$$
\begin{aligned}
\dot{\Delta}(t) \;=\; & -\left(\alpha EI_b-\frac{\alpha\eta EI_b}{\sigma_3}-\sigma_6\alpha L^4\right)\int_0^L[w''(x,t)]^2dx\\
&-\left(\alpha GJ-\frac{\alpha\eta GJ}{\sigma_4}-\sigma_7\alpha L^2x_ac\right)\int_0^L[\theta'(x,t)]^2dx,\\
&+(\alpha m+2\alpha m x_e c\sigma_5)\int_0^L[\dot{w}(x,t)]^2dx\\
&+\left(\alpha I_p+\frac{2\alpha m x_e c}{\sigma_5}\right)\int_0^L[\dot{\theta}(x,t)]^2dx\\
&+\sigma_3\alpha\eta EI_b\int_0^L[\dot{w}''(x,t)]^2dx\\
&+\sigma_4\alpha\eta GJ\int_0^L[\dot{\theta}'(x,t)]^2dx\\
&-\beta m x_e c\int_0^L\left[\dot{w}(x,t)\ddot{\theta}(x,t)+\ddot{w}(x,t)\dot{\theta}(x,t)\right]dx\\
&+\alpha\theta(L,t)\left[M(t)+\eta\dot{M}(t)\right]\\
&-\alpha w(L,t)\left[F(t)+\eta\dot{F}(t)\right]\\
&+\left(\frac{\alpha}{\sigma_6}+\frac{\alpha x_a c}{\sigma_7}\right)LF_{b\max}^2. \qquad (54)
\end{aligned}
$$

Therefore, we have the derivative of the Lyapunov function candidate as

$$
\begin{aligned}
\dot{V}(t) \;\le\; & -[\alpha w(L,t)+\beta\dot{w}(L,t)]\left[F(t)+\eta\dot{F}(t)\right]\\
&+\left[\alpha\theta(L,t)+\beta\dot{\theta}(L,t)\right]\left[M(t)+\eta\dot{M}(t)\right]\\
&-\left(\alpha EI_b-\frac{\alpha\eta EI_b}{\sigma_3}-\sigma_6\alpha L^4\right)\int_0^L[w''(x,t)]^2dx\\
&-\left(\alpha GJ-\frac{\alpha\eta GJ}{\sigma_4}-\sigma_7\alpha L^2x_ac\right)\int_0^L[\theta'(x,t)]^2dx\\
&-\left(\frac{\beta\eta EI_b}{2L^4}-\sigma_1\beta-\alpha m-2\alpha m x_e c\sigma_5\right)\\
&\times\int_0^L[\dot{w}(x,t)]^2dx\\
&-\left(\frac{\beta\eta GJ}{2L^2}-\sigma_2\beta x_ac-\alpha I_p-\frac{2\alpha m x_e c}{\sigma_5}\right)\\
&\times\int_0^L[\dot{\theta}(x,t)]^2dx\\
&-\left(\frac{\beta\eta EI_b}{2}-\sigma_3\alpha\eta EI_b\right)\int_0^L[\dot{w}''(x,t)]^2dx
\end{aligned}
$$

$$-\left(\frac{\beta\eta GJ}{2}-\sigma_4\alpha\eta GJ\right)\int_0^L[\dot\theta'(x,t)]^2dx$$
$$+\left(\frac{\beta}{\sigma_1}+\frac{\beta x_a c}{\sigma_2}+\frac{\alpha L}{\sigma_6}+\frac{\alpha x_a cL}{\sigma_7}\right)LF_{b\max}^2. \quad (55)$$

Let $U(t)=F(t)+\eta\dot F(t)$ and $V(t)=M(t)+\eta\dot M(t)$ are the new control variables, and we design the control laws as

$$U(t) = k_1\left[\alpha w(L,t)+\beta\dot w(L,t)\right], \quad (56)$$
$$V(t) = -k_2\left[\alpha\theta(L,t)+\beta\dot\theta(L,t)\right], \quad (57)$$

where $k_1\geq 0$ and $k_2\geq 0$ are the control parameters. Let $\frac{\beta\eta EI_b}{2}-\sigma_3\alpha\eta EI_b\geq 0$, and $\frac{\beta\eta GJ}{2}-\sigma_4\alpha\eta GJ\geq 0$, we further have

$$\begin{aligned}\dot V(t) \leq& -\mu_1\int_0^L[\dot w(x,t)]^2dx-\mu_2\int_0^L[\dot\theta(x,t)]^2dx\\ &-\mu_3\int_0^L[w''(x,t)]^2dx-\mu_4\int_0^L[\theta'(x,t)]^2dx+\varepsilon\\ \leq& -\lambda_3\kappa(t)+\varepsilon, \end{aligned}\quad (58)$$

