1 Modeling

A rigid-flexible wing is shown in Figure.1. The wing is consisted of a rigid link and a flexible link. And the controller are installed at the joints of wing to suppress the vibration of the flexible link and to drive the links to the desired angular positions.

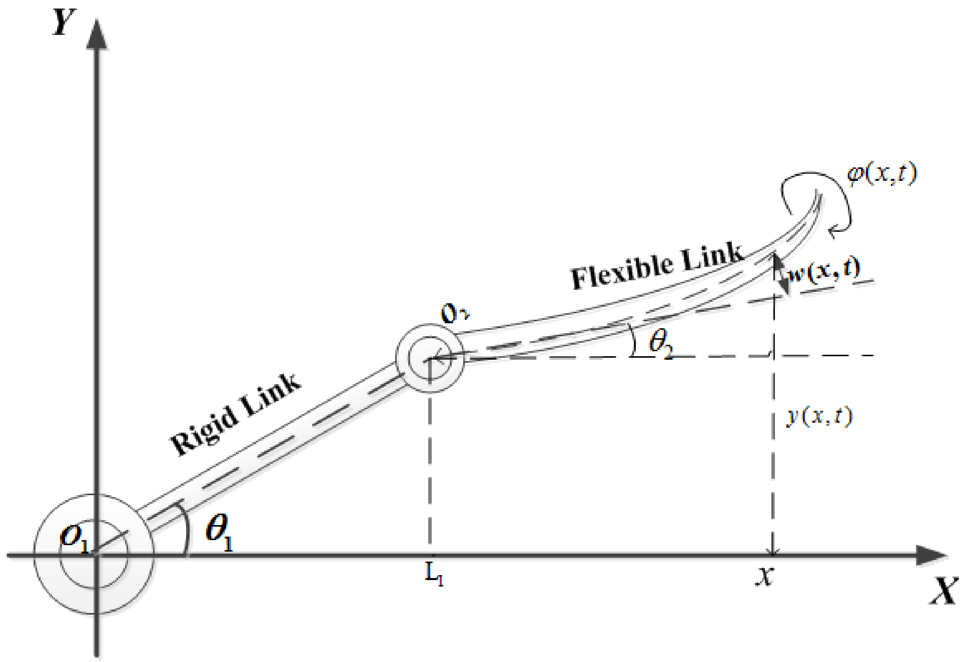

Figure 1: Rigid-Flexible Wing

$$
\begin{aligned}
y(x,t) &= w(x,t) + (x - L_1)\tan\theta_2(t) + L_1 \tan\theta_1(t) \\
&= w(x,t) + (x - L_1)\theta_2(t) + L_1\theta_1(t) \\
&= w(x,t) + x\theta_2(t) + L_1[\theta_1(t) - \theta_2(t)]
\end{aligned}
$$

1.1 Rigid Link

The kinetic energy of the rigid link $E_R(t)$ is

$$
\begin{aligned}
E_R(t) &= \frac{1}{2} I_R \left[\dot{\theta}_1(t)\right]^2 + \frac{1}{2} m_P \left[\dot{y}(L_1,t)\right]^2 \\
&= \frac{1}{2}(I_R + m_P L_1^2)\left[\dot{\theta}_1(t)\right]^2
\end{aligned}
$$

According to the Lagrangian form

$$\begin{aligned}\frac{d}{dt}\Big[\frac{\partial E_R(t)}{\partial \dot{\theta}_1(t)}\Big]-\frac{\partial E_R(t)}{\partial \theta_1(t)} &= F_{u1}(t)\\(I_R+m_P L_1^2)\ddot{\theta}_1(t) &= F_{u1}(t)\end{aligned}$$

1.2 Flexible Link

The kinetic energy of the flexible link $E_k(t)$ is

$$E_k(t)=\frac{1}{2}\rho\int_{L_1}^{L}\big[\dot{y}(x,t)\big]^2dx+\frac{1}{2}I_b\int_{L_1}^{L}\big[\dot{\varphi}(x,t)\big]^2dx+\frac{1}{2}I_h\big[\dot{\theta}_2(t)\big]^2$$

The potential energy of the flexible link $E_p(t)$ is

$$E_p(t)=\frac{1}{2}EI_b\int_{L_1}^{L}\big[w''(x,t)\big]^2dx+\frac{1}{2}GJ\int_{L_1}^{L}\big[\varphi'(x,t)\big]^2dx$$

