Experiments

Claudius Gräbner-Radkowitsch

2025-06-13

Table of contents

1 Introduction **2**

2 The data used **3**
2.1 The between leadership study . . . 3
2.2 The within leadership study . . . 4
2.3 The communication study . . . 4
2.4 The factorial study . . . 5

3 Part 2: Simple Experiments - t-tests **6**
3.1 Descriptive statistics . . . 6
3.2 Assumption Checking . . . 9
3.3 Independent t-test . . . 11
3.4 Effect Size Calculation . . . 11
3.5 Paired t-test Example . . . 12

4 Part 3: Multi-Group Experiments - ANOVA (25 minutes) **13**
4.1 Descriptive statistics . . . 14
4.2 ANOVA Assumptions . . . 15
4.3 One-Way ANOVA . . . 18
4.4 Detour: categorical variables in Regression . . . 19
4.5 Effect Size for ANOVA . . . 21
4.6 Post-Hoc Comparisons . . . 21
4.7 Detour: Standardized differences across groups . . . 22

5 Part 4: Factorial Designs **23**
5.1 Two-Way ANOVA - Checking assumptions . . . 25
5.2 Two-Way ANOVA: Interpretation . . . 27

6 Part 5: Power Analysis and Sample Size Planning **30**
6.1 General aspects of power analysis . . . 30
6.2 t-Tests . . . 31
6.3 ANOVA . . . 32

7 Summary and Key Takeaways **33**
7.1 Key take-aways . . . 33

7.2 Key decisions to be made 33

1 Introduction

In this lab we will learn how to analyze data obtained from experiments. We will complement the lecture by also introducing some additional, practically relevant concepts.

More precisely, we focus on the following aspects:

- Import and explore datasets as typically produced by experiments
- Conduct t-tests for simple experimental comparisons
- Perform ANOVA for multi-group comparisons
- Understand how ANOVA is a special case of linear regression
- Analyze factorial experimental designs
- Calculate and interpret effect sizes
- Create visualizations of experimental results

Throughout the tutorial we will use the following packages:

```
library(dplyr)         # Data manipulation
library(ggplot2)       # Data visualization
library(ggdist)        # More visualization options
library(readr)         # Simple data import
library(broom)         # Extract model data
library(effectsize)    # Effect size calculations
library(car)           # Advanced ANOVA functions
library(emmeans)       # Post-hoc comparisons
library(knitr)         # For nice tables
library(kableExtra)    # For enhanced table formatting
library(patchwork)     # For aligning multiple plots
library(pwr)           # For power analysis and sample size planning
```

We will use the following data sets, which are available for download from the lab webpage.

```
leadership_study_between <- read_csv(
  file = "leadership_study_between.csv", show_col_types = FALSE)
leadership_study_within <- read_csv(
  file = "leadership_study_within.csv", show_col_types = FALSE)
communication_study <- read_csv(
  file = "communication_study.csv", show_col_types = FALSE)
factorial_study <- read_csv(
  file = "factorial_study.csv", show_col_types = FALSE)
```

For each main analysis tool - t-test, one-way ANOVA, two-way ANOVA - there are a number of steps to be taken:

1. Get an exploratory overview over the data

2. Check the assumptions of the analysis tool
3. Implement the analysis tool
4. Calculate standardized effect sizes
5. Conduct post-hoc tests, if necessary

Since the commands are basically the same for all tools, we discuss these steps more extensively in the beginning when dealing with t-tests, and only provide example code later on.

2 The data used

As usual, it is a good idea to start with looking at the data sets, such that you know what the data looks like:[1]

2.1 The between leadership study

Results of an experiment where the treatment group has received leadership training, and the control group did not. Both groups participated in an exercise where their performance was assessed.

```
head(leadership_study_between) |>
  kable()
```

participant_id	group	team_performance
1	control	69.39524
2	control	72.69823
3	control	90.58708
4	control	75.70508
5	control	76.29288
6	control	92.15065

```
summary(leadership_study_between) |>
  kable()
```

participant_id	group	team_performance
Min. : 1.00	Length:60	Min. : 55.33
1st Qu.:15.75	Class :character	1st Qu.: 70.71
Median :30.50	Mode :character	Median : 79.33
Mean :30.50	NA	Mean : 79.16
3rd Qu.:45.25	NA	3rd Qu.: 87.32

[1] I use the function `kable()` for nicer output in the html file. When you replicate the code in R-Studio its best to skip the part `|> kable()`.

participant_id	group	team_performance
Max. :60.00	NA	Max. :103.69

2.2 The within leadership study

The effectiveness of a leadership training was assessed by testing the performance of a group before and after the training.

```
head(leadership_study_within) |>
  kable()
```

participant_id	pre_performance	post_performance
31	77.94803	86.26464
32	74.38718	79.04929
33	71.90985	90.95126
34	64.97675	90.78133
35	80.88522	90.21581
36	65.19792	88.88640

```
summary(leadership_study_within) |>
  kable()
```

participant_id	pre_performance	post_performance
Min. :31.00	Min. :56.66	Min. : 66.51
1st Qu.:38.25	1st Qu.:64.46	1st Qu.: 78.97
Median :45.50	Median :68.07	Median : 82.48
Mean :45.50	Mean :69.25	Mean : 83.78
3rd Qu.:52.75	3rd Qu.:73.72	3rd Qu.: 89.57
Max. :60.00	Max. :87.50	Max. :103.69

2.3 The communication study

An experiment where employees communicated only by one of three possible ways (face_to_face, via video calls or via email) and then their work satisfaction was assessed.

```
head(communication_study) |>
  kable()
```

participant_id	communication_method	satisfaction_score	task_completion_time
1	face_to_face	5.587774	22.63382

participant_id	communication_method	satisfaction_score	task_completion_time
2	face_to_face	7.946131	21.65397
3	face_to_face	8.161050	31.79263
4	face_to_face	5.533329	32.06459
5	face_to_face	6.342772	21.33613
6	face_to_face	6.811127	24.59724

```
summary(communication_study) |>
  kable()
```

	participant_id	communication_method	satisfaction_score	task_completion_time
	Min. : 1.00	Length:90	Min. :2.743	Min. :15.57
	1st Qu.:23.25	Class :character	1st Qu.:5.840	1st Qu.:23.38
	Median :45.50	Mode :character	Median :6.784	Median :27.63
	Mean :45.50	NA	Mean :6.752	Mean :28.43
	3rd Qu.:67.75	NA	3rd Qu.:7.789	3rd Qu.:32.65
	Max. :90.00	NA	Max. :9.936	Max. :47.41

2.4 The factorial study

Novice and expert employees received either critical or positive feedback. Afterwards, their performance improvement was assessed.

```
head(factorial_study) |>
  kable()
```

participant_id	feedback_type	experience_level	performance_improvement
1	positive	novice	9.572290
2	positive	expert	1.217696
3	positive	novice	7.940961
4	positive	expert	8.549420
5	positive	novice	6.915946
6	positive	expert	6.546548

```
summary(factorial_study) |>
  kable()
```

participant_id	feedback_type	experience_level	performance_improvement
Min. : 1.00	Length:120	Length:120	Min. :-1.882
1st Qu.: 30.75	Class :character	Class :character	1st Qu.: 4.206
Median : 60.50	Mode :character	Mode :character	Median : 6.937
Mean : 60.50	NA	NA	Mean : 6.367
3rd Qu.: 90.25	NA	NA	3rd Qu.: 8.757
Max. :120.00	NA	NA	Max. :12.748

Short recap: How have these data sets been created? How do they connect to the experimental designs discussed in the lecture?