where

$$\mu_1 = \frac{\beta\eta EI_b}{2L^4}-\sigma_1\beta-\alpha m-2\alpha m x_e c\sigma_5>0, \quad (59)$$
$$\mu_2 = \frac{\beta\eta GJ}{2L^2}-\sigma_2\beta x_a c-\alpha I_p-\frac{2\alpha m x_e c}{\sigma_5}>0, \quad (60)$$
$$\mu_3 = \alpha EI_b-\frac{\alpha\eta EI_b}{\sigma_3}-\sigma_6\alpha L^4>0, \quad (61)$$
$$\mu_4 = \alpha GJ-\frac{\alpha\eta GJ}{\sigma_4}-\sigma_7\alpha L^2 x_a c>0, \quad (62)$$
$$\lambda_3 = \min(\mu_1,\mu_2,\mu_3,\mu_4)>0, \quad (63)$$
$$\varepsilon = \left(\frac{\beta}{\sigma_1}+\frac{\beta x_a c}{\sigma_2}+\frac{\alpha L}{\sigma_6}+\frac{\alpha x_a cL}{\sigma_7}\right)LF_{b\max}^2. \quad (64)$$

Combining (21) and (58), we have

$$\dot V(t) \leq -\lambda V(t)+\varepsilon, \quad (65)$$

where $\lambda=\lambda_3/\lambda_1$.

From the above statement, the control design for the flexible wing subjected to external loads can be summarized in the following theorem.

Theorem 1: For the dynamical system described by governing equations (1), (2) and boundary conditions (3) - (5), under Assumption 1 and the proposed boundary control (56) and (57), if the initial conditions are bounded, then the closed-loop system is uniformly bounded.

Proof: Following the similar procedures in our previous work [28], integrating of the inequality (65), we obtain

$$\begin{aligned}V(t) \leq& \left(V(0)-\frac{\varepsilon}{\lambda}\right)e^{-\lambda t}+\frac{\varepsilon}{\lambda}\\ \leq& V(0)e^{-\lambda t}+\frac{\varepsilon}{\lambda}\in\mathcal{L}_\infty,\end{aligned}\quad (66)$$

which implies $V(t)$ is bounded. Utilizing inequalities (7) and (10), we have

$$\begin{aligned}\frac{1}{L^3}w^2(x,t) \leq& \frac{1}{L^2}\int_0^L[w'(x,t)]^2dx\leq\int_0^L[w''(x,t)]^2dx\\ \leq& \kappa(t)\leq\frac{1}{\lambda_2}V(t)\in\mathcal{L}_\infty\end{aligned}\quad (67)$$

$$\frac{1}{L}\theta^2(x,t)\leq\int_0^L[\theta'(x,t)]^2dx \leq \kappa(t)\leq\frac{1}{\lambda_2}V(t)\in\mathcal{L}_\infty \quad (68)$$

Appropriately rearranging the terms of (66) - (68), we obtain that $w(x,t)$ and $\theta(x,t)$ are uniformly bounded as follows

$$|w(x,t)| \leq \sqrt{\frac{L^3}{\lambda_2}\left(V(0)e^{-\lambda t}+\frac{\varepsilon}{\lambda}\right)}, \quad (69)$$
$$|\theta(x,t)| \leq \sqrt{\frac{L}{\lambda_2}\left(V(0)e^{-\lambda t}+\frac{\varepsilon}{\lambda}\right)}. \quad (70)$$

Furthermore, as t tends to infinity, we have

$$|w(x,t)|\leq\sqrt{\frac{L^3\varepsilon}{\lambda_2\lambda}}, \quad \forall x\in[0,L], \quad (71)$$
$$|\theta(x,t)|\leq\sqrt{\frac{L\varepsilon}{\lambda_2\lambda}}, \quad \forall x\in[0,L]. \quad (72)$$

IV. Simulation

In order to present the effectiveness of the proposed control scheme, finite-difference approximation method is introduced for approximating the system described by (1) and (2) in numerical simulations. Parameters of the system are listed in Table I.