The virtual work done by the bending and torsion rigidity coupling $\delta W_c(t)$ is

$$\delta W_c(t)=\rho x_e c\int_{L_1}^{L}\ddot{w}(x,t)\delta\varphi(x,t)dx+\rho x_e c\int_{L_1}^{L}\ddot{\varphi}(x,t)\delta w(x,t)dx$$

The virtual work done by the Kelvin-Voigt damping on the flexible link $\delta W_d(t)$ is

$$\delta W_d(t)=-\eta EI_b\int_{L_1}^{L}\dot{w}''(x,t)\delta w''(x,t)dx-\eta GJ\int_{L_1}^{L}\dot{\varphi}'(x,t)\delta\varphi'(x,t)dx$$

The virtual work done by the distributed disturbance $\delta W_f(t)$ is

$$\delta W_f(t)=\int_{L_1}^{L}F_b(x,t)\delta y(x,t)dx-x_a c\int_{L_1}^{L}F_b(x,t)\delta\varphi(x,t)dx$$

The virtual work done by the control force $\delta W_{u2}(t)$ is

$$\delta W_{u2}(t)=F_{u2}(t)\delta\theta_2(t)+M_{u2}(t)\delta\varphi(L_1,t)$$

The total virtual work $\delta W(t)$ is

$$\delta W(t) = \delta\big[W_c(t) + W_d(t) + W_f(t) + W_{u2}(t)\big]$$

Integrating $E_k(t)$ with respect to t yield

$$\begin{aligned}\int_{t_1}^{t_2} \delta E_k(t)dt &= \rho\int_{t_1}^{t_2}\int_{L_1}^{L} \dot{y}(x,t)\delta\dot{y}(x,t)dxdt + I_b\int_{t_1}^{t_2}\int_{L_1}^{L}\dot{\varphi}(x,t)\delta\dot{\varphi}(x,t)dxdt + I_h\int_{t_1}^{t_2}\dot{\theta}_2(t)\delta\dot{\theta}_2(t)dt \\ &= -\rho\int_{t_1}^{t_2}\int_{L_1}^{L} \ddot{y}(x,t)\delta y(x,t)dxdt - I_b\int_{t_1}^{t_2}\int_{L_1}^{L}\ddot{\varphi}(x,t)\delta\varphi(x,t)dxdt - I_h\int_{t_1}^{t_2}\ddot{\theta}_2(t)\delta\theta_2(t)dt\end{aligned}$$

We can also obtain the variation about the potential energy

$$\begin{aligned}\int_{t_1}^{t_2} \delta E_p(t)dt &= EI_b\int_{t_1}^{t_2}\int_{L_1}^{L} w''(x,t)\delta w''(x,t)dxdt + GJ\int_{t_1}^{t_2}\int_{L_1}^{L}\varphi'(x,t)\delta\varphi'(x,t)dxdt \\ &= EI_b\int_{t_1}^{t_2}\big[w''(x,t)\delta w'(x,t)\big]\Big|_{L_1}^{L}dt - EI_b\int_{t_1}^{t_2}\big[w'''(x,t)\delta w(x,t)\big]\Big|_{L_1}^{L}dt \\ &\quad + EI_b\int_{t_1}^{t_2}\int_{L_1}^{L} w''''(x,t)\delta w(x,t)dxdt + GJ\int_{t_1}^{t_2}\big[\varphi'(x,t)\delta\varphi(x,t)\big]\Big|_{L_1}^{L}dt \\ &\quad - GJ\int_{t_1}^{t_2}\int_{L_1}^{L}\varphi''(x,t)\delta\varphi(x,t)dxdt\end{aligned}$$

The variation about stiffness coupling

$$\int_{t_1}^{t_2} \delta W_c(t)dt = \rho x_e c\int_{t_1}^{t_2}\int_{L_1}^{L}[\ddot{w}(x,t)\delta\varphi(x,t) + \ddot{\varphi}(x,t)\delta w(x,t)]dxdt$$

