# Growth Models

Development Economics, Lecture 4

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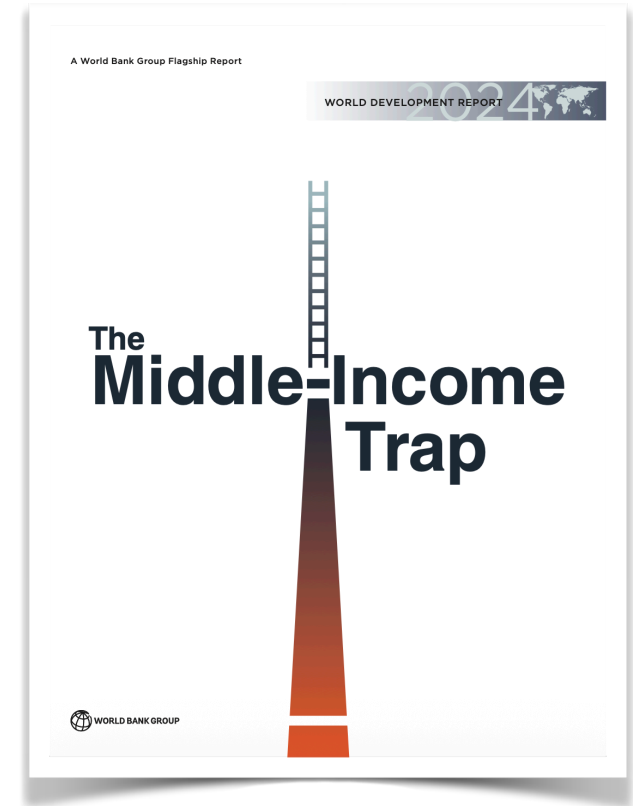
Institute for Comprehensive Analysis of the Economy

# Outlook

- Mandatory reading illustrates practical relevance
- **Vantage point:** growth important for development
  - Which factors determine the growth rate of an economy?
  - Why do countries' growth rates differ?
  - Can we expect convergence?
- Which aspects do growth models **highlight or obscure**?
- Outline:
  - Part 1: Classical views of growth
  - Part 2: The Solow-Swan Model
  - Part 3: Growth accounting and endogenous growth



# Part 1: Classical Views of Growth

# Historical foundations

- Today: Growth as growth of national income a.k.a. GDP
- GDP as such emerged in the mid of the 20th century
  - First ideas: 17th century by William Petty
  - Important contributions: Colin Clark (1932), Simon Kuznets (1933)
  - First time a political goal in US elections of 1938

> "We must start again on a long, steady, upward incline in national income."

Franklin D. Roosevelt (1882-1945)

- Few contributions on growth before 1930s/40s: "Statistics without theory"
  - Key contributions: Harrod & Domar (1939, 1946), Arthur Lewis (1954), Trevor Swan & Robert Solow (1956)