Possible answers

- **Dataset 1**: Classic randomized controlled trial (RCT) with treatment and control groups
- **Dataset 2**: One-way experimental design with three conditions (between-subjects)
- **Dataset 3**: 2×2 factorial design allowing us to test main effects and interactions
- **Connection to lecture**: These represent the three main experimental designs we discussed - simple, multi-group, and factorial

---

3 Part 2: Simple Experiments - t-tests

Assume we are asking the following research question:

> Does leadership training improve team performance?

One way to tackle this question is to compare a treatment group, which has received a leadership training, to a control group, which has not received such training. If the groups are otherwise similar, then this setting should help us to identify the causal effect of the leadership training.[2]

3.1 Descriptive statistics

For this task, we will use the first data set. Let us first compute the standard statistics:

[2] At this point we assume that the groups were similar before the training. In practice, it would be good to first make sure the performances of the groups before the training were similar.

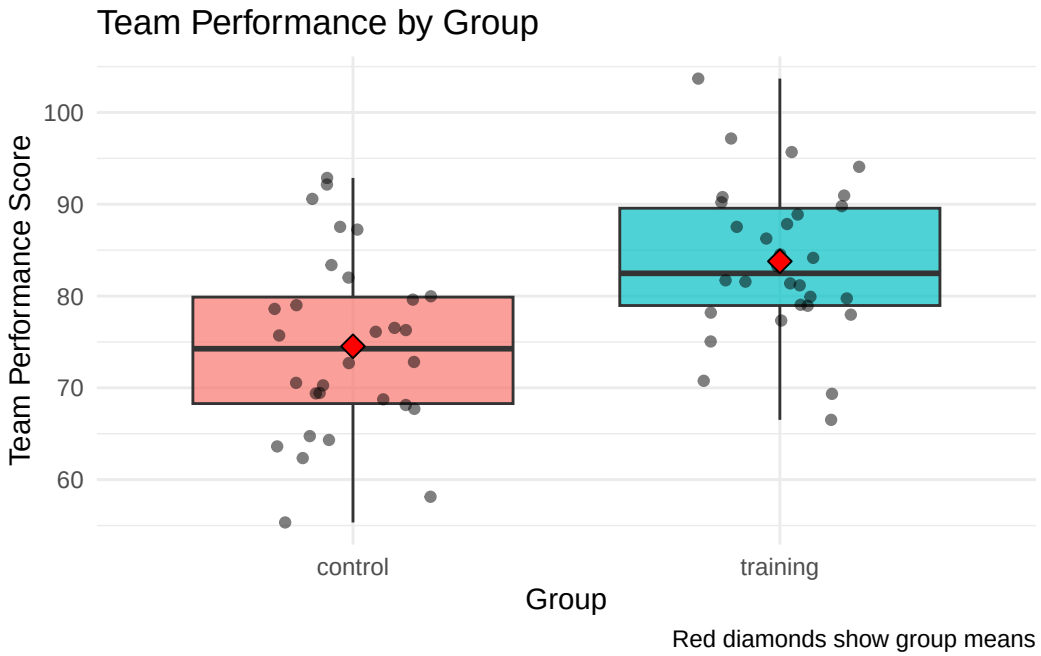
```r
descriptive_stats <- leadership_study_between %>%
  group_by(group) %>%
  summarise(
    n = n(),
    mean = mean(team_performance),
    sd = sd(team_performance),
    median = median(team_performance),
    .groups = 'drop'
  )

kable(descriptive_stats, digits = 2)
```

group	n	mean	sd	median
control	30	74.53	9.81	74.26
training	30	83.78	8.35	82.48

As usual, it is also strongly recommended to complement the quantitative info with a visualization. Data such as those is often presented using boxplots:

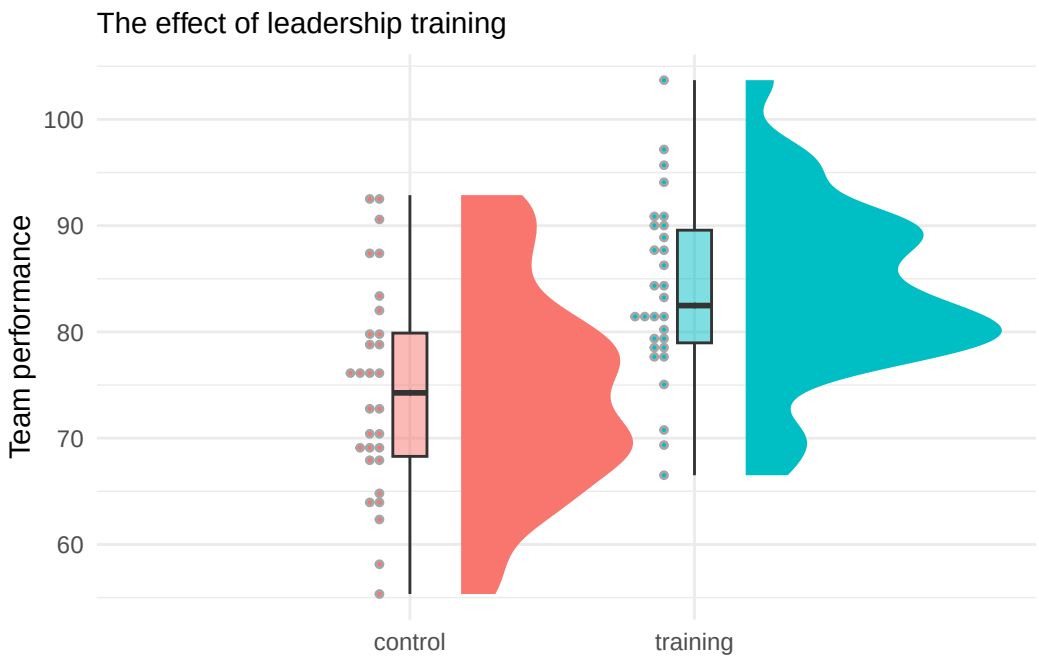
```r
ggplot(leadership_study_between, aes(x = group, y = team_performance, fill = group)) +
  geom_boxplot(alpha = 0.7) +
  geom_jitter(width = 0.2, alpha = 0.5) +
  stat_summary(fun = mean, geom = "point", shape = 23, size = 3, fill = "red") +
  labs(title = "Team Performance by Group",
       x = "Group",
       y = "Team Performance Score",
       caption = "Red diamonds show group means") +
  theme_minimal() +
  theme(legend.position = "none")
```



But boxplots might shallow important distributional info, so you should use them carefully or complement them with other tools. Below is an alternative that provides more information on the distribution of the data. For more on this issue see Holtz (2025).

```
ggplot(
  data = leadership_study_between,
  mapping = aes(x = group, y = team_performance, fill = group)
  ) +
    stat_halfeye(
    adjust = 0.5,
    justification = -0.2,
    .width = 0,
    point_colour = NA
  ) +
  geom_boxplot(
    width = 0.12,
    outlier.color = NA,
    alpha = 0.5
  ) +
  stat_dots(
    side = "left",
    justification = 1.1,
    binwidth = 0.85
  ) +
  labs(
    title = "The effect of leadership training",
    y = "Team performance") +
  theme_minimal() +
```

```
  theme(
    legend.position = "none",
    plot.title = element_text(size = 11),
    axis.title.x = element_blank()
  )
```



3.2 Assumption Checking

In the following we want to compare the means across independent groups. To this end, we may use a t-test.

