

SOEE1475 Statistics and Data Analysis

Lecture 7: Directional statistics



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Today

- Complexities of circular data
- Resultant
- von Mises distribution

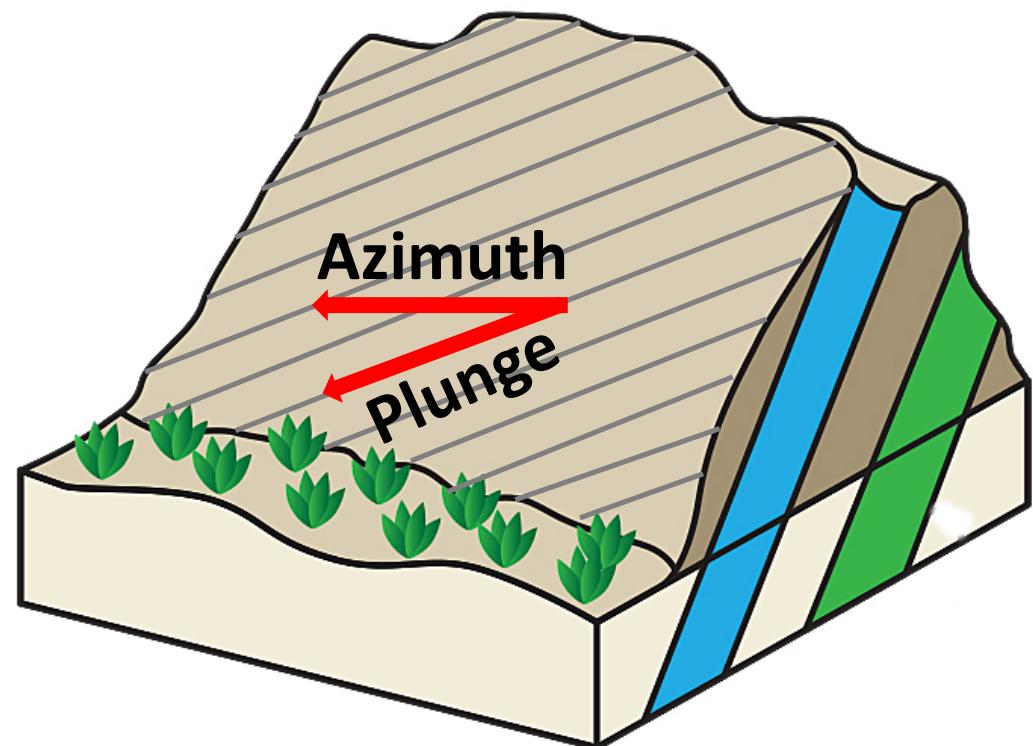
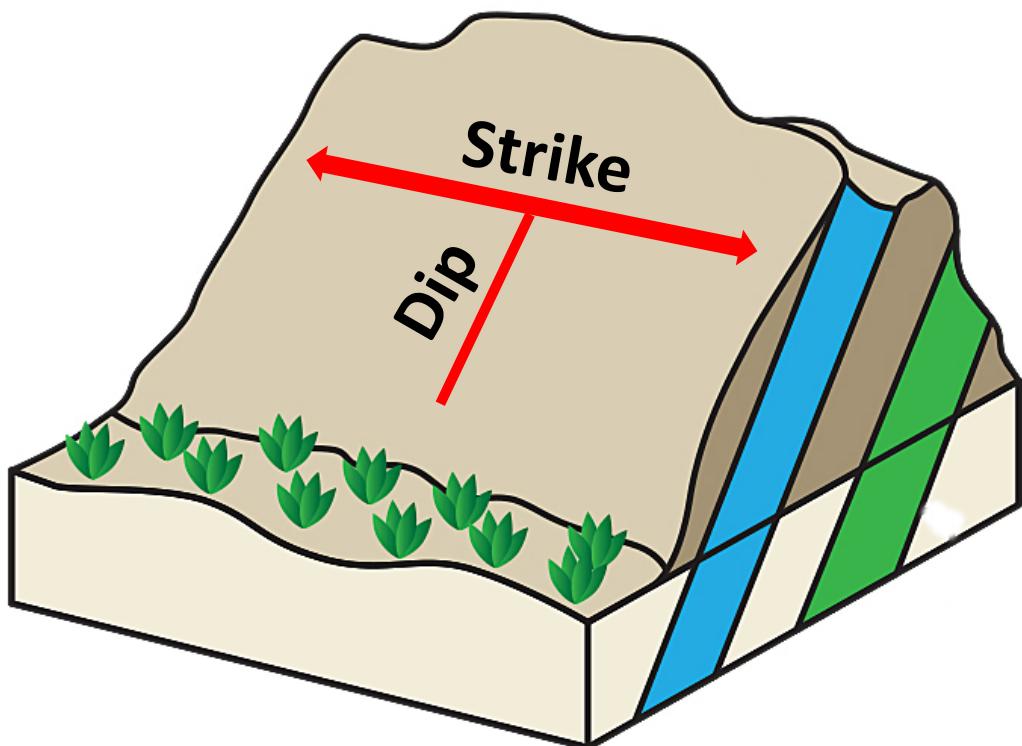