The variation of Kelvin-Voigt damping on the wing

$$\begin{aligned}\int_{t_1}^{t_2} \delta W_d(t)dt &= -\eta EI_b\int_{t_1}^{t_2}\int_{L_1}^{L} \dot{w}''(x,t)\delta w''(x,t)dxdt - \eta GJ\int_{t_1}^{t_2}\int_{L_1}^{L}\dot{\varphi}'(x,t)\delta\varphi'(x,t)dxdt \\ &= -\eta EI_b\int_{t_1}^{t_2}\big[\dot{w}''(x,t)\delta w'(x,t)\big]\Big|_{L_1}^{L}dt + \eta EI_b\int_{t_1}^{t_2}\big[\dot{w}'''(x,t)\delta w(x,t)\big]\Big|_{L_1}^{L}dt \\ &\quad - \eta EI_b\int_{t_1}^{t_2}\int_{L_1}^{L} \dot{w}''''(x,t)\delta w(x,t)dxdt - \eta GJ\int_{t_1}^{t_2}\big[\dot{\varphi}'(x,t)\delta\varphi(x,t)\big]\Big|_{L_1}^{L}dt \\ &\quad + \eta GJ\int_{t_1}^{t_2}\int_{L_1}^{L}\dot{\varphi}''(x,t)\delta\varphi(x,t)dxdt\end{aligned}$$

The variation about distributed disturbance and control force

$$\begin{aligned}\int_{t_1}^{t_2} \delta W_f(t)dt &= \int_{t_1}^{t_2}\int_{L_1}^{L} F_b(x,t)\delta y(x,t)dxdt - x_a c\int_{t_1}^{t_2}\int_{L_1}^{L} F_b(x,t)\delta\varphi(x,t)dxdt \\ \int_{t_1}^{t_2} \delta W_{u2}(t)dt &= \int_{t_1}^{t_2} F_{u2}(t)\delta\theta_2(t)dt + \int_{t_1}^{t_2} M_{u2}(t)\delta\varphi(L_1,t)dt\end{aligned}$$

Using the Hamilton's principle $\int_{t_1}^{t_2}\delta\big[E_k(t) - E_p(t) + W(t)\big]dt = 0$, at the same time, $\delta w(x,t) = 0$, $\delta\varphi(x,t) = 0$, at $t = t_1, t_2$.

The governing equation as:

$$\begin{aligned}\rho\ddot{y}(x,t) + EI_b w''''(x,t) + \eta EI_b \dot{w}''''(x,t) - \rho x_e c\ddot{\varphi}(x,t) &= F_b(x,t) \\ I_b\ddot{\varphi}(x,t) - GJ\varphi''(x,t) - \eta GJ\dot{\varphi}''(x,t) - \rho x_e c\ddot{w}(x,t) &= -x_a c F_b(x,t)\end{aligned}$$

The boundary conditions for the u_2 actuation as:

$$\begin{aligned}w(L_1,t) = w'(L_1,t) = w''(L,t) &= 0 \\ w'''(L_1,t) + \eta\dot{w}'''(L_1,t) &= 0 \\ w'''(L,t) + \eta\dot{w}'''(L,t) &= 0 \\ \varphi'(L,t) + \eta\dot{\varphi}'(L,t) &= 0 \\ I_h\ddot{\theta}_2(t) - EI_b w''(L_1,t) - \eta EI_b\dot{w}''(L_1,t) &= F_{u2}(t) \\ -GJ\varphi'(L_1,t) - \eta GJ\dot{\varphi}'(L_1,t) &= M_{u2}(t)\end{aligned}$$