But such statistical tests make specific assumptions about the data. If these assumptions were violated, the results would be unreliable or incorrect. Therefore, it is important to check the adequacy of these assumptions for the data at hand first.

And no worries if the assumptions for one test are violated - usually there are alternatives available.

In the present case, we want to use a simple t-test. This test makes two assumptions:

1. The two groups each are normally distributed.
2. The variances of both groups are the same.

To test the first assumption, we can use the **Shapiro-Wilk Test for Normality**. Here we test the following hypothesis:

- H_0 : The data is normally distributed
- H_1 : The data is not normally distributed

Thus, we we get $p > 0.05$, we cannot reject H_0. But for smaller p-values, we should reject H_0 and need to look for alternatives to the standard t-test.

```
leadership_study_between |>
  filter(group=="control") |>
  pull(team_performance) |>
  shapiro.test()
```

```
    Shapiro-Wilk normality test

data:  pull(filter(leadership_study_between, group == "control"), team_performance)
W = 0.97894, p-value = 0.7966
```

```
leadership_study_between |>
  filter(group=="training") |>
  pull(team_performance) |>
  shapiro.test()
```

```
    Shapiro-Wilk normality test

data:  pull(filter(leadership_study_between, group == "training"), team_performance)
W = 0.98662, p-value = 0.9614
```

Good! We cannot reject the hypothesis of normally distributed data as the p-value is much larger than 0.05.

The next step is to test, whether both groups have the same variance. Levene's test can be used to do exactly this. It tests:

- H_0: The variances are equal across groups
- H_1: The variances are not equal across groups

If $p > 0.05$, we do not reject H_0 and we can use a simple t-test. If we have to reject H_0, however, it would be better to use the more robust Welch test.

```
car::leveneTest(team_performance ~ group, data = leadership_study_between)
```

```
Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  1  1.0763 0.3038
      58
```

Again, the p-value is much larger than 0.05, so we can safely continue with the standard t-test.

3.3 Independent t-test

The standard independent t-test is the common tool to compare means between two groups when we have continuous data and want to test if there's a statistically significant difference.

```
t_test_result <- t.test(
  team_performance ~ group,
  data = leadership_study_between,
  var.equal = TRUE # Use FALSE if variances unequal
  )
t_test_result
```

```
    Two Sample t-test

data:  team_performance by group
t = -3.9344, df = 58, p-value = 0.0002256
alternative hypothesis: true difference in means between group control and group training
95 percent confidence interval:
 -13.962870  -4.545972
sample estimates:
 mean in group control mean in group training
              74.52896               83.78338
```

The very small p-value indicates that the difference of -9.25 is highly significant!

3.4 Effect Size Calculation

The previous result tells us that the difference in means between the groups appeaers to be about -9.25. But is this a lot? Effect sizes tell us about the practical significance of our findings by relating the absolute numbers to the scale of the measurement.

Unlike p-values, effect sizes are not influenced by sample size and help us understand if our statistically significant result is also practically meaningful.

The standard measure is **Cohen's d**: This standardized effect size tells us how many standard deviations apart the two group means are. Its interpretation follows a convention:

- **Small effect**: $d \approx 0.2$ (groups overlap about 85%)
- **Medium effect**: $d \approx 0.5$ (groups overlap about 67%)
- **Large effect**: $d \approx 0.8$ (groups overlap about 53%)

The implementation in R is trivial:

```
cohens_d <- effectsize::cohens_d(team_performance ~ group, data = leadership_study_betwee
print(cohens_d)
```

```
Cohen's d |         95% CI
--------------------------
-1.02     | [-1.55, -0.47]

- Estimated using pooled SD.
```

The key value here is Cohen's d of -1.02! This suggests a large and practically meaningful effect!

3.5 Paired t-test Example

Next, we might want to look at our research question from a slightly different angle. Rather than the between-subject design from above, we now take a *within-subject* view: to this end, we want to check whether the training had an effect on those people who were in the training (treatment) group by comparing their performance before and after the training.

To this end, we use the data set `leadership_study_within`, and then use the function `t.test()` with the argument `paired = TRUE`. This makes sure we are using the version of the test for the within-subjects context:

```
paired_result <- t.test(
  leadership_study_within$post_performance,
  leadership_study_within$pre_performance,
  paired = TRUE)
print(paired_result)
```

```
    Paired t-test

data:  leadership_study_within$post_performance and leadership_study_within$pre_performa
t = 7.4774, df = 29, p-value = 3.059e-08
alternative hypothesis: true mean difference is not equal to 0
95 percent confidence interval:
 10.55898 18.51001
sample estimates:
mean difference
       14.53449
```

As above, we should also compute standardized effect sizes. There are two options: using the t-test object directly, or using the raw data. If you do the latter, make sure to set `paired=TRUE` to use the correct version for the within context:

```
cohens_d_within_a <- effectsize::cohens_d(paired_result)
print(cohens_d_within_a)
```

```
Cohen's d |       95% CI
------------------------
1.37      | [0.86, 1.86]
```

```r
cohens_d_within_b <- effectsize::cohens_d(
  leadership_study_within$post_performance,
  leadership_study_within$pre_performance,
  paired = TRUE)
print(cohens_d_within_b)
```

```
Cohen's d |       95% CI
------------------------
1.37      | [0.86, 1.86]
```

Exercise (5 minutes): Interpret the results of the test above. What can we conclude about the effectiveness of leadership training?

Possible answers

- **Statistical significance**: If $p < 0.05$, training significantly improved performance; since $p \approx 0$, the training has a highly significant effect
- **Effect size interpretation**: Cohen's d is large, so we have a large effect. This suggests the effect of the training is also practically meaningful.
- **Confidence interval**: If the CI doesn't include 0, we're confident there's a real difference; even if we are very conservative, we would still expect a 10 point improvement of the training.
- **Business implication**: Training appears effective and worth the investment

---

4 Part 3: Multi-Group Experiments - ANOVA (25 minutes)

Let us now turn to the following research question:

> Which communication method (face-to-face, video call, email) leads to highest satisfaction?

Note that this time we not only compare one group to another as in the previous section, but we need to compare three groups with each other as we have three different communication methods. Therefore, we cannot use simple t-tests, but need to use an ANOVA.