Circular measurements





Circular measurements

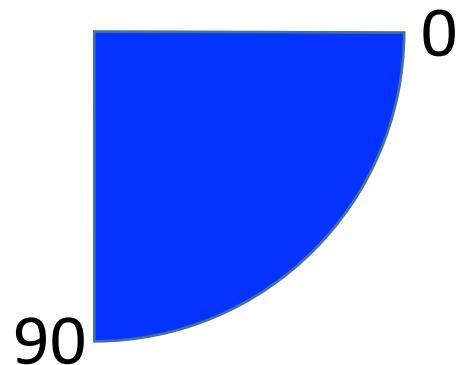




Circular measurements

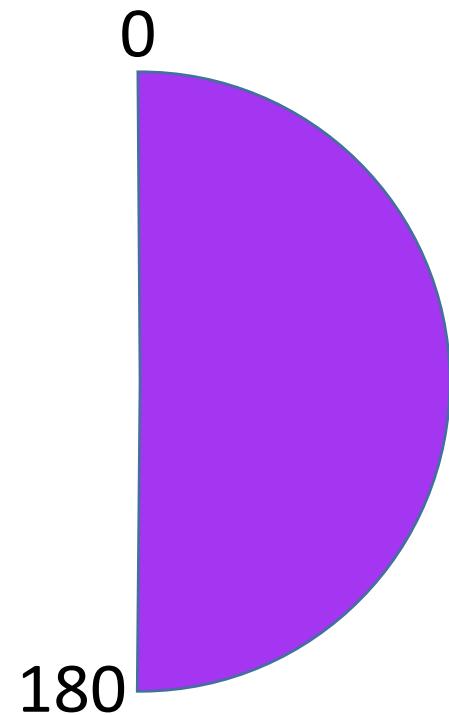
Dip/plunge

Closed, 0-90



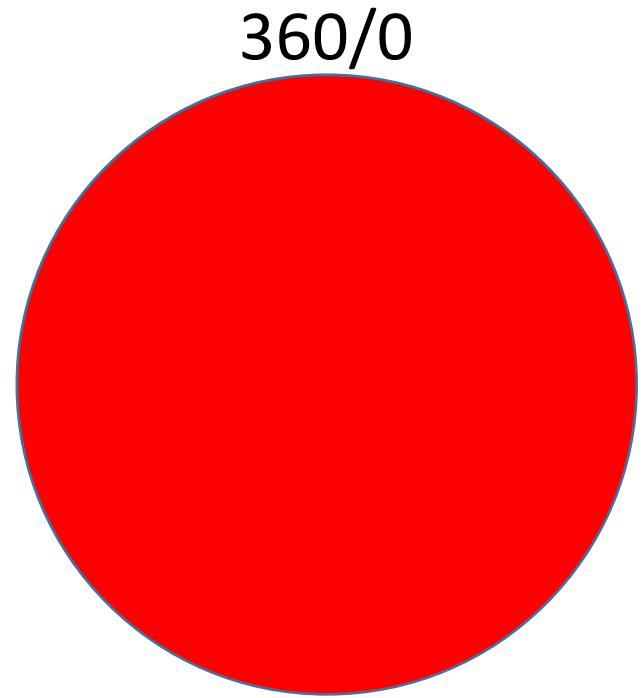
Strike

Modulo, 0-180



Azimuth

Modulo, 0-360



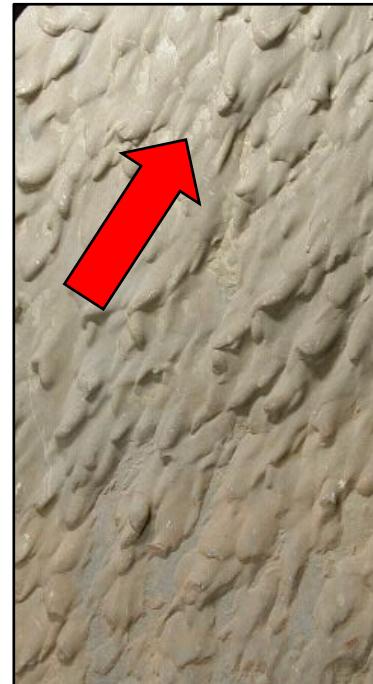


Directional measurements

Slickensides



Flute casts



Oriented fossils