2 Control Design

$$
\begin{aligned}
V(t) &= V_1(t) + V_2(t) + V_3(t) \\
V_1(t) &= \frac{1}{2}\alpha\rho \int_{L_1}^{L} [\dot{y}(x,t)]^2 dx + \frac{1}{2}\alpha I_b \int_{L_1}^{L} [\dot{\varphi}(x,t)]^2 dx + \frac{1}{2}\alpha EI_b \int_{L_1}^{L} [w''(x,t)]^2 dx + \frac{1}{2}\alpha GJ \int_{L_1}^{L} [\varphi'(x,t)]^2 dx \\
V_2(t) &= \frac{1}{2}(I_R + m_P L_1^2)[u_{1a}(t)]^2 + \frac{1}{2}(k_1 + k_2)[\theta_1(t) - \theta_{1d}]^2 + \frac{1}{2}\alpha I_h[\dot{\theta}_2(t)]^2 + \frac{1}{2}\alpha I_h [u_{2a}(t)]^2 \\
&\quad + \frac{1}{2}\alpha(p_1 + p_2)[\theta_2(t) - \theta_{2d}]^2 \\
V_3(t) &= \beta\rho \int_{L_1}^{L} \dot{y}(x,t) w(x,t) dx + \beta I_b \int_{L_1}^{L} \dot{\varphi}(x,t)\varphi(x,t) dx - \beta\rho x_e c \int_{L_1}^{L} \Big[\dot{y}(x,t)\varphi(x,t) + \dot{\varphi}(x,t) w(x,t)\Big] dx \\
&\quad - \alpha\rho x_e c \int_{L_1}^{L} \dot{y}(x,t)\dot{\varphi}(x,t) dx \\
F_{u1}(t) &= -(I_R + m_P L_1^2)\dot{\theta}_1(t) + k u_{1a}(t) \\
&\quad - k_1 \dot{\theta}_1(t) - k_2[\theta_1(t) - \theta_{1d}] \\
F_{u2}(t) &= -I_h \dot{\theta}_2(t) + p u_{2a}(t) - p_1 \dot{\theta}_2(t) - p_2[\theta_2(t) - \theta_{2d}] \\
&\quad - EI_b w''(L_1,t) - \eta EI_b \dot{w}''(L_1,t) \\
M_{u2}(t) &= -q[\alpha\dot{\varphi}(L_1,t) + \beta\varphi(L_1,t)] \\
u_{1a}(t) &= -\dot{\theta}_1(t) - [\theta_1(t) - \theta_{1d}] \\
u_{2a}(t) &= -\dot{\theta}_2(t) - [\theta_2(t) - \theta_{2d}]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_1\kappa_1(t) &\leqslant V_1(t) \leqslant \gamma_2\kappa_1(t), \kappa_1(t) = \int_{L_1}^{L} \Big\{[\dot{y}(x,t)]^2 + [\dot{\varphi}(x,t)]^2 + [w''(x,t)]^2 + [\varphi'(x,t)]^2\Big\} dx \\
\gamma_1 &= \frac{\alpha}{2}\min\{\rho, I_b, EI_b, GJ\}, \gamma_2 = \frac{\alpha}{2}\max\{\rho, I_b, EI_b, GJ\} \\
\gamma_3\kappa_2(t) &\leqslant V_2(t) \leqslant \gamma_4\kappa_2(t), \kappa_2(t) = [u_{1a}(t)]^2 + [\theta_1(t) - \theta_{1d}]^2 + [\dot{\theta}_2(t)]^2 + [u_{2a}(t)]^2 + [\theta_2 - \theta_{2d}]^2 \\
\gamma_3 &= \frac{1}{2}\min\{(I_R + m_p L_1^2), (k_1 + k_2), \alpha I_h, (p_1 + p_2)\}, \gamma_4 = \frac{1}{2}\max\{(I_R + m_p L_1^2), (k_1 + k_2), \alpha I_h, (p_1 + p_2)\} \\
\Big|V_3(t)\Big| &\leqslant \beta\rho \int_{L_1}^{L} [\dot{y}(x,t)]^2 dx + \beta\rho \int_{L_1}^{L} [w(x,t)]^2 dx + \beta I_b \int_{L_1}^{L} [\dot{\varphi}(x,t)]^2 dx + \beta I_b \int_{L_1}^{L} [\varphi(x,t)]^2 dx \\
&\quad + \beta\rho x_e c \int_{L_1}^{L} [\dot{y}(x,t)]^2 dx + \beta\rho x_e c \int_{L_1}^{L} [\varphi(x,t)]^2 dx + \beta\rho x_e c \int_{L_1}^{L} [\dot{\varphi}(x,t)]^2 dx + \beta\rho x_e c \int_{L_1}^{L} [w(x,t)]^2 dx \\
&\quad + \alpha\rho x_e c \int_{L_1}^{L} [\dot{y}(x,t)]^2 dx + \alpha\rho x_e c \int_{L_1}^{L} [\dot{\varphi}(x,t)]^2 dx \\
&\leqslant (\beta\rho + \beta\rho x_e c + \alpha\rho x_e c) \int_{L_1}^{L} [\dot{y}(x,t)]^2 dx + (\beta I_b + \beta\rho x_e c + \alpha\rho x_e c) \int_{L_1}^{L} [\dot{\varphi}(x,t)]^2 dx \\
&\quad + (\beta\rho + \beta\rho x_e c) L_2^4 \int_{L_1}^{L} [w''(x,t)]^2 dx + (\beta I_b + \beta\rho x_e c) L_2^2 \int_{L_1}^{L} [\varphi'(x,t)]^2 dx \\
&\leqslant \gamma_5\kappa_1(t), \gamma_5 = \max\left(\beta\rho + \beta\rho x_e c + \alpha\rho x_e c), (\beta I_b + \beta\rho x_e c + \alpha\rho x_e c), (\beta\rho + \beta\rho x_e c) L_2^4, (\beta I_b + \beta\rho x_e c) L_2^2\right. \\
-\gamma_5\kappa_1(t) &\leqslant V_3(t) \leqslant \gamma_5\kappa_1(t)
\end{aligned}
$$