4.1 Descriptive statistics

But first, let us again look at the data:

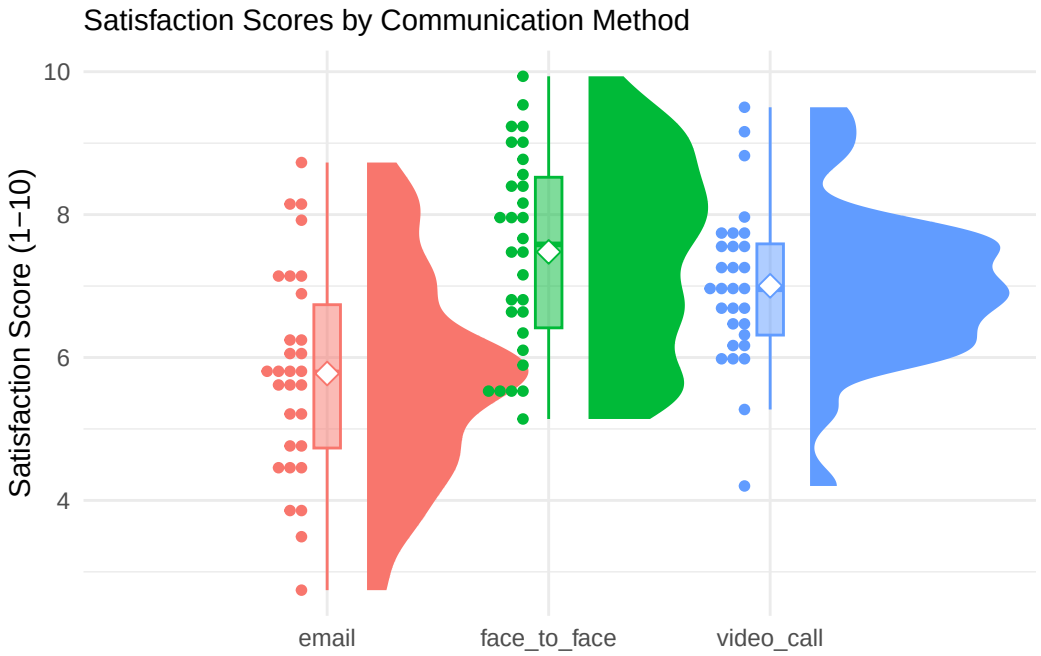
```
communication_study %>%
  group_by(communication_method) %>%
  summarise(
    n = n(),
    mean = mean(satisfaction_score),
    sd = sd(satisfaction_score),
    min = min(satisfaction_score),
    max = max(satisfaction_score),
    .groups = 'drop'
  ) |>
  kable(digits = 2)
```

communication_method	n	mean	sd	min	max
email	30	5.78	1.46	2.74	8.73
face_to_face	30	7.48	1.39	5.14	9.94
video_call	30	7.00	1.10	4.20	9.50

And complement this by a visualization:

```
ggplot(
  data = communication_study,
  mapping = aes(
    x = communication_method,
    y = satisfaction_score,
    fill = communication_method,
    color = communication_method)
  ) +
    stat_halfeye(
    adjust = 0.5,
    justification = -0.2,
    .width = 0,
    point_colour = NA
  ) +
  geom_boxplot(
    width = 0.12,
    outlier.color = NA,
    alpha = 0.5
  ) +
  stat_dots(
    side = "left",
    justification = 1.1,
    binwidth = 0.15
```

```
  ) +
  stat_summary(fun = mean, geom = "point", shape = 23, size = 3, fill = "white") +
  labs(
    title = "Satisfaction Scores by Communication Method",
    x = "Communication Method",
    y = "Satisfaction Score (1-10)") +
  theme_minimal() +
  theme(
    legend.position = "none",
    plot.title = element_text(size = 11),
    axis.title.x = element_blank()
  )
```



4.2 ANOVA Assumptions

ANOVA is more robust than t-tests but still requires certain conditions to be met for valid results. In fact, we are testing the same assumptions as in the t-test case:

- **Normality of Residuals**: For ANOVA, we check if the residuals (not the raw data) are normally distributed.
- **Homogeneity of Variances**: ANOVA assumes that the variance of the dependent variable is equal across all groups.

Let us start with testing the normality of the residuals. We again use the Shapiro test, which tests the following hypothesis:

- H_0: Residuals are normally distributed
- H_1: Residuals are not normally distributed

Thus, if $p > 0.05$, the Null cannot be rejected and we can assume the residuals to follow a normal distribution. If $p \leq 0.05$, however, the hypothesis of normally distributed residuals must be rejected and we need to consider transforming the data or using a non-parametric test.

```
aov_model <- aov(satisfaction_score ~ communication_method, data = communication_study)

shapiro.test(residuals(aov_model))
```

```
    Shapiro-Wilk normality test

data:  residuals(aov_model)
W = 0.99149, p-value = 0.8342
```

Since $p > 0.05$ we are on the save side!

We then check the equality of variances and again use Levene's test with the following hypotheses:

- H_0: Variances are equal across all groups
- H_1: Variances are not equal across groups

Thus, if $p > 0.05$, the Null cannot be rejected and we can assume the variances to be equal. If $p \leq 0.05$, however, the hypothesis of equal variances must be rejected and we need to consider transforming the data or using Welch's ANOVA.

```
car::leveneTest(satisfaction_score ~ communication_method, data = communication_study)
```

```
Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  2  1.8801 0.1587
      87
```

Again, the Null cannot be rejected and we can continue with the ANOVA as planned.

Before we enter the actual analysis we can also create the common diagnostics plots:

```
model_data <- augment(aov_model) %>%
  mutate(
    sqrt_abs_resid = sqrt(abs(.std.resid)),
    obs_number = row_number()
  )

residuals_vs_fitted <- model_data %>%
  ggplot(aes(x = .fitted, y = .resid)) +
  geom_point(alpha = 0.7) +
  geom_smooth(method = "loess", se = FALSE, color = "red") +
```