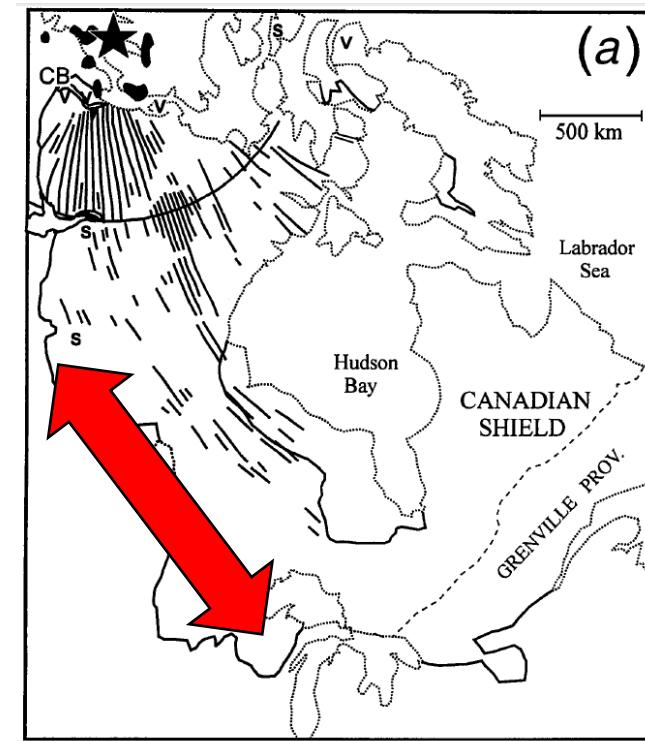


Orientation measurements

Ripple crests



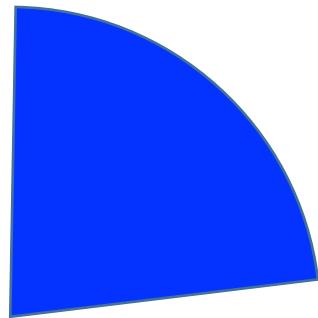
Dike swarms



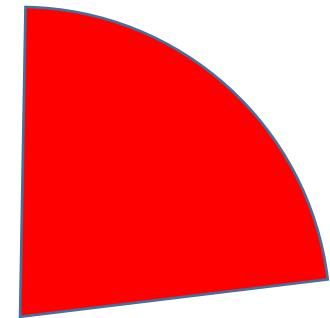


Degrees and radians

$$\text{angle in degrees} \times \frac{\pi}{180} = \text{angle in radians}$$



57.2958 degrees



1 radian



R

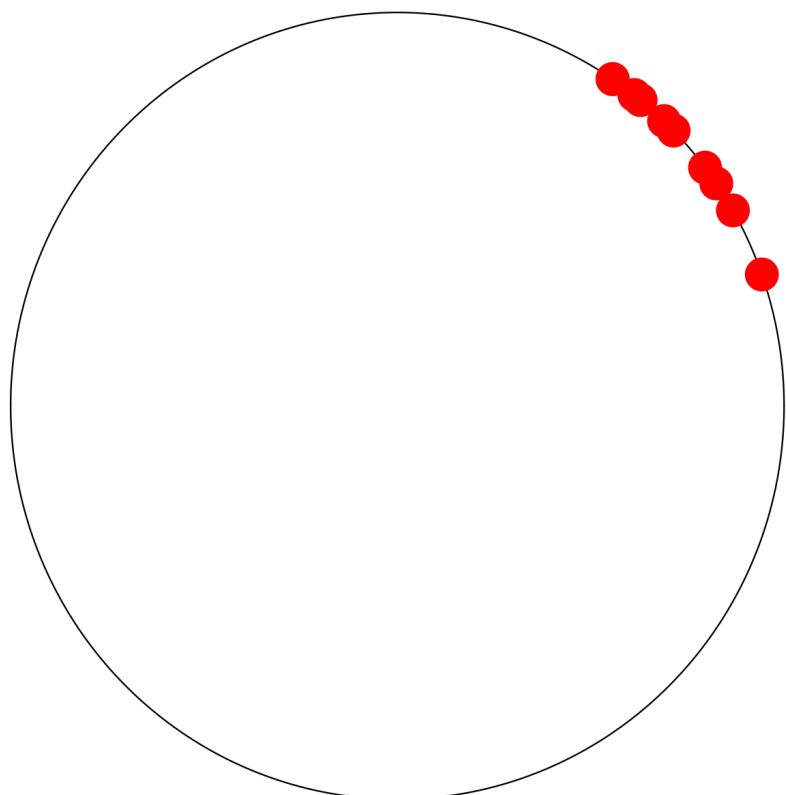


library(circular)

