Sampling theory

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Prologue:

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Feedback and exercises

- XX of you filled out the feedback survey. Main take-aways:
  - TBA
- What were the main problems with the exercises?

Learning Goals

- Understand the difference between a sample and a population
- Learn about the central terminology of sampling theory
- Learn how to do a Monte Carlo simulation in R and understand its usefulness

Motivation

Why sampling?

- The goal of scientists is often to learn something about phenomena that involve a great number of subjects
  - Marketing research want to know how customers respond to certain ads
  - Sociologists want to understand how the attitudes of people on climate change relate to their socio-economic backgrounds
  - Economists want to understand what makes firms competitive
  - Political scientists want to understand whom people vote for and why
- In all these cases, the subjects from a very large (or even unknown) **population**
- Since we cannot study the entire population, we study subsets of this population, and try to make inferences about the whole population

→ These subsets are called **samples**, and when and how the **inference** from a sample to a population works will be the subject of the upcoming sessions

Motivating example

- We begin the study of sampling theory with a stylised example
- Suppose we have bought a ball pid with grey and white balls
- We now want to know how many of the bally are grey, and how many are white
- We could either do an exhaustive count
  - But if the seller is correct, the ball pid contains 5.000 balls → too much work

  - Alternatively, we could remove a sample of 50 balls, count them, and make an inference about the original ball pid
    - This would save a lot of work...

Motivating example

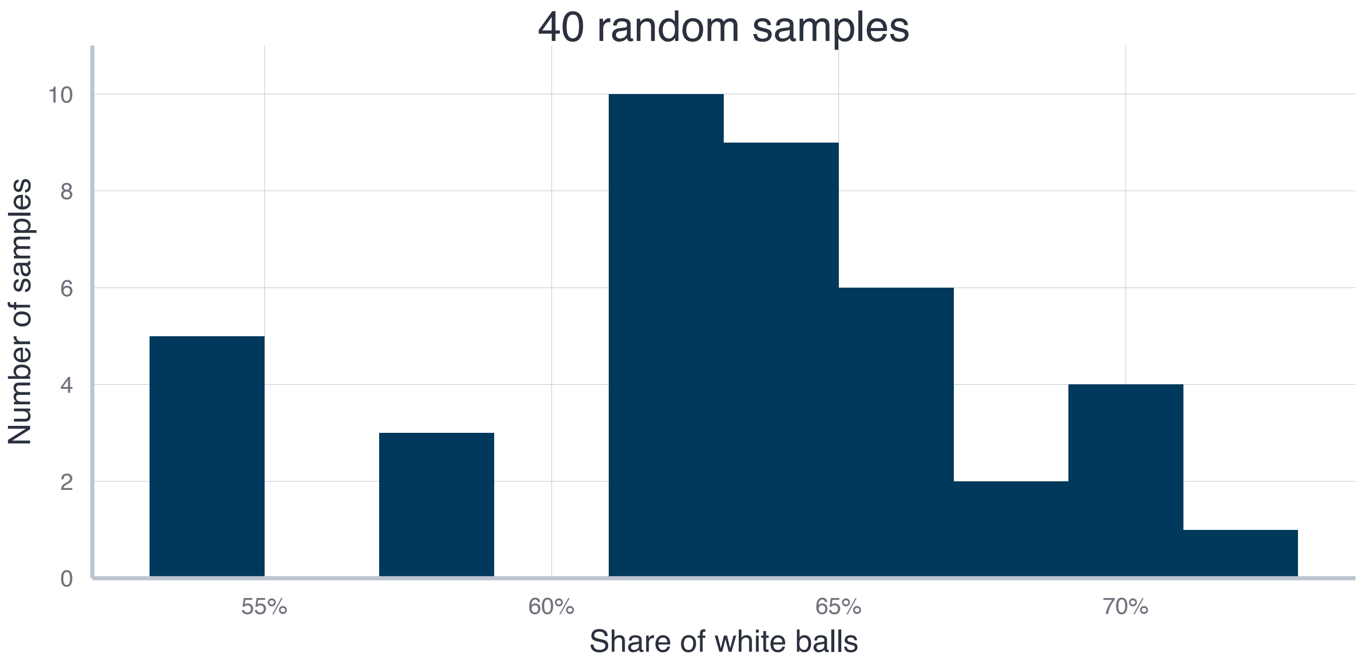
- Suppose we take a sample of 50 balls and find that 64% of the balls were white, does this mean that 64% of all balls are white?