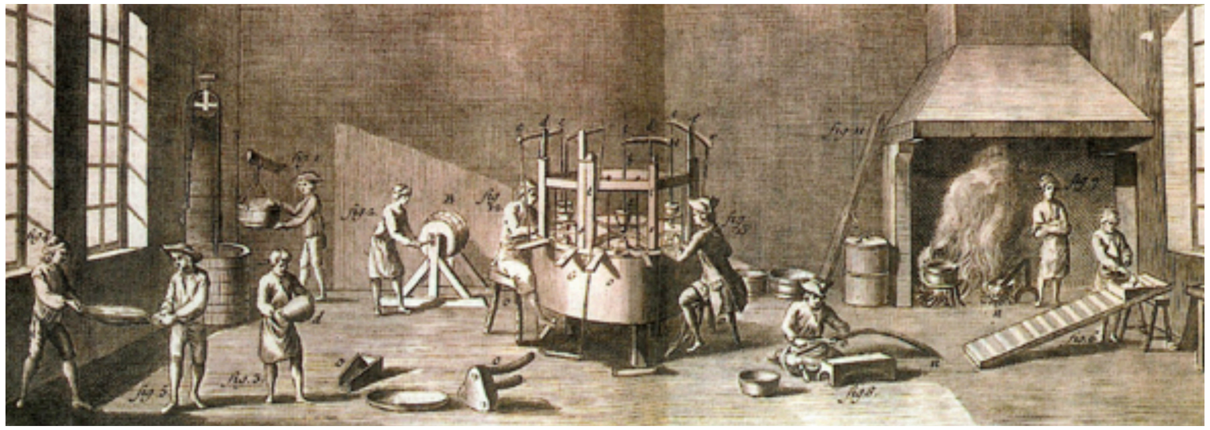
# Classical economics and growth

## Adam Smith: market size as a limit

- Classical economists more concerned with capitalist dynamics
    - Focus was on limits to development, not growth mechanisms
- Smith: **division of labor** as driver of the wealth of nations
    - Division of labor limited by the size of the market

**Smith's pin factory**

- One worker: 20 pins/day
- 10 workers: 48,000 pins/day
- Specialization drives productivity

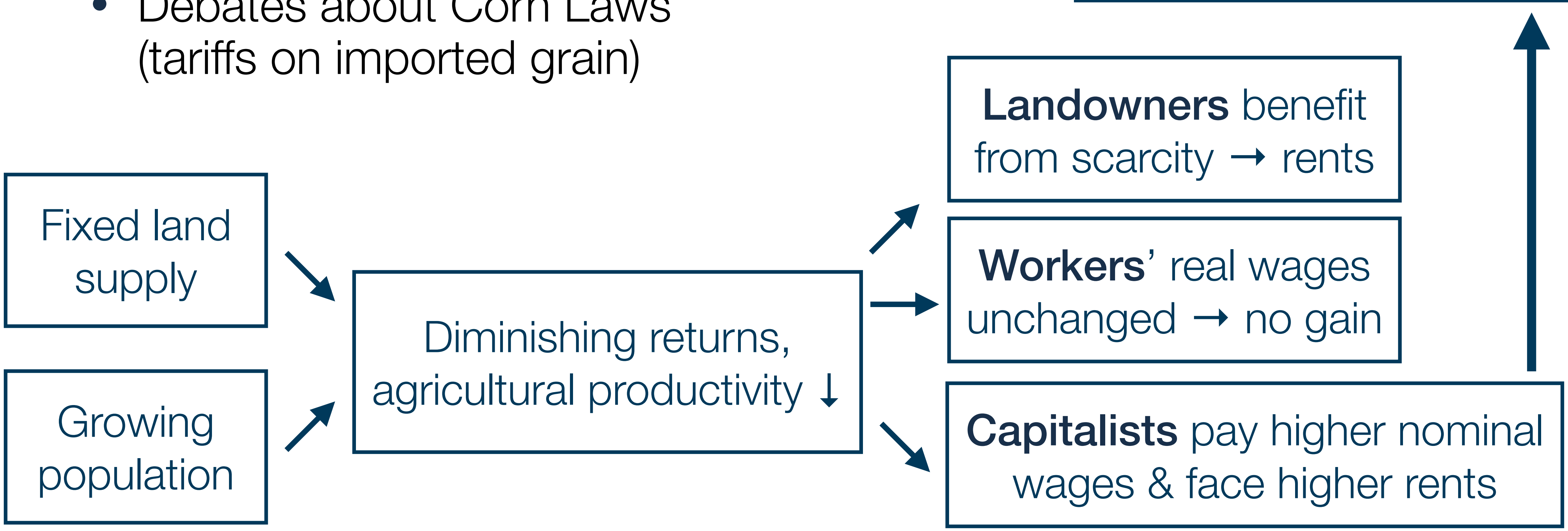


- Small markets do not support high specialization

# Classical economics and growth
## David Ricardo: land as a limit

- Key concern: Can UK feed its growing population when land supply is fixed
- Historical environment:
  - Rapid population growth
  - Rising grain prices
  - Debates about Corn Laws (tariffs on imported grain)

### Stationary state
- No incentive to invest (no returns to capital)
- Capital accumulation stops → Economic growth ceases
- All surplus captured by landowners as rent



# Classical economics and growth
## Karl Marx: the falling rate of profit

- Expanded reproduction a la Marx:

**Exploitation**: capitalists extract surplus value from workers → Re-invest surplus into more capital → **accumulation** → **Expansion**

- **Internal contradiction** of capitalism

Competition among capitalists enforces **mechanization** → Economy becomes more **capital-intensive** ($c/v$ rises) → **Falling rate of profit**

