evolMC: a package for Monte-Carlo simulation

W Burchett E Roualdes G Weyenberg

Fall 2013

- We wish to sample from a distribution $f(x|t) \propto \exp\{-H(x)/t\}$, where $t \geq 1$ is called the *temperature* and $H(x)$, called the *fitness function*, corresponds to the negative log-density of x, up to a constant.
- A *population* $\mathbf{x}$ consists of N *individuals* x_i, $i = 1, \ldots, N$, and we also define an associated set of temperatures $\mathbf{t} = (t_1, \ldots, t_N)$. ($x_i \in \mathbb{R}^d$ and the t_i are in descending order.)
- Each individual x_i is independently sampled from the distribution $f_i(x_i) \propto f(x_i|t_i)$.

Algorithm 1 Evolutionary Monte Carlo

procedure EMC
  with prob p_m ▷ p_m is the *mutation probability*.
    MUTATE
  otherwise
    CROSSOVER
  end w/prob
  EXCHANGE
end procedure

Algorithm 2 A random-walk *mutation*.

procedure MUTATE
  Copy the current population to x.
  for all individuals in x **do**
    $y \leftarrow \mathcal{N}_d(x_i, t_i\sigma^2 I)$
    with prob $\min\{1, \exp(\cdots)\}$
      $x_i \leftarrow y$
    end w/prob
  end for
  Set current population to x.
end procedure

Algorithm 3 The fitness-weighted *crossover*.

procedure CROSSOVER
  Copy the current population to x.
  for all individuals in x **do**
    $w_i \leftarrow \exp(-H(x_i))$
  end for
  Select k uniformly from $\{1 : d\}$.
  Select i from $\{1 : N\}$ with weights proportional to $\{w_\cdot\}$.
  Select j uniformly from $\{1 : N\} \backslash \{i\}$.
  In x, swap elements $k : d$ of individuals i and j.
  with prob $\min\{1, \exp(-H(x_i)/t_i + H(x_j)/t_j + \cdots)\}$
    Set the current population to x.
  end w/prob
end procedure

Algorithm 4 The *exchange* attempts to swap individuals between neighboring temperature states.

procedure EXCHANGE
  Copy the current population to x.
  Select i uniformly from $\{1\colon N\}$.
  Select j uniformly from $\{i \pm 1\} \cap \{1\colon N\}$.
  Swap individuals i, j of x.
  with prob $\min\{1, \exp(-H(x_i)/t_i + H(x_j)/t_j)\}$
    Set the current population to x.
  end w/prob
end procedure