- Not really, our sample was drawn randomly → **random sample**

- This means if we repeat the process we are likely to observe a different share of white balls

    - Suppose we draw 40 such random samples and write down the share of white balls each time

    - We could visualise our results using a histogram



- The fact that the different random samples differ from each other is referred to as the concept of **sample variation**

Monte Carlo simulations

Monte Carlo Simulations

- At this point we want to learn more about how sampling works
- One excellent way to do this is to use simulations → simulate the act of drawing samples from a population on the computer
  - The act of drawing a random sample is a random process
  - Simulations used to study properties of random processes by repeating them many times are called **Monte Carlo simulations** (MCS)
- MCS help us understand determinants & implications of sampling variation
  - Useful even though in reality we usually only draw one single sample
- In an MCS we create the population ourselves and know everything about it
  - In reality we do not know the true properties of the population, but in the MCS context this is necessary to answer the questions above

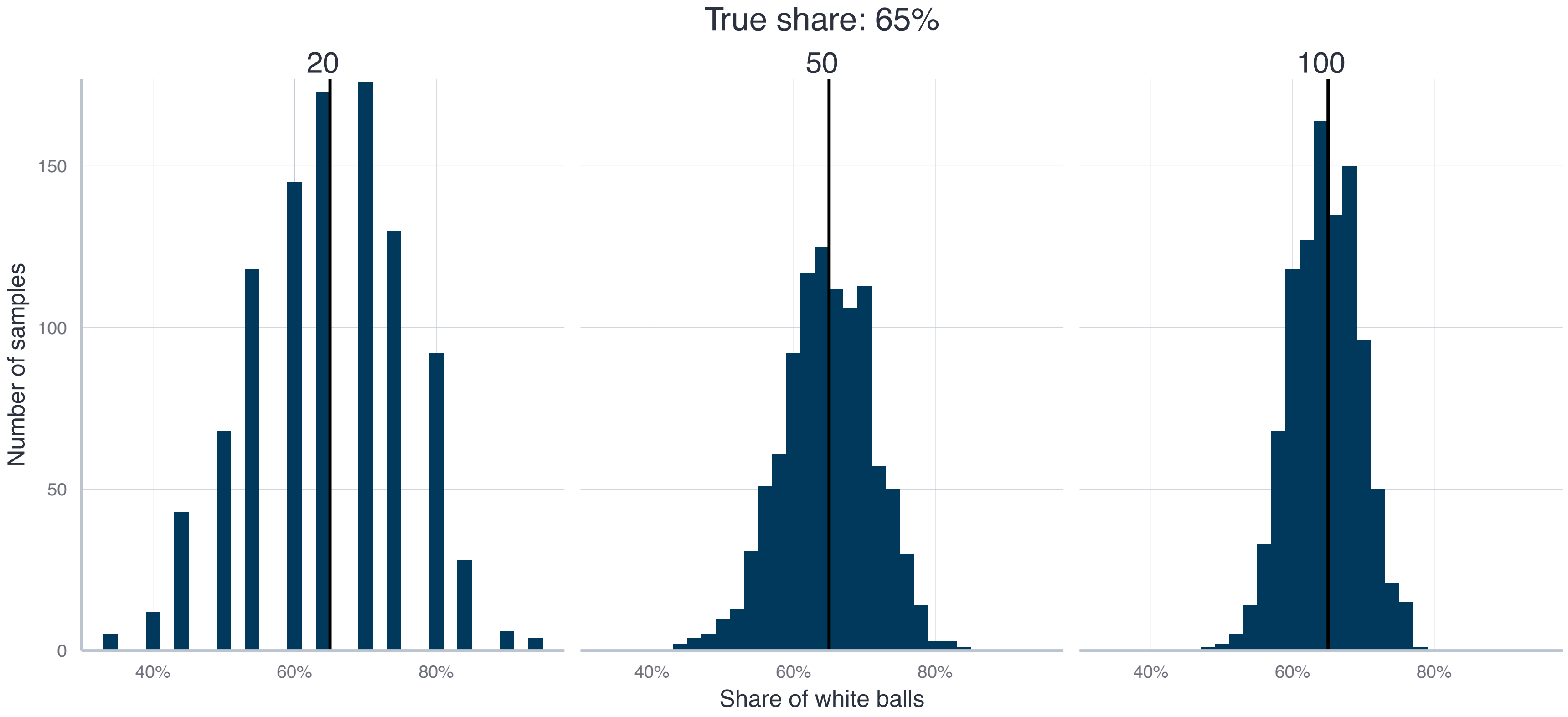
Monte Carlo Simulations

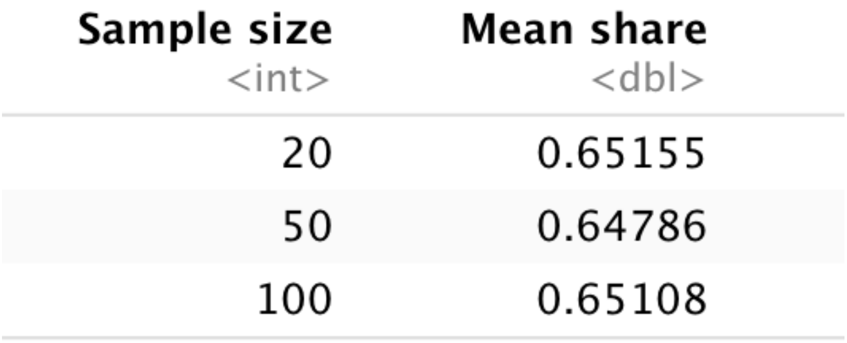
- The general idea is to create an artificial population for which we have all the relevant information
- Then draw samples from this population and study questions such as:
  - Are properties of samples similar to that of the population?
  - What determines sample variation?
  - What is the effect of different sample sizes or sampling iterations?
- Conducting an MCS always involves the same steps

For the practical implementation see the tutorial!