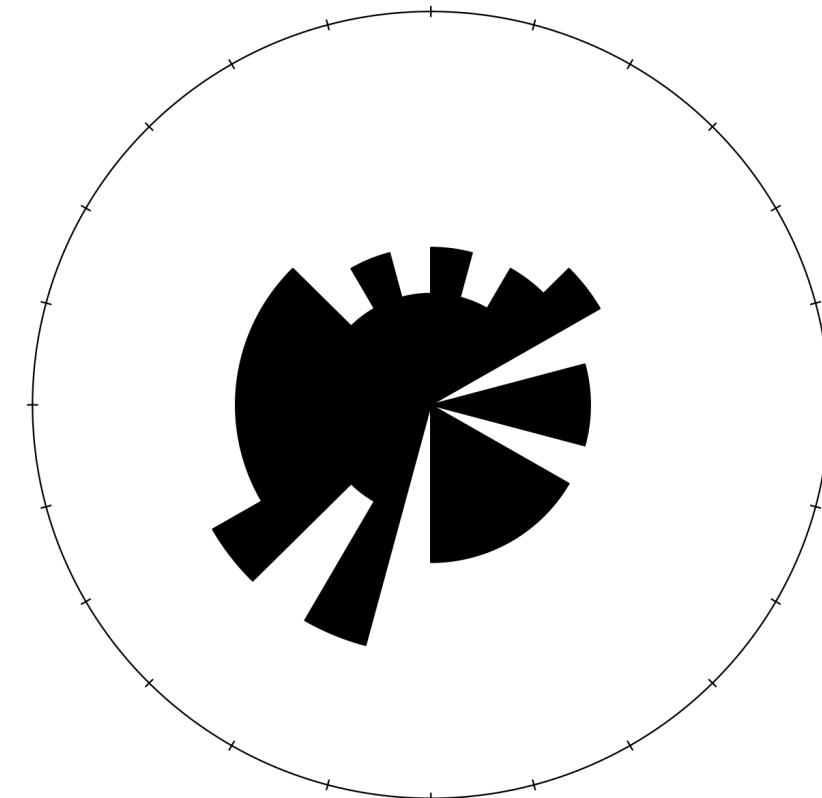


Visualisations

Circle plot



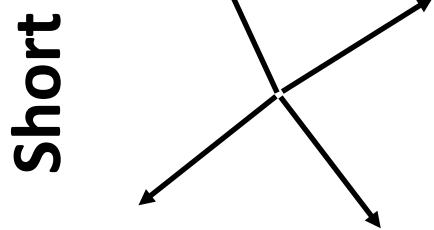
Rose diagram



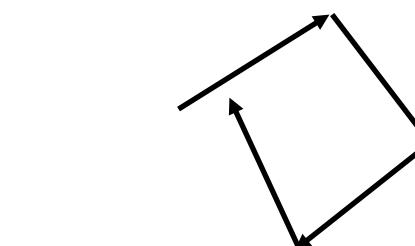


Resultant