- The falling rate of profit leads to:
  - Declining investment and stagnation
  - Intensifying competition and increasing exploitation
  - Imperialism, war, system-wide crises and revolution

# Classical economics and growth
## Summary

- Smith, Ricardo and Marx were concerned with **systemic dynamics**
  - No growth theory in the contemporary sense → broader view on society
- Smith and especially Ricardo: **external limits**
- Marx: **internal contradictions** of capitalism
- All highlight **distributional conflicts** of growth and development
- **Technology** as problem solver for Smith & Ricardo,
  - Marx: endogeneous to class conflict and source of further problems
- Smith and Ricardo: **limits and stagnation**
- Marx: **endogeneous self-destruction** of campitalism

# Part 2: The Solow-Swan model

# Towards the Solow-Swan Model
## The historical vantage point

- First mathematical growth model by Harrod (1939) and Domar (1946)
  - Context: Great Depression, Keynesian revolution, Sputnik
- Key relationships:
  - $Y = (1/v) \times K$
  - $I = \Delta K$
  - $S = s \times Y$
  - $S = I$

$\Rightarrow$

  - $g$: actual growth rate
  - $g_w$: warranted growth rate
  - $g_n$: natural growth rate
- Steady growth only if $g = g_w = g_n$, otherwise **cumulative instability**
- **Conclusion:** no self-correction, inherent instability, need for intervention

# Towards the Solow-Swan Model
## Additional info for the Harrod Domar Model

- $Y$ : Output
- $K$ : Capital stock
- $v$ : capital-output ratio (constant)
- $S$ : total savings
- $s$ : savings rate
- $I$ : total investment

- The three growth rates:
  - $g = \Delta Y/Y$ : actual growth rate (i.e. empirical observation)
  - $g_w = s/v$ : warranted growth rate
    - The growth rate at which entrepreneurs' expectations are fulfilled and they maintain their desired capital-output ratio
    - Full utilitzation of capital, desired investment equals actual investment
  - $g_n = n + \lambda$ : natural growth rate
    - $n$ : growth of labor force; $\lambda$ : rate of labor-augmenting tech change
    - Exogeneous and determined by supply side fundamentals
    - Maximum sustainable growth rate given labor force growth and technological progress

# The Solow-Swan Model

- Solved the instability 'problem' of Harrod-Domar
  - **Key innovations**: factor substitution and diminishing returns
  - **Implication**: stable growth possible without intervention
  - **Microfoundations**: coefficients not exogeneous, but possible derived from microeconomics
- Harrod-Domar: $Y = (1/v) \times K$
- Solow-Swan: $Y = F(K, L) = K^{\alpha} \times L^{1-\alpha}$
  - Allows for **substituting** $K$ for $L$ and vice versa
  - **Diminishing returns** for both factors

Robert Solow (1924-2023)