Because of $V(t) = V_1(t) + V_2(t) + V_3(t)$

$$
\begin{aligned}
(\gamma_1 - \gamma_5)\kappa_1(t) &\leqslant V_1(t) + V_3(t) \leqslant (\gamma_2 + \gamma_5)\kappa_1(t) \\
(\gamma_1 - \gamma_5)\kappa_1(t) + \gamma_3\kappa_2(t) &\leqslant V(t) \leqslant (\gamma_2 + \gamma_5)\kappa_1(t) + \gamma_4\kappa_2(t) \\
0 \leqslant \lambda_1\kappa(t) &\leqslant V(t) \leqslant \lambda_2\kappa(t), \lambda_1 = \min\{\gamma_1 - \gamma_5, \gamma_3\}, \lambda_2 = \max\{\gamma_2 + \gamma_5, \gamma_4\}, \kappa(t) = \kappa_1(t) + \kappa_2(t)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_1(t) &= \alpha\rho\int_{L_1}^{L}\dot{y}(x,t)\ddot{y}(x,t)dx+\alpha I_b\int_{L_1}^{L}\dot{\varphi}(x,t)\ddot{\varphi}(x,t)dx+\alpha EI_b\int_{L_1}^{L}w''(x,t)\dot{w}''(x,t)dx\\
&\quad+\alpha GJ\int_{L_1}^{L}\varphi'(x,t)\dot{\varphi}'(x,t)dx\\
&\leqslant -\frac{\alpha\eta EI_b}{2}\int_{L_1}^{L}[\dot{y}''(x,t)]^2dx+\alpha\sigma_1\int_{L_1}^{L}[\dot{y}(x,t)]^2dx-\Big(\frac{\alpha\eta EI_b}{2L_2^2}-\alpha\sigma_2x_ac\Big)\int_{L_1}^{L}[\dot{\varphi}(x,t)]^2dx\\
&\quad-\frac{\alpha\eta EI_b}{2}\int_{L_1}^{L}[\dot{w}''(x,t)]^2dx-\frac{\alpha\eta GJ}{2}\int_{L_1}^{L}[\dot{\varphi}'(x,t)]^2dx+\Big(\frac{\alpha}{\sigma_1}+\frac{\alpha x_ac}{\sigma_2}\Big)L_2F_{bmax}^2\\
&\quad+\alpha\big[-EI_bw''(L_1,t)-\eta EI_b\dot{w}''(L_1,t)\big]\dot{\theta}_2(t)+\alpha\big[-GJ\varphi'(L_1,t)-\eta GJ\dot{\varphi}'(L_1,t)\big]\dot{\varphi}(L_1,t)\\
&\quad+\alpha\rho x_ec\int_{L_1}^{L}\Big[\dot{y}(x,t)\ddot{\varphi}(x,t)+\dot{\varphi}(x,t)\ddot{w}(x,t)\Big]dx\\
&\leqslant -(\frac{\alpha\eta EI_b}{2L_2^4}-\alpha\sigma_1)\int_{L_1}^{L}[\dot{y}(x,t)]^2dx-\Big(\frac{\alpha\eta EI_b}{2L_2^2}-\alpha\sigma_2x_ac\Big)\int_{L_1}^{L}[\dot{\varphi}(x,t)]^2dx\\
&\quad-\frac{\alpha\eta EI_b}{2}\int_{L_1}^{L}[\dot{w}''(x,t)]^2dx-\frac{\alpha\eta GJ}{2}\int_{L_1}^{L}[\dot{\varphi}'(x,t)]^2dx+\Big(\frac{\alpha}{\sigma_1}+\frac{\alpha x_ac}{\sigma_2}\Big)L_2F_{bmax}^2+\frac{C}{L_2^4}\\
&\quad+\alpha\big[-EI_bw''(L_1,t)-\eta EI_b\dot{w}''(L_1,t)\big]\dot{\theta}_2(t)+\alpha\big[-GJ\varphi'(L_1,t)-\eta GJ\dot{\varphi}'(L_1,t)\big]\dot{\varphi}(L_1,t)\\