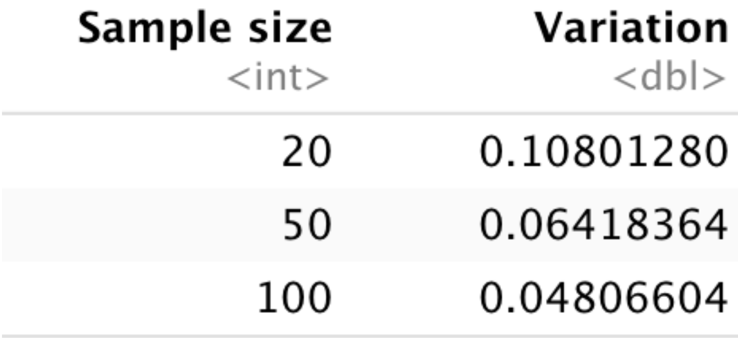
Monte Carlo Simulation - central results

- Here is a summary of our central results:





Sample size	Mean share
<int>	<dbl>
20	0.65155
50	0.64786
100	0.65108



Sample size	Variation
<int>	<dbl>
20	0.10801280
50	0.06418364
100	0.04806604

We measure the variation via the standard deviation

- And these are the central take-aways:

  I. All distributions have a very similar mean of about 65%

  II. The larger the sample, the smaller the sample variation

Exercise 1: Monte Carlo Simulation

- Assume you want to compute the average height of students of the Europa-University Flensburg

- Assume that the data set `DataScienceExercises::EUFstudents` contains the result of a census among EUF students

I am stuck and have a question!

Finished!

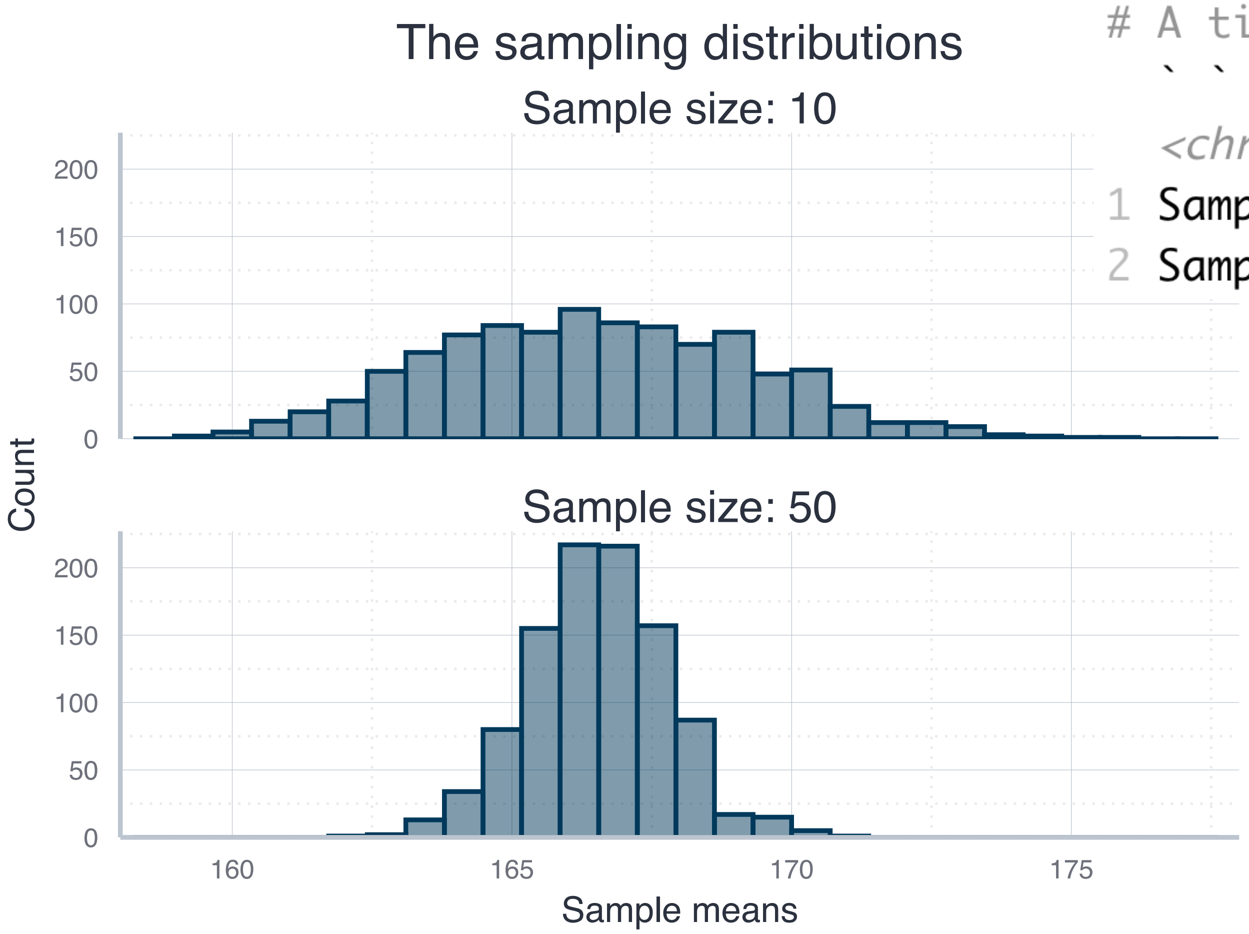
I am working on it, leave me alone!