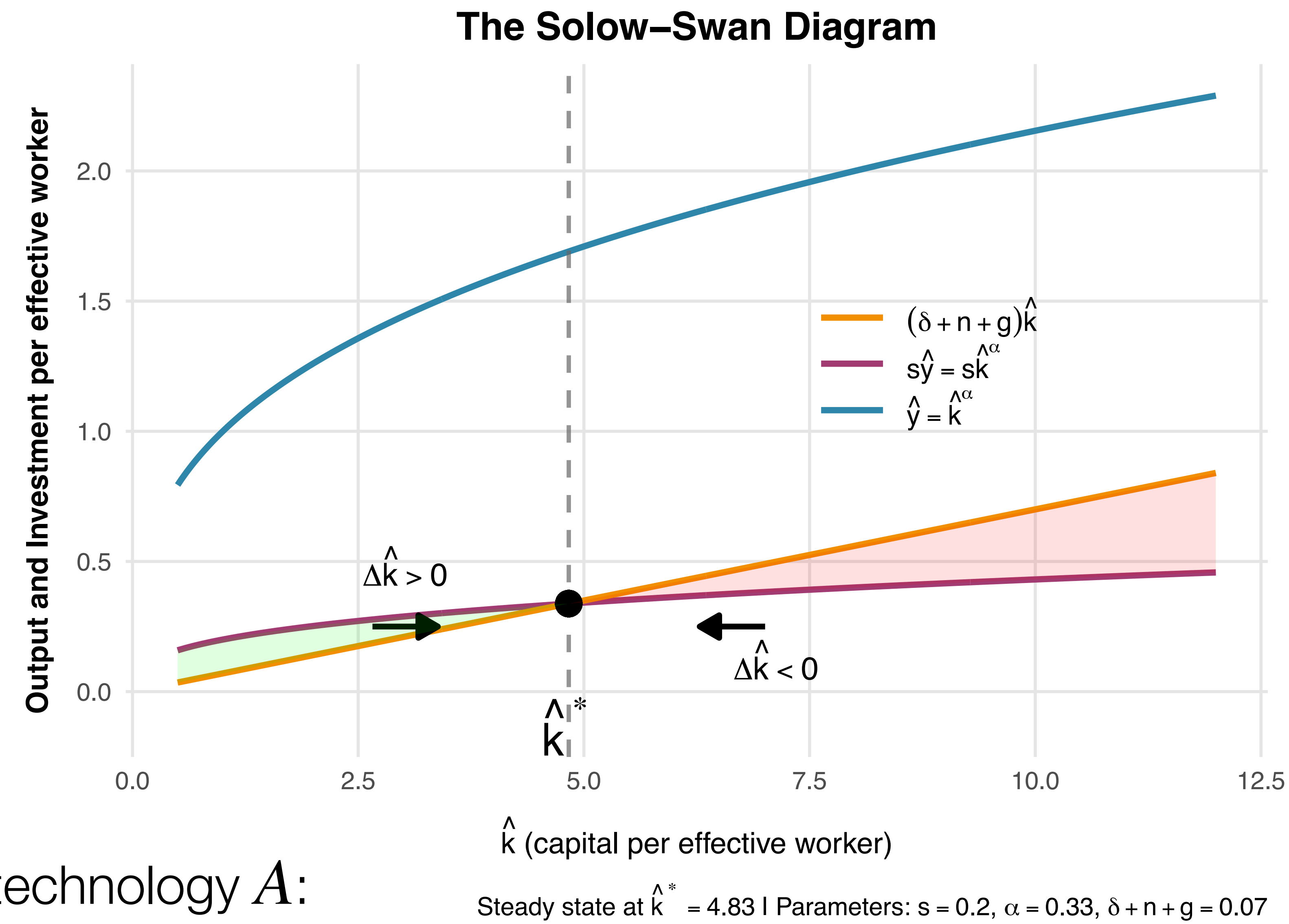
Trevor Swan (1918-1989)

# The Solow-Swan Model
## Model details

- **Absolute terms**: $Y = F(K, L) = K^{\alpha} \times L^{1-\alpha}$
- **Per capita terms**: $y = F(k) = k^{\alpha}$
  - $y \equiv Y/L$ (output per worker) and $k \equiv K/L$ (capital per worker)
  - Dividing everything by $L \rightarrow L/L = 1$
  - $0 < \alpha < 1$: diminishing returns to capital
- $\Delta k$ : Investment minus depreciation $\delta \rightarrow$ **equation of capital accumulation**
- $\Delta k = sy - \delta k = sk^{\alpha} - \delta k$
  - Investment $sk^{\alpha}$ adds to capital stock, depreciation $\delta k$ reduces it
  - **Steady state**: investment equals depreciation

# The Solow-Swan Model
## Finding the steady state

- Set $\Delta k = 0$
- Then: $sk^{\alpha} = \delta k$
- Solve for $k^* = (s/\delta)^{1/(1-\alpha)}$
- At $k^*$, $y^*$ is constant
  → no per capita growth
- Absolute growth determined by population growth $n$