Table I: Parameters of the flexible wing of a robotic aircraft

Parameter	Description	Value
L	Length of the wing	2 m
m	Mass per unit span	10 kg/m
I_p	Polar moment of inertia of the wing cross section	1.5 kgm
EI_b	Bending stiffness	0.12 Nm^2
GJ	Torsion stiffness	0.2 Nm^2
$x_e c$	Distance between the center of mass and the shear center of wing	0.25 m
$x_a c$	Distance between the aerodynamic center and shear center	0.05 m
η	Kelvin-Voigt damping coefficient	0.05

Under the initial conditions, given as $w(x,0)=\frac{x}{L},\theta(x,0)=\frac{\pi x}{2L},\dot w(x,0)=0$ and $\dot\theta(x,0)=0$, the flexible wing is excited by the inevitable spatiotemporally varying distributed load $F_b(x,t)$, which is defined as $F_b(x,t)=[1+\sin(\pi t)+3\cos(3\pi t)]x$.

From Figs. 2 and 3, it can be seen that the bending deflection of the flexible wing $w(x,t)$ is far beyond expectation, as large as 1 m. Considering the length of the wing with only 2 m, it will result in system mechanical damage, which is unacceptable in practice. Besides the twist deflection $\theta(x,t)$ can also be driven to reach the undesired position, which is harmful to the system physical structure and should be avoided as well.

On account of the unsatisfactory system performances, the control target is to suppress the vibrations in both $w(x,t)$ and $\theta(x,t)$. With the proposed control (56) and (57), the deflections of the wing system are reduced significantly, as shown in Figs. 4 and 5, where the parameters of the controllers are chosen as: $\alpha=50$, $\beta=1$, $k_1=50$ and $k_2=1.5\times10^2$.

Under the impact of the tip force controller $F(t)$, the bending deflection $w(x,t)$ continues to decrease from the initial position, and remains in a small range around zero about 60 seconds later. Meanwhile with the twisting moment controller $M(t)$, the twist deflection $w(x,t)$ can also be restrained and regulated to remain in an acceptable small scope around zero after about 50 seconds.

To certify the effectiveness of the proposed control scheme more clearly, we focus on the states of the end-point position bending and twist deflection $w(L,t)$ and $\theta(L,t)$ when the system is operated without and with controllers implemented, shown in Figs. 6 and 7, which demonstrate that compared to the freely vibrating situation, the designed controllers can make the flexible wing system perform well, free from the influence of the disturbance $F_b(x,t)$ and the vibration along with it as the deflections can be controlled in small limited extents. And the control signal is depicted in Fig. 8.

In conclusion, the proposed control scheme is effective and feasible in dealing with the vibration problems existing in the system, and the stability of the closed-loop system can also be ensured.

V. Conclusion

This paper has presented the control design and stability analysis for flexible wings for a robotic aircraft. The flexible wing is modeled as a coupled twist-bending system with the unknown spatiotemporally varying distributed load. Boundary control schemes have been proposed to control the deformation of the wing. Based on the Lyapunov direct method, the stability has been proved. Numerical examples have illustrated the performance of the control system.

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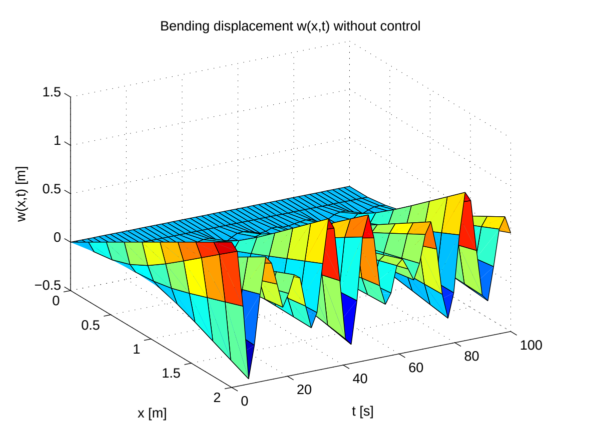