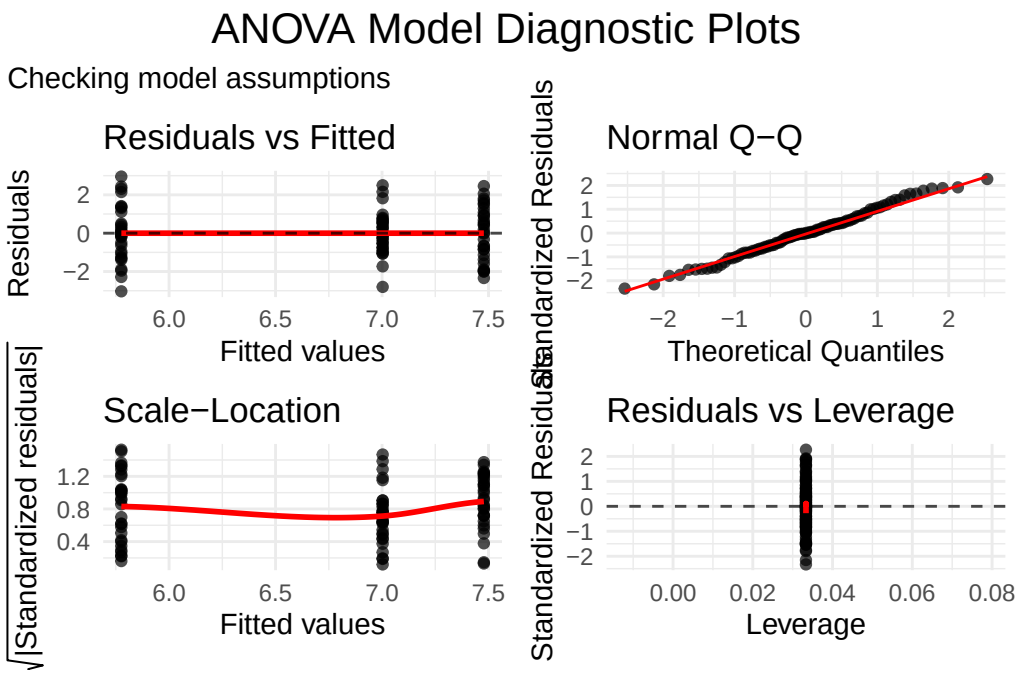
```
  geom_hline(yintercept = 0, linetype = "dashed", alpha = 0.7) +
  labs(title = "Residuals vs Fitted", x = "Fitted values", y = "Residuals") +
  theme_minimal()

qq_plot <- model_data %>%
  ggplot(aes(sample = .std.resid)) +
  stat_qq(alpha = 0.7) +
  stat_qq_line(color = "red") +
  labs(title = "Normal Q-Q", x = "Theoretical Quantiles", y = "Standardized Residuals") +
  theme_minimal()

scale_location <- model_data %>%
  ggplot(aes(x = .fitted, y = sqrt_abs_resid)) +
  geom_point(alpha = 0.7) +
  geom_smooth(method = "loess", se = FALSE, color = "red") +
  labs(title = "Scale-Location", x = "Fitted values",
       y = expression(sqrt("|Standardized residuals|"))) +
  theme_minimal()

residuals_vs_leverage <- model_data %>%
  ggplot(aes(x = .hat, y = .std.resid)) +
  geom_point(alpha = 0.7) +
  geom_smooth(method = "loess", se = FALSE, color = "red") +
  geom_hline(yintercept = 0, linetype = "dashed", alpha = 0.7) +
  labs(title = "Residuals vs Leverage", x = "Leverage", y = "Standardized Residuals") +
  theme_minimal()

combined_patchwork <- (residuals_vs_fitted | qq_plot) /
                          (scale_location | residuals_vs_leverage) +
  plot_annotation(
    title = "ANOVA Model Diagnostic Plots",
    subtitle = "Checking model assumptions",
    theme = theme(plot.title = element_text(size = 16, hjust = 0.5))
  )
combined_patchwork
```



- **Residuals vs Fitted**: Should show random scatter (no patterns)
- **Q-Q plot**: Points should follow the diagonal line (normality)
- **Scale-Location**: Should show random scatter (equal variances)
- **Residuals vs Leverage**: Identifies influential outliers

4.3 One-Way ANOVA

Since we compare three groups we do not use t-tests but an ANOVA.

As you know, ANOVA is actually a special case of linear regression. Therefore, we can get the ANOVA results in two equivalent ways.

The first option is to use the classical `aov()` function:

```
anova_result <- aov(satisfaction_score ~ communication_method, data = communication_study
summary(anova_result)
```

```
                    Df Sum Sq Mean Sq F value   Pr(>F)
communication_method  2  46.29  23.146    13.2 9.81e-06 ***
Residuals            87 152.50   1.753
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The second option is to use `lm()` as we know it from linear regression:

```
lm_model <- lm(satisfaction_score ~ communication_method, data = communication_study)
anova(lm_model)
```

```
Analysis of Variance Table

Response: satisfaction_score
                     Df  Sum Sq Mean Sq F value    Pr(>F)
communication_method  2  46.292 23.1460  13.205 9.813e-06 ***
Residuals            87 152.497  1.7528
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- The overall p-value is extremely small, so the Null hypothesis of no difference between the groups should definitely be rejected
- In other words: Communication method significantly affects satisfaction scores
- This means that at least one communication method produces significantly different satisfaction scores than the others
- But the result does not tell us which specific methods differ from each other (need post-hoc tests, see below)
- The result also does not contain information about the direction of differences (which method is best/worst) or the effect size

4.4 Detour: categorical variables in Regression

When you add a categorial variable as a predictor to your regression, R automatically creates dummy variables for categorical predictors. The first level alphabetically becomes the reference group:

```
summary(lm_model)$coefficients |>
  kable(digits = 3)
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	5.776	0.242	23.895	0.000
communication_method-face_to_face	1.702	0.342	4.980	0.000
communication_method-video_call	1.227	0.342	3.590	0.001

Since 'email' comes first alphabetically, it is the reference group.

- **Intercept** = mean of reference group (email, comes first alphabetically)
- **communication_methodface_to_face** = difference between face_to_face and email
- **communication_methodvideo_call** = difference between video_call and email

We can verify this manually:

```
communication_study %>%
  group_by(communication_method) %>%
  summarise(mean = mean(satisfaction_score), .groups = 'drop') |>
  kable(digits = 2)
```

communication_method	mean
email	5.78
face_to_face	7.48
video_call	7.00

4.4.1 Detour: When to Use `lm()` vs `aov()`

As shown above, both approaches give identical results. Still, they offer different perspectives:

Use aov() when: - You want traditional ANOVA output - Focus is on group comparisons - Need post-hoc tests such as `TukeyHSD()`, which take the `aov`-model as an input

Use lm() when: - You want to see specific contrasts - Planning to add continuous covariates later - Want regression-style interpretation - Building toward more complex models

Example: From ANOVA to ANCOVA

If we add a continuous variable to an ANOVA, we get an ANCOVA:

```
ancova_model <- lm(satisfaction_score ~ communication_method + task_completion_time,
                   data = communication_study)
summary(ancova_model)
```

```
Call:
lm(formula = satisfaction_score ~ communication_method + task_completion_time,
    data = communication_study)

Residuals:
     Min       1Q   Median       3Q      Max
-3.07895 -0.87806 -0.00768  0.78530  2.90678

Coefficients:
                                Estimate Std. Error t value Pr(>|t|)
(Intercept)                     5.628267   0.807830   6.967 6.15e-
10 ***
communication_methodface_to_face 1.737394   0.389664   4.459 2.48e-
05 ***
communication_methodvideo_call   1.253306   0.369901   3.388  0.00106 **
task_completion_time            0.004472   0.023342   0.192  0.84853
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.331 on 86 degrees of freedom
Multiple R-squared:  0.2332,    Adjusted R-squared:  0.2064
F-statistic: 8.718 on 3 and 86 DF,  p-value: 4.108e-05
```

In the example, we now control for individual differences in task completion time. While trivial in the `lm()`-context, this would be much harder to do with the aov() approach!

4.5 Effect Size for ANOVA

η^2 tells us what proportion of the total variance in the dependent variable is explained by the independent variable and, as explained above, serves as a standardized measure for comparing effect sizes:

Remember that:

- **Small effect**: ^{2} 0.01 (1% of variance)
- **Medium effect**: ^{2} 0.06 (6% of variance)
- **Large effect**: ^{2} 0.14 (14% of variance)

In our case:

```
eta_squared <- effectsize::eta_squared(anova_result)
print(eta_squared)
```

```
# Effect Size for ANOVA

Parameter             | Eta2 |       95% CI
-----------------------------------------
communication_method | 0.23 | [0.11, 1.00]

- One-sided CIs: upper bound fixed at [1.00].
```

$\eta = 0.23$ means communication method explains 23 % of the variance of satisfaction scores.

4.6 Post-Hoc Comparisons

The ANOVA tells us there's a difference somewhere among the groups, but not which specific groups differ. This is why we need post-hoc tests: they provide these pairwise comparisons while controlling for multiple testing.

> **The multiple testing problem**
>
> If we do multiple t-tests (e.g. one for each pairwise comparison), our Type I error rate will inflate. Post-hoc tests adjust critical values to maintain overall $\alpha = 0.05$.

The most common post-hoc test has a nice name: Tukey's Honestly Significant Difference (HSD). It takes the fitted ANOVA model as its input:

```
tukey_result <- TukeyHSD(anova_result)
print(tukey_result)
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = satisfaction_score ~ communication_method, data = communication_study)

$communication_method
                              diff        lwr       upr     p adj
face_to_face-email       1.702240  0.8871253 2.5173543 0.0000095
video_call-email         1.227135  0.4120202 2.0422492 0.0015714
video_call-face_to_face -0.475105 -1.2902195 0.3400094 0.3506328
```

This suggests that the following differences exist:

- Face-to-face > Email: 1.70 points higher satisfaction ($p < 0.001$)
- Video call > Email: 1.23 points higher satisfaction ($p = 0.002$)

But there are no significant differences between video calls and face-to-face ($p = 0.35$)

Thus, both face-to-face and video call communication methods produce significantly higher satisfaction scores than email, but face-to-face and video call don't differ significantly from each other.

4.7 Detour: Standardized differences across groups

Now we might want to standardize the differences between groups. Otherwise it is hard to judge whether they are also practically meaningful. Unfortunately, I am not aware of a function in any of the common packages that does this.

But we can do it manually. We first need the mean squared error from the ANOVA summary object:

```
summary(anova_result)
```

```
                     Df Sum Sq Mean Sq F value   Pr(>F)
communication_method  2  46.29  23.146    13.2 9.81e-06 ***
Residuals            87 152.50   1.753
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

This is the `1.753`! The square root is the pooled standard deviation:

```
pooled_sd <- sqrt(1.753)
```

This is how we normalize the differences from our Tukey table above:

```
tukey_differences <- c(
  face_to_face_vs_email = tukey_result$communication_method[1],
  video_call_vs_email = tukey_result$communication_method[2],
  video_call_vs_face_to_face = tukey_result$communication_method[3]
)
pairwise_cohens_d_values <- tukey_differences / pooled_sd

tibble(
  "Comparison"=names(pairwise_cohens_d_values),
  "Standardized difference" = pairwise_cohens_d_values
  ) |>
  kable()
```

Comparison	Standardized difference
face_to_face_vs_email	1.2856708
video_call_vs_email	0.9268326
video_call_vs_face_to_face	-0.3588382

So the two statistically significant differences are large effects (so also practically significant), whereas the insignificant difference would only have counted as small to medium effect.

5 Part 4: Factorial Designs

For this last part we consider the following research question:

> How do feedback type and experience level interact to affect performance improvement?

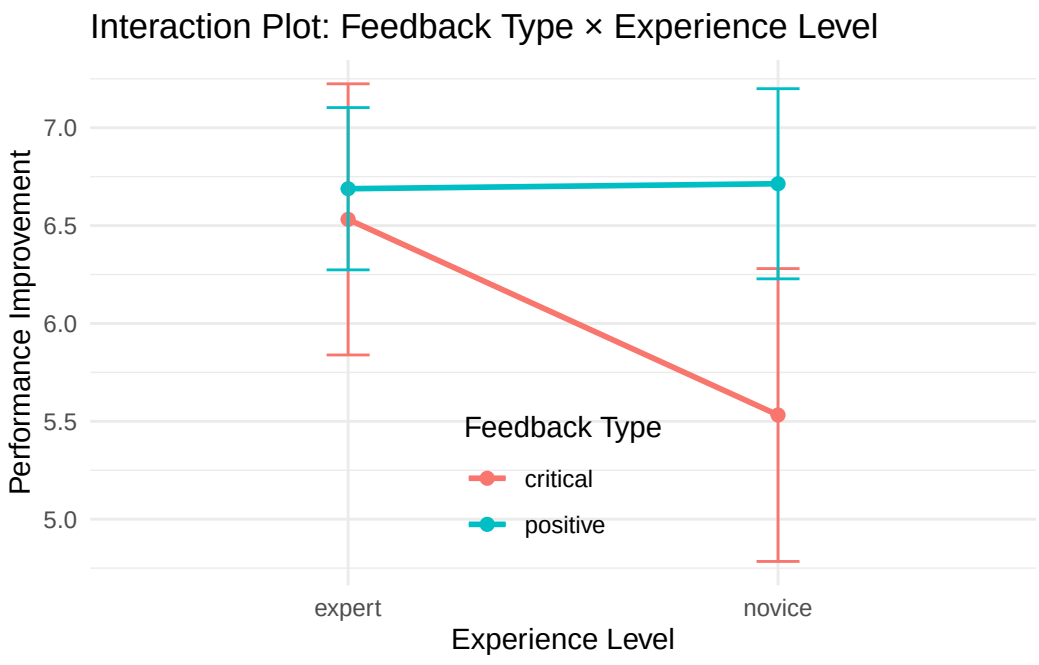
We use the data stored as `factorial_study`:

```
factorial_study %>%
  group_by(feedback_type, experience_level) %>%
  summarise(
    n = n(),
    mean = mean(performance_improvement),
    sd = sd(performance_improvement),
    .groups = 'drop'
  ) |>
  kable(digits = 2)
```

feedback_type	experience_level	n	mean	sd
critical	expert	30	6.53	3.79
critical	novice	30	5.53	4.10
positive	expert	30	6.69	2.27
positive	novice	30	6.71	2.66

We then visualize the relationship using an interaction plot as discussed in the lecture:

```
ggplot(
  data = factorial_study,
  mapping = aes(
    x = experience_level,
    y = performance_improvement,
    color = feedback_type,
    group = feedback_type)
  ) +
  stat_summary(fun = mean, geom = "point", size = 2) +
  stat_summary(fun = mean, geom = "line", linewidth = 1) +
  stat_summary(fun.data = mean_se, geom = "errorbar", width = 0.1) +
  guides(color = guide_legend(position = "inside")) +
  labs(
    title = "Interaction Plot: Feedback Type × Experience Level",
    x = "Experience Level",
    y = "Performance Improvement",
    color = "Feedback Type") +
  theme_minimal() +
  theme(legend.position.inside = c(0.5, 0.2))
```



This already gives us a good visual impression of the results, but we also want to analyze the results quantitatively.

5.1 Two-Way ANOVA - Checking assumptions

The factorial design allows us to consider interaction effects among factors. But to detect such interaction, we must use a two-way ANOVA, not the traditional one!