- Study the process of sampling by conducting an MCS in which you draw random samples from this population of sizes 10 or 50.

- For your MCS, set the number of repetitions to 1000

- What do you observe for the differente sample sizes?
  - Note: a quick-and-dirty way to represent your results is the function `hist()`

Exercise - MCS

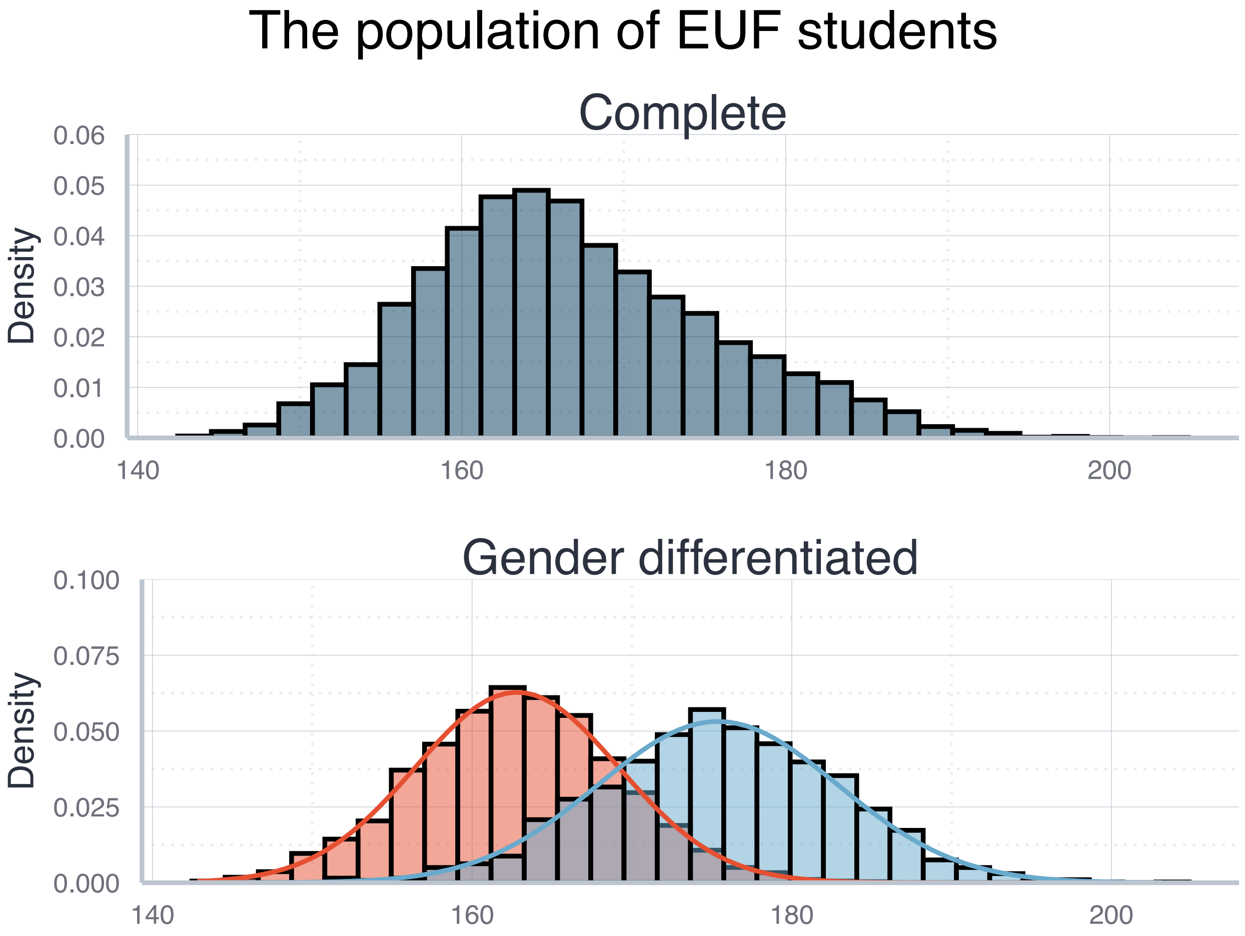


Sample statistics

```
# A tibble: 2 × 3
  ``                   Mean Variation
  <chr>               <dbl>     <dbl>
1 Sample size: 10      166.      2.83
2 Sample size: 50      167.      1.24
```

Population statistics

```
# A tibble: 3 × 3
  Gender  Mean    SD
  <chr>  <dbl> <dbl>
1 female  163.  6.35
2 male    175.  7.50
3 total   167.  8.85
```



- Note: the student population of the EUF is asymmetric in terms of gender
  - While the population is not normally distributed, the sampling distributions tend to be normal → take up later

Terminology

Terminology

- In the following we introduce the fundamental terminology that we use when talking about anything that has to do with sampling
- We will cover the following areas:

Populations

Samples

Methodology

- Most of these concepts are also of prime importance in the context of estimation and inference

Population terminology

A **population** is a collection of individuals or objects that are of interest.