&\quad+\alpha\rho x_ec\int_{L_1}^{L}\Big[\dot{y}(x,t)\ddot{\varphi}(x,t)+\dot{\varphi}(x,t)\ddot{w}(x,t)\Big]dx
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_2(t) &= (I_R+m_pL_1^2)u_{1a}(t)[-\ddot{\theta}_1(t)-\dot{\theta}_1(t)]+(k_1+k_2)[\theta_1(t)-\theta_{1d}]+\alpha I_h\dot{\theta}_2(t)\ddot{\theta}_2(t)\\
&\quad+\alpha I_hu_{2a}(t)[-\ddot{\theta}_2(t)-\dot{\theta}_{2d}]+\alpha(p_1+p_2)[\theta_2(t)-\theta_{2d}]\dot{\theta}_2(t)\\
&\leqslant -k[u_{1a}(t)]^2-k_1[\dot{\theta}_1(t)]^2-k_2[\theta_1(t)-\theta_{1d}]^2\\
&\quad-(\alpha p-\alpha p\mu_1)[u_{2a}(t)]^2-(\alpha I_h+2\alpha p_1)[\dot{\theta}_2(t)]^2-(\alpha p_2-\alpha p_2\mu_2)[\theta_2(t)-\theta_{2d}]^2\\
&\quad+\alpha[EI_bw''(L_1,t)+\eta EI_b\dot{w}''(L_1,t)]\dot{\theta}_2(t)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_3(t) &= \beta\rho\int_{L_1}^{L}\ddot{y}(x,t)w(x,t)dx+\beta\rho\int_{L_1}^{L}\dot{y}(x,t)\dot{w}(x,t)dx+\beta I_b\int_{L_1}^{L}\ddot{\varphi}(x,t)\varphi(x,t)dx+\beta I_b\int_{L_1}^{L}[\dot{\varphi}(x,t)]^2dx\\
&\quad-\beta\rho x_e c\int_{L_1}^{L}\ddot{y}(x,t)\varphi(x,t)dx-\beta\rho x_e c\int_{L_1}^{L}\dot{y}(x,t)\dot{\varphi}(x,t)dx-\beta\rho x_e c\int_{L_1}^{L}\ddot{\varphi}(x,t)w(x,t)dx\\
&\quad-\beta\rho x_e c\int_{L_1}^{L}\dot{\varphi}(x,t)\dot{w}(x,t)dx-\alpha\rho x_e c\int_{L_1}^{L}\ddot{y}(x,t)\dot{\varphi}(x,t)dx-\alpha\rho x_e c\int_{L_1}^{L}\dot{y}(x,t)\ddot{\varphi}(x,t)dx\\
&\leqslant (\beta\rho+\beta\rho L_1\sigma_5+2\beta\rho x_e c\sigma_{10})\int_{L_1}^{L}[\dot{y}(x,t)]^2dx\\
&\quad+\Big(\beta I_b+\beta\rho x_e cL_2\sigma_6+\beta\rho x_e cL_1\sigma_7+\frac{2\beta\rho x_e c}{\sigma_{10}}\Big)\int_{L_1}^{L}[\dot{\varphi}(x,t)]^2dx\\
&\quad-\Big(\beta EI_b-\beta\sigma_3L_2^4-\frac{\beta\eta EI_b}{\sigma_8}\Big)\int_{L_1}^{L}[w''(x,t)]^2dx-\Big(\beta GJ-\beta\sigma_4x_a cL_2^2-\frac{\beta\eta GJ}{\sigma_9}\Big)\int_{L_1}^{L}[\varphi'(x,t)]^2dx\\