In a first step we need to again check the assumptions of the two-way ANOVA, which are in fact the same as in the previous case:

1. Normality of *residuals* (not raw data)
2. Homogeneity of variances (across *all* groups)

Regarding the first assumption, we again do the Shapiro-Wilk test on the residuals. To get the residuals we need to fit the model first. For this purpose, we use the same function `aov()` (or `lm()`) as previously, but add the additional factor to the formula:

```
factorial_model <- aov(
  formula = performance_improvement ~ feedback_type * experience_level,
  data = factorial_study)
```

Two-way ANOVA and linear regression

While the classical ANOVA was the same as simple linear regression with categorial variables, two-way ANOVA is the same as *multiple* regression with interaction effects:

```
lm_factorial <- lm(performance_improvement ~ feedback_type * experience_level,
                   data = factorial_study)
anova(lm_factorial)
```

```
Analysis of Variance Table

Response: performance_improvement
                                Df  Sum Sq Mean Sq F value Pr(>F)
feedback_type                    1   13.43 13.4301  1.2373 0.2683
experience_level                 1    7.12  7.1155  0.6556 0.4198
feedback_type:experience_level   1    7.88  7.8766  0.7257 0.3960
Residuals                      116 1259.07 10.8541
```

is the same as:

```
summary(factorial_model)
```

```
                                Df Sum Sq Mean Sq F value Pr(>F)
feedback_type                    1   13.4  13.430   1.237  0.268
experience_level                 1    7.1   7.115   0.656  0.420
feedback_type:experience_level   1    7.9   7.877   0.726  0.396
Residuals                      116 1259.1  10.854
```

We then extract the residuals using the function `residuals()` and pass them to the `shapiro.test()`:

```
shapiro.test(x = residuals(factorial_model))
```

```
    Shapiro-Wilk normality test

data:  residuals(factorial_model)
W = 0.98225, p-value = 0.1153
```

Since we test the same assumptions as above, $p > 0.05$ means that we cannot reject the H_0 of normally distributed residuals.

Let us then turn to the second assumption: Homogeneity of variances across *all* groups. The syntax remains basically the same as above:

```
car::leveneTest(
  performance_improvement ~ feedback_type * experience_level,
  data = factorial_study)
```

```
Levene's Test for Homogeneity of Variance (center = median)
       Df F value    Pr(>F)
group   3  6.2621 0.0005615 ***
      116
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Oh no! This time $p < 0.05$, so we need to reject H_0 of equal variances, meaning that some cells (in the sense of factor-combinations) have more variation than others.

From this it becomes clear that the variability for the groups receiving critical feedback is much higher!

> Understanding the Variance Pattern
>
> We could have a look at which cells are affected like this:
>
> ```
> factorial_study %>%
>   group_by(feedback_type, experience_level) %>%
>   summarise(
>     n = n(),
>     variance = var(performance_improvement),
>     sd = sd(performance_improvement),
>     .groups = 'drop'
>   ) %>%
>   kable(digits = 3)
> ```

feedback_type	experience_level	n	variance	sd
critical	expert	30	14.392	3.794
critical	novice	30	16.791	4.098
positive	expert	30	5.156	2.271
positive	novice	30	7.078	2.660

This suggests that some people respond very well to criticism, others very poorly and that there are important differences in how people handle criticism:

- Critical feedback cells: High variance (14-17) suggests people respond very differently to criticism
- Positive feedback cells: Low variance (5-7) suggests people respond more consistently to praise

This example shows how testing assumptions is not only relevant for its own sake, but also hints an interesting results themselfes.
The higher variance in the critical feedback cells suggests that critical feedback is "riskier" - it might help some people a lot but hurt others, but that positive feedback is more predictable in its effects.

In effect this means that the results of the standard two-way ANOVA would be less reliable. But that mean our analysis must end? Fortunately not! There are versions of the two-way ANOVA that are robust to unequal variances!

To implement such robust version we use the function `Anova()` from the `car` package, which takes the resul of the `aov()` function above. But it is important to specify `type = "III"` as this tells `R` to use the version that takes into account unequal variances:

```
factorial_model_robust <- car::Anova(factorial_model, type = "III")
```

5.2 Two-Way ANOVA: Interpretation

Let us now inspect the results of the two-way ANOVA. We will pay particular attention to the following aspects:

1. **Main effect of feedback_type**: Overall difference between positive vs critical
2. **Main effect of experience_level**: Overall difference between novice vs expert
3. **Interaction**: Does the effect of feedback depend on experience level?

```
factorial_model_robust
```

```
Anova Table (Type III tests)

Response: performance_improvement
                              Sum Sq  Df  F value Pr(>F)
(Intercept)                 1279.89   1 117.9186 <2e-16 ***
```

```
feedback_type                      0.37   1   0.0339 0.8542
experience_level                  14.98   1   1.3803 0.2424
feedback_type:experience_level     7.88   1   0.7257 0.3960
Residuals                       1259.07 116
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Dummy coding in the two-way context

With two factors, R creates dummy variables for each factor plus their interaction:

```
coefficients(lm_factorial)
```

```
                                   (Intercept)
                                     6.5317033
                          feedback_typepositive
                                     0.1566833
                         experience_levelnovice
                                    -0.9994123
feedback_typepositive:experience_levelnovice
                                     1.0247960
```

Their interpretation is as follows:

- **Intercept** = mean of reference group (critical feedback + expert)
- **feedback_typepositive** = main effect of positive vs critical for experts only
- **experience_levelnovice** = main effect of novice vs expert for critical feedback only
- **interaction** = additional effect of being novice AND receiving positive feedback

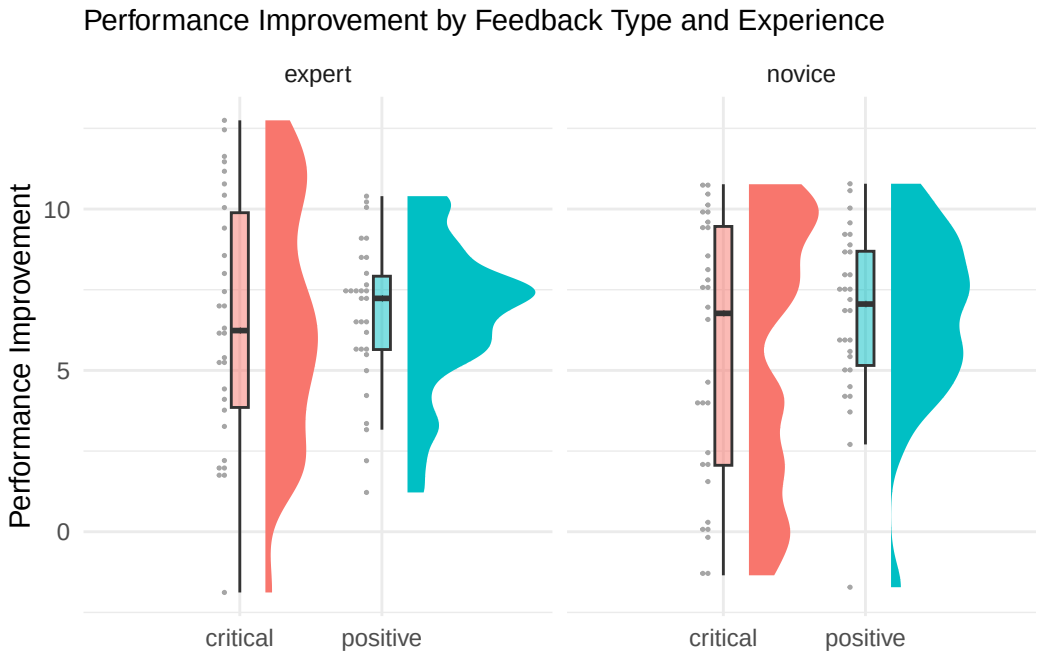
From this output we see that all three effects (both main effects and the interaction) are non-significant, suggesting that:

- Neither feedback type nor experience level significantly affects performance improvement
- The combination of these factors doesn't create any synergistic effects
- The observed differences in your sample means are likely due to random variation rather than true population differences

In fact, this is also what a visual inspection would suggest:

```
ggplot(
  data = factorial_study,
  mapping = aes(
    x = feedback_type,
    y = performance_improvement,
    fill = feedback_type)
```

```
  ) +
    stat_halfeye(
    adjust = 0.5,
    justification = -0.2,
    .width = 0,
    point_colour = NA
  ) +
  geom_boxplot(
    width = 0.12,
    outlier.color = NA,
    alpha = 0.5
  ) +
  stat_dots(
    side = "left",
    justification = 1.1,
    binwidth = 0.15
  ) +
  facet_wrap(~ experience_level) +
  labs(
    title = "Performance Improvement by Feedback Type and Experience",
    x = "Feedback Type",
    y = "Performance Improvement") +
  theme_minimal() +
  theme(
    legend.position = "none",
    plot.title = element_text(size = 11),
    axis.title.x = element_blank()
  )
```



6 Part 5: Power Analysis and Sample Size Planning

Statistical power is the probability of detecting an effect when it truly exists. You know from the lecture that the decision about sample sizes determines in part statistical power. Power analysis helps us plan adequate sample sizes and evaluate our study's sensitivity.

6.1 General aspects of power analysis

Remember the **components of power analysis:**

- **Power**: Probability of detecting effect (usually we want 0.8)
- **Effect size**: How big a difference we want to detect
- **Sample size**: Number of participants needed
- **Alpha level**: Type I error rate (usually 0.05)

We can use power analysis in two different ways:

1. **Post-hoc (observed)**: What was our power given the sample size we had?
2. **A priori (prospective)**: How many participants do we need to detect an effect?

Regarding the first, we may ask: what was our power to detect the effect we found in the leadership study?

```
observed_power <- pwr.t.test(n = 30, d = as.numeric(cohens_d$Cohens_d), sig.level = 0.05)
print(observed_power)
```

```
     Two-sample t test power calculation

              n = 30
              d = 1.015848
      sig.level = 0.05
          power = 0.9718339
    alternative = two.sided

NOTE: n is number in *each* group
```

Lets turn to the *a priori power analysis*, i.e. what you should do BEFORE collecting data to determine how many participants you need.

In a first step, you always need to specify your research parameters:

- What effect size do you want to detect?
- What power level do you want? (typically 0.8 or 0.9)
- What alpha level will you use? (typically 0.05)

The remaining steps depend on the analysis method we wish to employ:

6.2 t-Tests

Assume we want to plan a new leadership training study, similar to the one above. We want to detect a medium effect ($d = 0.5$) with 80% power.

```
sample_size_medium <- pwr.t.test(d = 0.5, power = 0.8, sig.level = 0.05)
print(sample_size_medium)
```

```

     Two-sample t test power calculation

              n = 63.76561
              d = 0.5
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

If instead we would like to identify a small effect (with $d = 0.5$). Everything else remains the same:

```
sample_size_small <- pwr.t.test(d = 0.2, power = 0.8, sig.level = 0.05)
print(sample_size_small)
```

```

     Two-sample t test power calculation

              n = 393.4057
              d = 0.2
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

And what would happen if we wanted higher power (90%) for a medium effect?

```
sample_size_high_power <- pwr.t.test(d = 0.5, power = 0.9, sig.level = 0.05)
print(sample_size_high_power)
```

```

     Two-sample t test power calculation

              n = 85.03128
              d = 0.5
      sig.level = 0.05
          power = 0.9
    alternative = two.sided

NOTE: n is number in *each* group
```

We see that small design choices can have huge effects:

```
power_results <- tibble(
  Scenario = c("Medium effect, 80% power", "Small effect, 80% power", "Medium effect, 90%
  Effect_Size = c(0.5, 0.2, 0.5),
  Power = c(0.8, 0.8, 0.9),
  Sample_per_Group = c(
    ceiling(sample_size_medium$n),
    ceiling(sample_size_small$n),
    ceiling(sample_size_high_power$n)
  ),
  Total_Sample = c(
    ceiling(sample_size_medium$n) * 2,
    ceiling(sample_size_small$n) * 2,
    ceiling(sample_size_high_power$n) * 2
  )
)

kable(power_results)
```

Scenario	Effect_Size	Power	Sample_per_Group	Total_Sample
Medium effect, 80% power	0.5	0.8	64	128
Small effect, 80% power	0.2	0.8	394	788
Medium effect, 90% power	0.5	0.9	86	172

6.3 ANOVA

Assume we are planning a communication study with 3 groups and we want to detect a medium effect (f = 0.25) with 80% power. Note that what was Cohen's d for the t-test case, has now become Cohens f for the ANOVA case:

```
sample_size_anova <- pwr.anova.test(
  k = 3, f = 0.25, sig.level = 0.05, power = 0.8)
print(sample_size_anova)
```

```
     Balanced one-way analysis of variance power calculation

              k = 3
              n = 52.3966
              f = 0.25
      sig.level = 0.05
          power = 0.8

NOTE: n is number in each group
```

Aside from that it works the same way as above!

7 Summary and Key Takeaways

7.1 Key take-aways

1. **Data exploration** is crucial before statistical testing
2. **Assumption checking** ensures valid results and guides method selection
3. **Effect sizes** provide practical significance context beyond p-values
4. **Visualization** aids interpretation and communication of results
5. **ANOVA is just regression** with categorical predictors
6. **Both aov() and lm()** give identical results but offer different perspectives
7. **Post-hoc tests** control for multiple comparisons when making pairwise comparisons
8. **Factorial designs** allow detection of interactions between factors

7.2 Key decisions to be made

- **Assumptions violated**: Choose appropriate alternatives (Welch's tests, transformations, non-parametric)
- **Multiple groups**: ANOVA preferred over multiple t-tests
- **Factorial designs**: Allow testing of interactions between factors but requires two-way ANOVA
- **Effect size interpretation**: Always consider practical alongside statistical significance
- **Post-hoc testing**: Required when ANOVA is significant to identify which groups differ

Holtz, Y. (2025) 'The boxplot and its pitfalls', *From Data to Viz*, available at https://www.data-to-viz.com/caveat/boxplot.html.