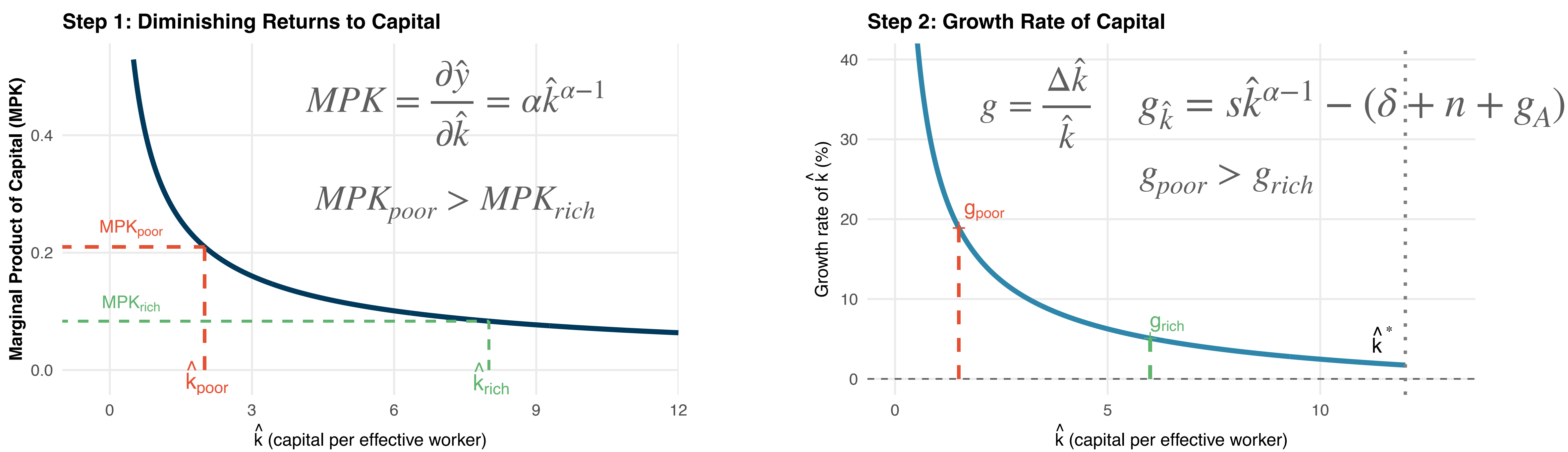

- The actual model also includes technology $A$:
  - $Y = K^{\alpha} \cdot AL^{1-\alpha}$
  - Fully exogenous → same effect as population growth
  - Determines total growth, but "falls from heaven"

# The Solow-Swan Model
## The Context

- **Harrod-Domar**: Great Depression, Keynesianism, planning affinity
- **Solow-Swan**:
  - Post-War prosperity → more consistent with empirical observations
  - **Factor substitution** happens in reality
  - Defence of **market capitalism** during the cold war
  - Free markets can be **self-correcting** without planning intervention
  - Contact to microeconomics → **microfoundations**
- Solow-Swan had then clear scientific advantages…
  - …but also fitted the then-dominant ideology much better
- Today, which of the two is ‘better’ is debated

# The Solow-Swan Model
## The view on global inequality

- The Solow-Swan model makes a strong prediction regarding **convergence**




- This prediction has sparked huge empirical literature
  - **Absolute convergence**: In the long run, all countries converge to the same per capita income (the Solow-Swan prediction)
  - **Conditional convergence**: In the long run, all countries *with the same fundamentals* converge to the same per capita income

# The Solow-Swan Model
## The view on global inequality

- Test absolute convergence: regress initial GDP on average growth rate:

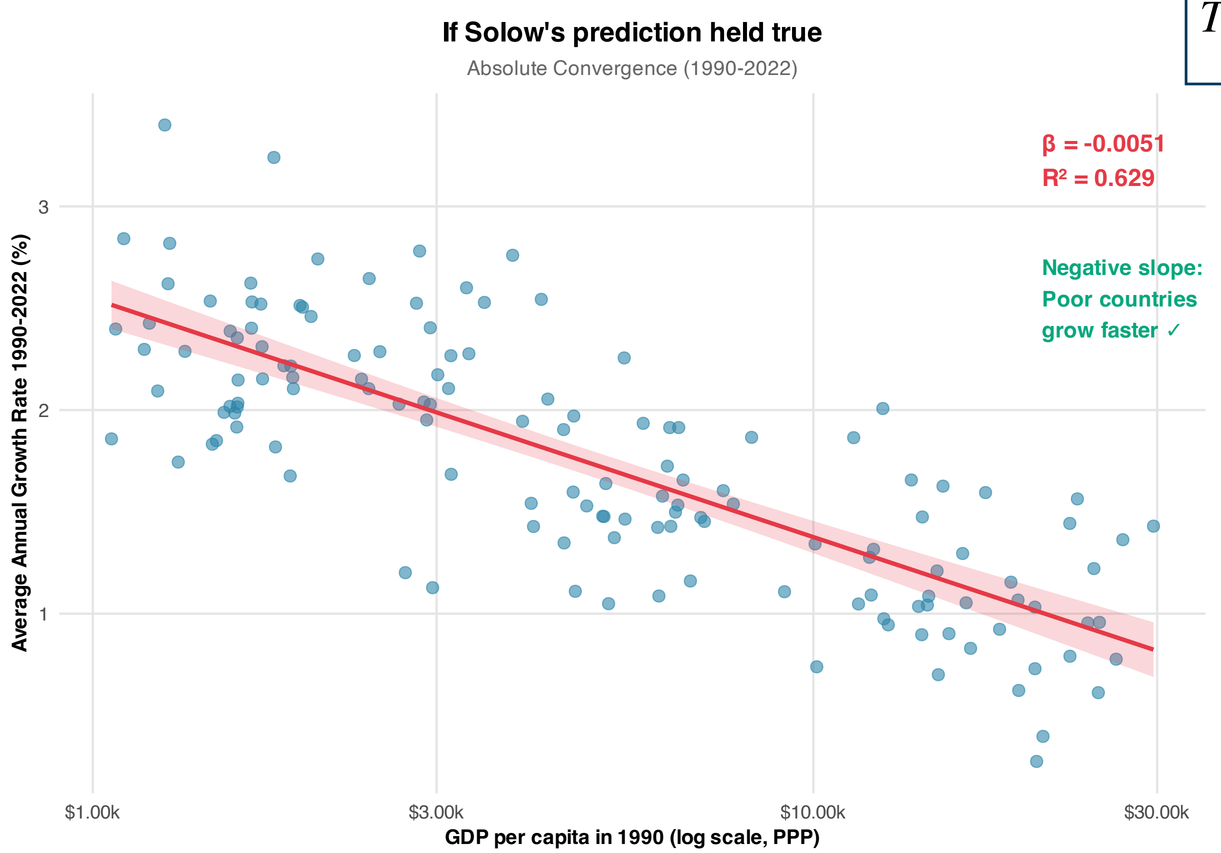
$$\frac{1}{T}\ln\left(\frac{y_{i,t+T}}{y_{i,t}}\right) = \alpha + \beta \ln\left(y_{i,t}\right) + \epsilon_i$$

$\frac{1}{T}\ln\left(\frac{y_{i,t+T}}{y_{i,t}}\right)$: Average annual growth rate of GDP per capita over time period $T$

$y_{i,t}$ : Initial GDP per capita

$\beta$ : Convergence parameter; absolute convergence if $\beta < 0$

$T, i$: Time period length and country index




**Your task**

Get data and test the absolute convergence claim yourself!
(See course website)

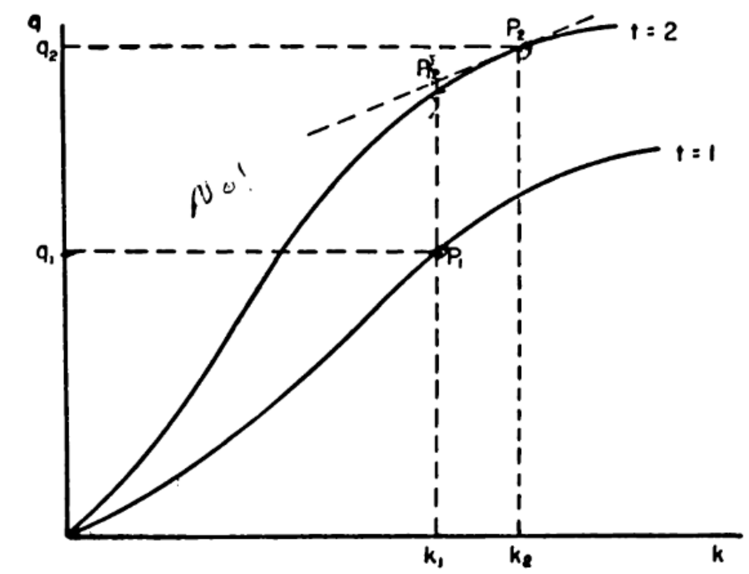
# Part 3: Growth Accounting and endogenous growth