Fig. 2. Bending displacement of the system $w(x,t)$ without control

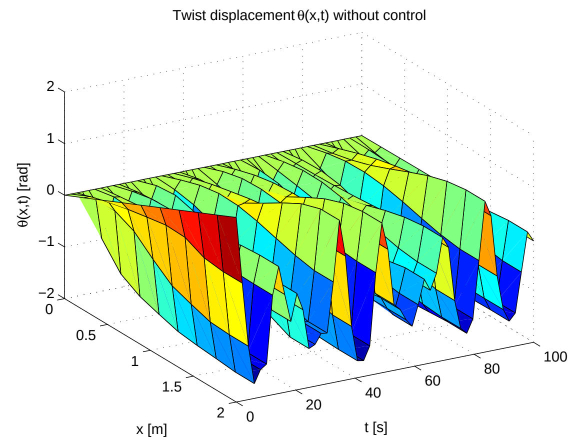

Fig. 3. Twist displacement of the system $\theta(x,t)$ without control

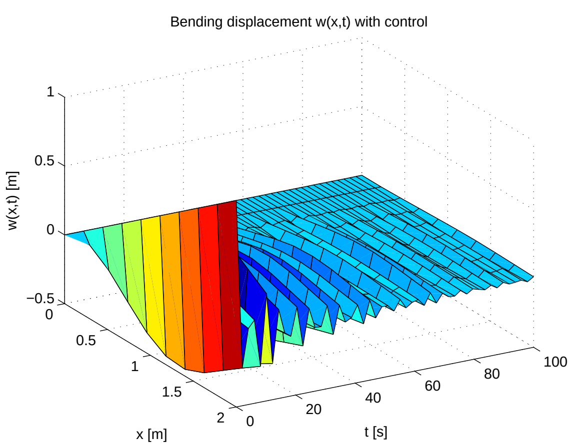

Fig. 4. Bending displacement of the system $w(x,t)$ with control

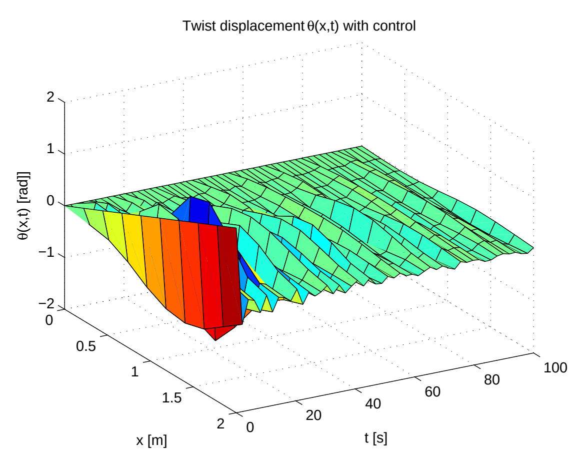

Fig. 5. Twist displacement of the system $w(x,t)$ with control

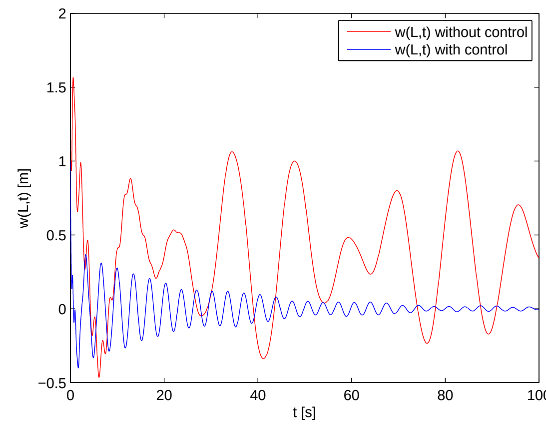

Fig. 6. End point bending displacement of the system $w(L,t)$

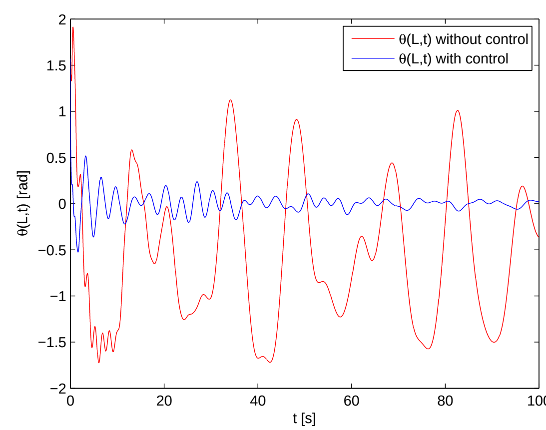

Fig. 7. End point twist displacement of the system $\theta(L,t)$

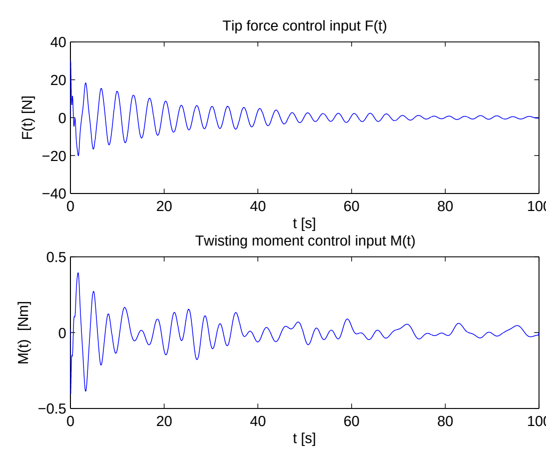

Fig. 8. Control input