Population size N: the number of individuals making up the population

A **population parameter** is a statistical property of population that is of interest.

A **census** is the act of studying each member of the population to determine the population parameter of interest exactly.

- **Example:** We are interested in the average height of all German women.
  - Population: all German women ($N \approx 42$M)
  - Population parameter: population mean
  - Census: measure all German women and compute the mean height

Sample terminology

A **sample** is a subset of the population. If the elements of the sample were selected randomly, we speak of a **random sample**.

The **sample size** is the number of its elements and denoted as $n < N$.

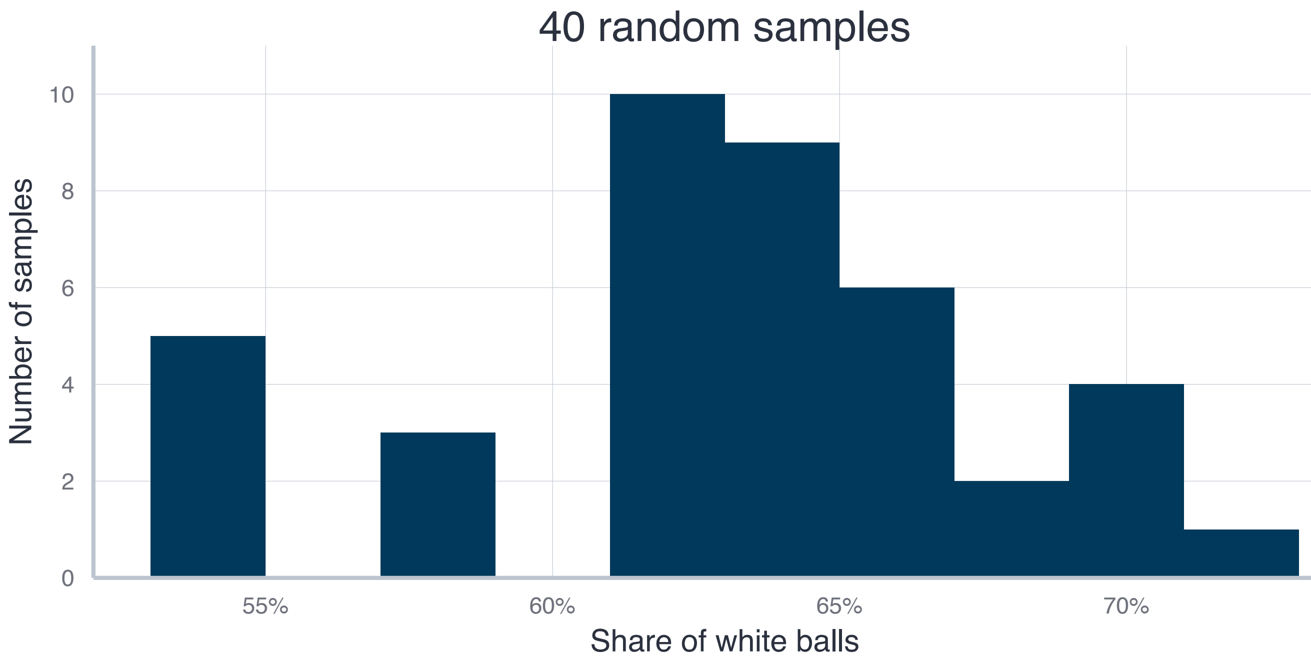
A **point estimate** or **sample statistic** is a statistic computed for the sample and that is to be used to **estimate** the population parameter of interest. It is written with a ^ on the symbol (e.g. $\hat{\beta}$).

- **Example:** We are interested in the average height of all German women.
  - (Random) sample: a group of (randomly selected) women in Germany
  - Sample statistic: the mean height of the women in the sample

Sample terminology

A **sampling distribution** is the distribution of a point estimate.

It formalises the effect of **sampling variation**, which originates from the random element of drawing a sample.

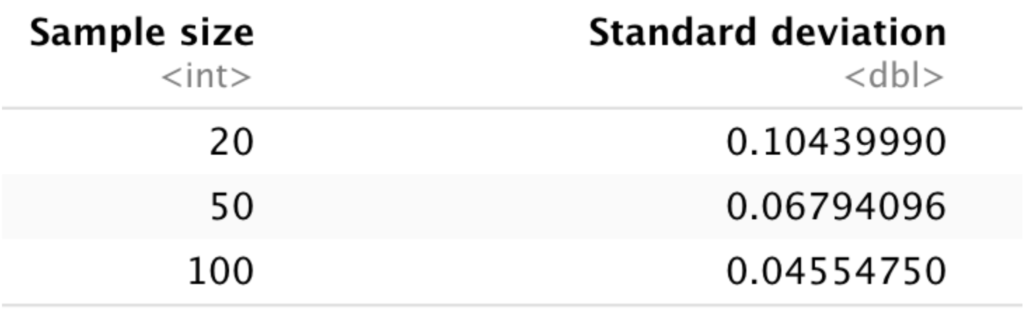


- **Note:** We considered the artificial case in which we drew many samples from the population. The distribution of the estimates is the sampling distribution.
  - In reality we draw only one sample → no direct access to the sampling distribution
  - We can still get information doubt the sampling distributions via **bootstrapping**

