HIGHER SYMPLECTIC SIGMA MODELS

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3.1. **The Boundary Theory.**

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4.1. **Exact Courant Algebroids and the A-model with H-flux.** Exact Courant algebroids are classified by their Severa class $H \in \Omega^3_{cl}(X)$. Twisting the canonical topological boundary condition via this class may give a description of the A-model with H-flux H. See R. Szabo's work or the JHEP article of Bonechi–Cattaneo–Iraso (RG: they get it as a certain gauged fixing for the Poisson sigma model where the Poisson structure is the inverse of a Kahler form).

4.2. **Dirac Structures.** Claim: For fixed E, topological boundary conditions (on the target) are given precisely by Dirac structures.

4.3. **Link Invariants.** Let E be a Courant algebroid and R a representation up to homotopy of the associated 2-symplectic Lie algebroid. Further, assume that R is equipped with an invariant trace. Wilson loop observables determine invariants for links in a 3-manifold source manifold (anomalies?).

5. HIGHER COURANT SIGMA MODELS

5.1. **Dimension 4.** Relate to R. Szabo's papers....

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5.2. **Generalized Dirac Structures.** Nothing too special...a Dirac structure is just a $(n-1)$ Lagrangian in a n symplectic Lie algebroid E. Is there a canonical one? Szabo claims there is for $n = 3$. Higher Severa classes.

RG: These are different than the n-plectic people consider...we want non-degeneracy! So not the same as Zambon or Ikeda's QP manifolds.

5.3. **Link Invariants.** Same as in the CSM case....

6. Topological Boundary Conditions and Quantum Groupoids

RG: This could be speculative...

6.1. **Quantizing Lie-bialgebroids.** Let E be the Courant algebroid determined by the Lie bi-algebroid $(L, L^\vee)$. Then L itself determines a topological boundary condition. The boundary observables of this theory may be related to the quantum groupoid quantizing $(L, L^\vee)$ as originally described by Ping Xu.

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