&\quad+\beta\sigma_8\eta EI_b\int_{L_1}^{L}[\dot{w}''(x,t)]^2dx+\beta\sigma_9\eta GJ\int_{L_1}^{L}[\dot{\varphi}'(x,t)]^2dx+\Big(\frac{\beta\rho}{\sigma_5}+\frac{\beta\rho x_e c}{\sigma_7}\Big)L_1[\dot{\theta}_1(t)]^2+\frac{\beta\rho x_e c}{\sigma_6}L_2[\dot{\theta}_2(t)]^2\\
&\quad+\Big(\frac{\beta}{\sigma_3}+\frac{\beta x_a c}{\sigma_4}\Big)L_2F_{bmax}^2\\
&\quad+\beta\big[-GJ\varphi'(L_1,t)-\eta GJ\dot{\varphi}'(L_1,t)\big]\varphi(L_1,t)\\
&\quad-\alpha\rho x_e c\int_{L_1}^{L}\big[\dot{y}(x,t)\ddot{\varphi}(x,t)+\ddot{w}(x,t)\dot{\varphi}(x,t)\big]dx
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}(t) &= \dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)\\
&\leqslant -\Big(\frac{\alpha\eta EI_b}{2L_2^4}-\alpha\sigma_1-\beta\rho-\beta\rho L_1\sigma_5-2\beta\rho x_e c\sigma_{10}\Big)\int_{L_1}^{L}[\dot{y}(x,t)]^2dx\\
&\quad-\Big(\frac{\alpha\eta EI_b}{2L_2^2}-\alpha\sigma_2x_a c-\beta I_b-\beta\rho x_e cL_2\sigma_6-\beta\rho x_e cL_1\sigma_7-\frac{2\beta\rho x_e c}{\sigma_{10}}\Big)\int_{L_1}^{L}[\dot{\varphi}(x,t)]^2dx\\
&\quad-\Big(\beta EI_b-\beta\sigma_3L_2^4-\frac{\beta\eta EI_b}{\sigma_8}\Big)\int_{L_1}^{L}[w''(x,t)]^2dx-\Big(\beta GJ-\beta\sigma_4x_a cL_2^2-\frac{\beta\eta GJ}{\sigma_9}\Big)\int_{L_1}^{L}[\varphi'(x,t)]^2dx\\
&\quad-k[u_{1a}(t)]^2-k_1[\dot{\theta}_1(t)]^2-k_2[\theta_1(t)-\theta_{1d}]^2\\
&\quad-(\alpha p-\alpha p\mu_1)[u_{2a}(t)]^2-(\alpha I_h+2\alpha p_1-\frac{\beta\rho x_e c}{\sigma_6}L_2)[\dot{\theta}_2(t)]^2-(\alpha p_2-\alpha p_2\mu_2)[\theta_2(t)-\theta_{2d}]^2\\
&\quad-\Big(\frac{\alpha\eta EI_b}{2}-\beta\sigma_8\eta EI_b\Big)\int_{L_1}^{L}[\dot{w}''(x,t)]^2dx-\Big(\frac{\alpha\eta GJ}{2}-\beta\sigma_9\eta GJ\Big)\int_{L_1}^{L}[\dot{\varphi}'(x,t)]^2dx\\
&\quad-\Big(k_1-\frac{\beta\rho}{\sigma_5}L_1-\frac{\beta\rho x_e c}{\sigma_7}L_1\Big)[\dot{\theta}_1(t)]^2-q[\alpha\dot{\varphi}(L_1,t)+\beta\varphi(L_1,t)]^2\\
&\quad+\Big(\frac{\alpha}{\sigma_1}+\frac{\alpha x_a c}{\sigma_2}+\frac{\beta}{\sigma_3}+\frac{\beta x_a c}{\sigma_4}\Big)L_2F_{bmax}^2+\frac{C}{L_2^4}\\
&\leqslant -\lambda_3\kappa(t)+\varepsilon\\
&\leqslant -\lambda V(t)+\varepsilon
\end{aligned}
$$

3 Conclusion

References