Sample terminology

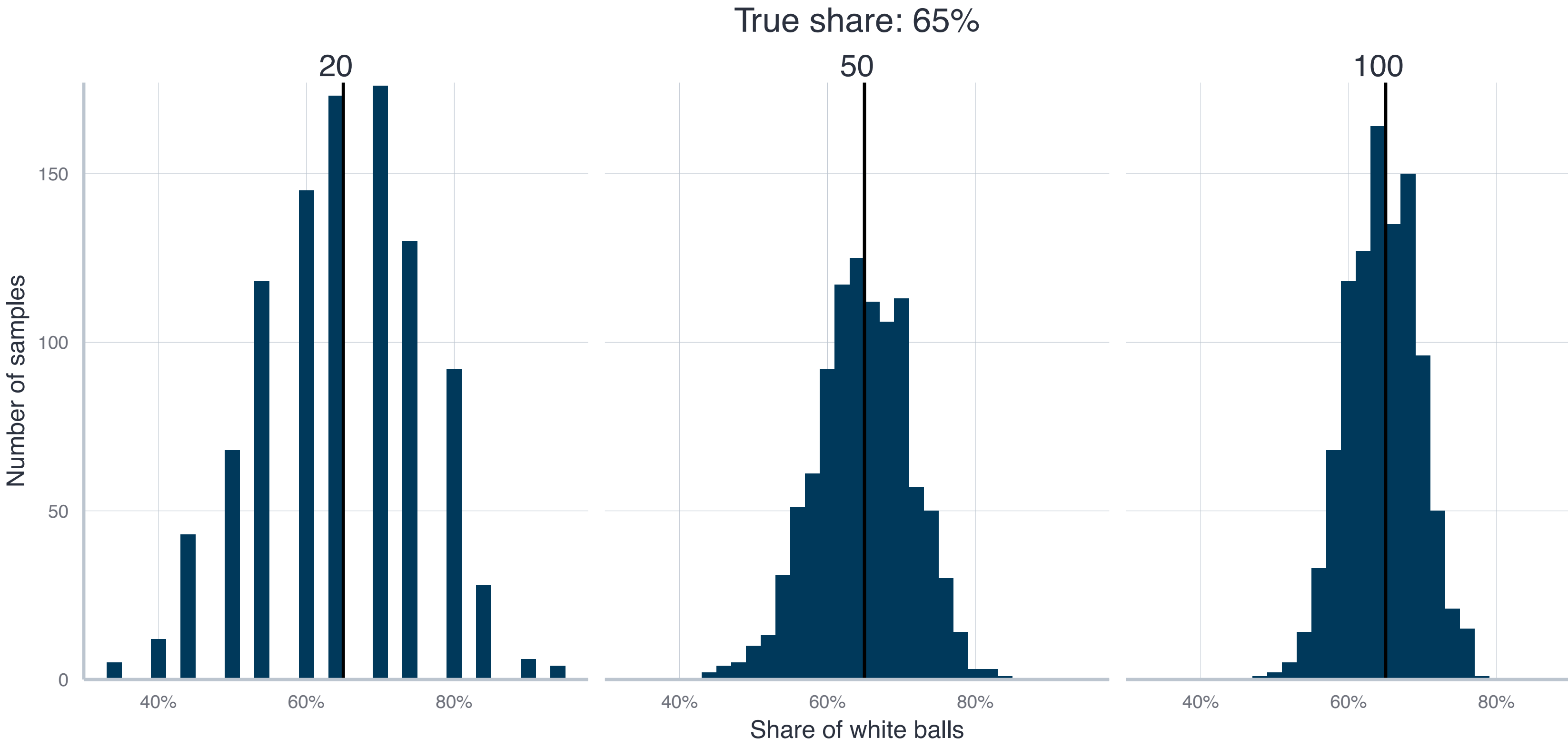
A **standard error** of a point estimate is the standard deviation of its sampling distribution.

It can be used as a measure for the precision of our estimation, and it decreases with sample size.



Sample size <int>	Standard deviation <dbl>
20	0.10439990
50	0.06794096
100	0.04554750

- **Example:** The standard error of our estimate for the share of white balls $\hat{p}$ is...
  - 0.1, 0.07, and 0.05 for sample sizes of 20, 50, and 100, respectively



Methodological concepts

- Building upon the notion of a sample, here are important sample properties:
  - A sample is **representative** for a population if it resembles the relevant properties of the latter
  - A sample is **generalisable** if results for the sample can be generalised into statements about the population
  - A sample is **unbiased** if each member of the population has the same probability to become a member of the sample
  - A **sample** that is not unbiased is called a biased sample
- To ensure that a sample is generalisable and unbiased we usually aim to do **random sampling**
- The act of inferring statistical properties of a population by using statistical properties of a sample is called **statistical inference**