# Growth Accounting

TECHNICAL CHANGE AND THE AGGREGATE PRODUCTION FUNCTION*

Robert M. Solow

CHART I




- Key contribution of Robert Solow (1957):
  - Decomposes output per worker growth into capital deepening and a residual ('technical change')
  - "Movement **along vs. shifts** of the production function"
- Methodological approach:
  - Assumes **constant returns to scale**: $Q = F(K, L; t) \rightarrow Q = A(t) \cdot f(K, L)$
  - Divide by $L$ for per capita perspective: $q = A(t) \cdot f\left(k\right)$ with $q = \frac{Q}{L}$ and $k = \frac{K}{L}$
  - Assume **competitive markets** → $w_K = \frac{\partial Q}{\partial K} \cdot \frac{K}{Q}$ and $w_L = \frac{\partial Q}{\partial L} \cdot \frac{L}{Q}$ as (observable) income shares
  - Then you get growth rates: $g_Q = g_A + w_K \cdot g_K + w_L \cdot g_L$
- Compute the 'residual': $g_A = g_Q - w_K \cdot g_K - w_L \cdot g_L$
  - "**Solow Residual**" ~ Technological change / Total Factor Productivity

# Solow's key findings

$$g_Q = g_A + w_K \cdot g_K + w_L \cdot g_L$$

- $w_K \cdot g_K$ and $w_L \cdot g_L$ represent movements **along** the production function
- The "Solow Residual" $g_A$ represents **shifts of** the production function

- Solow's findings for the US (1909 - 1949):
  - Focus on per capita: $g_q = g_A + w_K \cdot g_K$
  - $g_q$ doubled from 1909 to 1949
  - 12.5% can be attributed to capital deepening ($g_K$)
  - **87.5%** can be attributed to the **Solow Residual** $g_A$ a.k.a. technical change
  - Technical change was **neutral** over this period

# Growth Accounting after Solow (1957)

- Today's growth accounting strategies more complete and sophisticated
- Central result remains stable: **technical change** is the **key driver of growth**
- Unfortunately, it is the least understood component
  - "Solow residual as a **measure of our ignorance**"
- These findings motivated everything in modern mainstream growth theory
  - Endogeneous growth theory
  - It is essentially a theory about $g_A$

# A glimpse on endogeneous growth theory

- **Human Capital Models** (e.g., Lucas 1988)
  - $Y = A \cdot K^{\alpha} \times \left(hL\right)^{1-\alpha}$
  - $h$ as human capital → positive externalities and spillovers → sustained growth
- **R&D Models** (e.g., Romer 1990)
  - Ideas as drivers of technical change
  - Ideas are non-rival, partially excludable and feature increasing returns
- **Learning-by-doing models** (e.g., Arrow 1962)
  - Productivity improves with experience
  - Learning-by-doing potentially differs across sectors (manufacturing > agriculture)
- All these theories have been unsuccessful in fully explaining $g_A$
  - Heterodox approaches such as evolutionary economics more promising (?)
  - More details in the seminar Development Studies

# Part 4: Summary and Outlook

# Summary and outlook

- The Solow-Swan model was a catalyst for growth theory
  - Stable growth is feasible (contra Harrod-Domar)
  - Growth is driven by technical change → endogenous growth theory

**Highlights:**

- Key role of technological change
- Role of capital accumulation and investment
- Relevance of savings and productivity
- Mechanisms of (non-)convergence
- Instrumental value of education and innovation

**Obscures:**

- The mechanisms of technical change
- Distribution and inequality (≠ classics)
- International power relations and historical path dependencies
- Environmental implications and limits
- Institutions, sectoral composition, and heterogeneity of capital

# Outlook

- Mainstream growth models must be **complemented** by other approaches
- **Institutional and evolutionary approaches** (Sessions 5 and 7)
  - Institutions explain technological change and global inequality
- **Marxist and dependency approaches** (Session 6)
  - In capitalism, development depends on underdevelopment elsewhere
  - Imperialism, exploitation, and unequal exchange as systemic problems
- **Ecological economics and degrowth** (Session 8)
  - Systematic relationship between growth and environmental problems
  - Technical change might not save us, growth as problem rather than solution