Wraping up the terminology

- Based on our methodology, we can summarise the process of statistical inference as follows:

1. Draw a sample of size n from the study population of size N
2. If the sample is a **random** sample...
3. ...is is usually **unbiased** and **representative** of the population
4. Then results based on the sample can be **generalised** to the population
5. This implies that **sample statistics** are good **estimators** for the respective **population parameters** → no **census** necessary

Exercise - applying the terminology

- Consider the previous example where you studied the height of selected EUF students to make a statement about average height of all EUF students
- Describe the various elements using the terminology we have introduced above. Make sure you make use of the following concepts:
  - Population
  - Sample and sample size
  - Point estimate
  - Sampling distribution
  - Standard error
  - Properties of the sample and inference

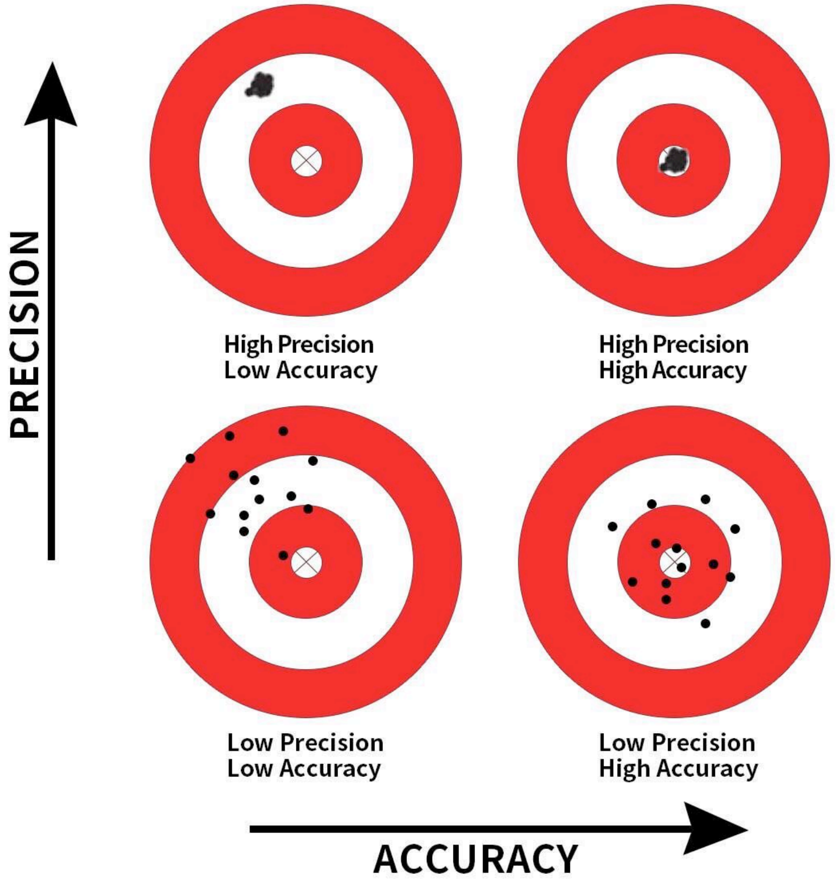
I am stuck and have a question!

Finished!

I am working on it, leave me alone!

Accuracy and precision are not the same

- Estimators that produce estimates that are correct on average are said to be **unbiased**
  - OLS, for instance, produces unbiased estimates for the intercept and slope of the regression line
  - Unbiasedness is often considered to be of prime importance, but it is also overrated



- This is especially the case in machine learning where both concepts relate to one of the fundamental challenge: the **bias-variance trade-off**
  - It means that for many prediction algorithms we can reduce variation by introducing some bias
  - We will learn more about this in the upcoming sessions

Figure: Ismay & Kim (2022)

The Central Limit Theorem

- Many of the experiments we did in this session were somehow artificial:
  - In reality we can only draw a single sample...
  - ...from a population to which we have no direct epistemic access
- This is why statistical inference is needed at all: we only have one sample to make a statement about the population
- But how come that statistical inference **is (often) possible**? The reason lies in the famous Central Limit Theorem

Central Limit Theorem (informal)

- When a sample becomes larger, its sampling distribution becomes narrower and more normally distributed (regardless of the population distribution)

The Central Limit Theorem

- The CLT links our singe sample and the population:
  - The point estimate based on our sample can be considered a draw from a normal distribution with the mean being the true population parameter...
  - ...and the standard deviation of this distribution corresponding to the standard error of our point estimate
- This is why sample size is so important: it makes our estimates more precise and leads to normal sampling distributions
- Again: even if the underlying distribution is not normal, the sampling distribution of the point estimates will still be normal!

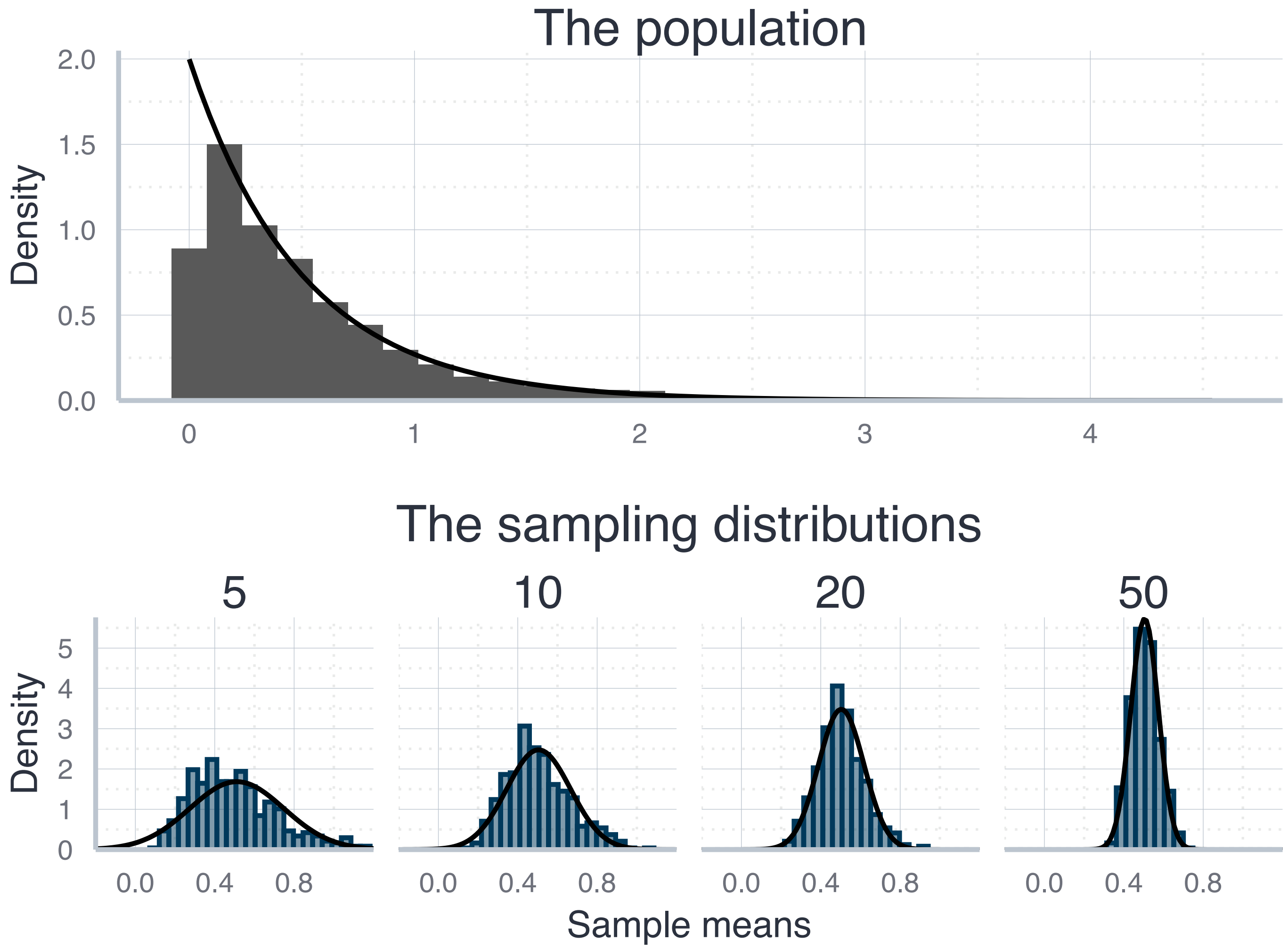
➡ **Check this out yourself!**

Illustration of the Central Limit Theorem

- Choose a distribution for your artificial population with $N = 5000$:
    - Create 5000 draws from a random distribution of your choice!
- Visualize the population using a histogram!
- Illustrate the CLT by conducting a MCS where you draw larger and larger samples from your population and visualize the sampling distribution of the sample means
- Upload your visualisations via Moodle - next week we will compare them and thereby appreciate the practical implications of the CLT more clearly!

Illustration of the Central Limit Theorem

- Here is my example with a exponentially distributed population:



- This is why we can assume a normal sampling distribution if our sample is 'large enough'

Summary & outlook

Summary

- Sampling theory provides tools to draw conclusions about unknown populations of interest by analysing only a sub-sample of this population
- The process of inferring population parameters of interest from samples using statistical techniques is called **statistical inference**
- We introduced all the necessary terminology to discuss the process of sampling and the methods of inference to be used
- To study how estimates are effected by sample variation we used **Monte Carlo Simulations** (MCS)
- This is a more general simulation tool to study random processes
  - Here is was useful to consider the artificial cases of drawing many samples from a known population → helps understanding how sampling works

Outlook

- Next session we will focus on the case where we only draw one sample from an unknown population
- We will learn how we can gather information about our sample by taking many samples from our single sample → **bootstrapping**
- This way we will be able to quantify our confidence in the estimates obtained from a sample

Münchhauſen O. Herrfurth pinx

Tasks until next week:

1. Fill in the **quick feedback survey** on Moodle
2. Read the **tutorials** and **lecture notes** posted on the course page
3. Do the **exercises** provided on the course page and **discuss problems** and difficulties via the Moodle forum

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