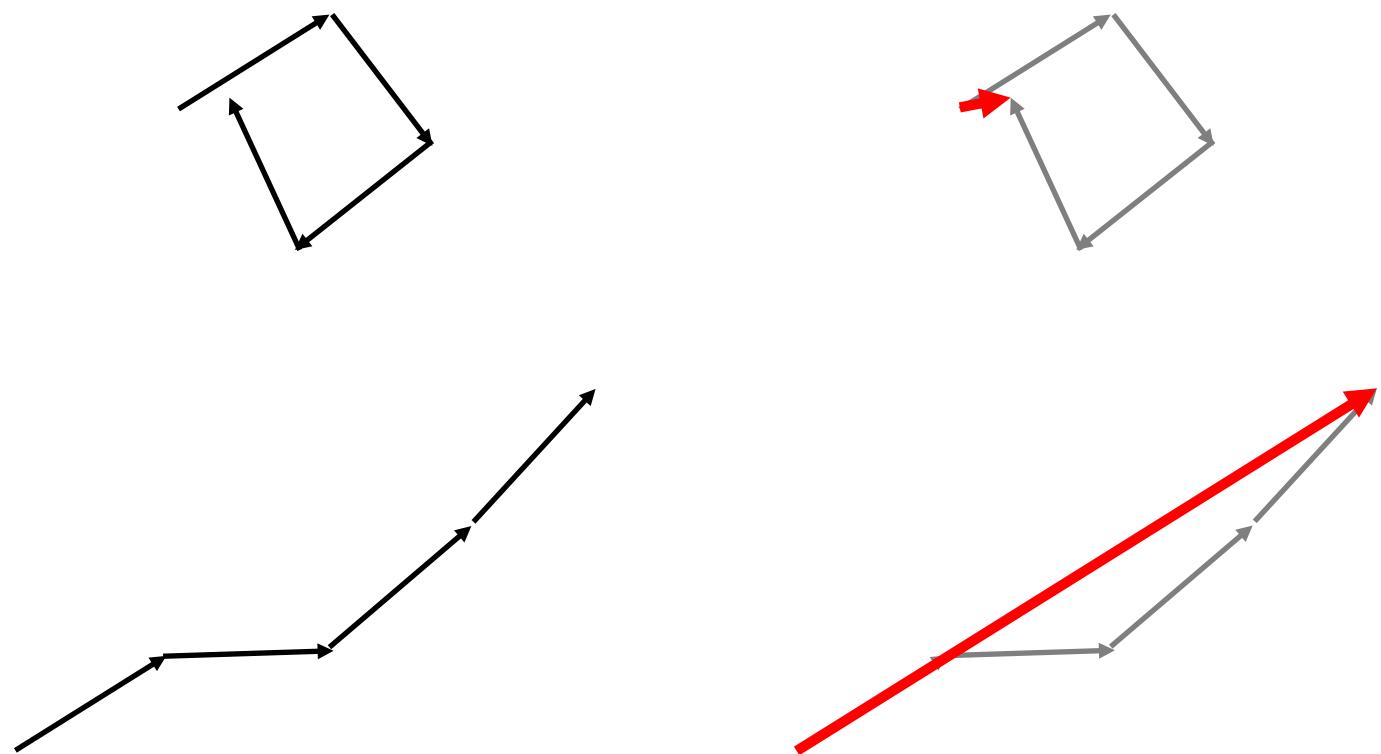
Unit vectors



Arrange as path



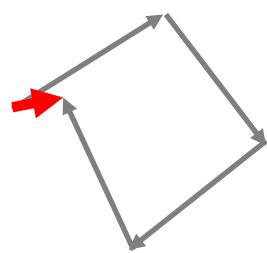
Resultant



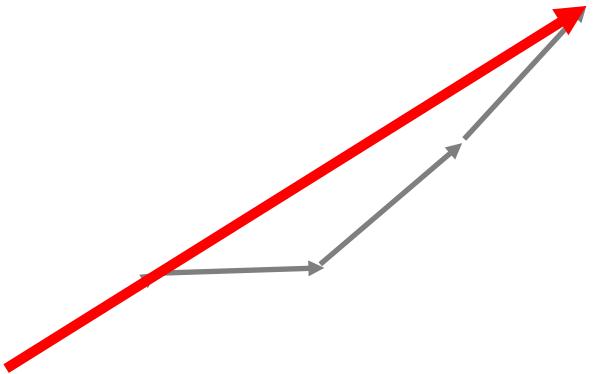


Resultant

Short



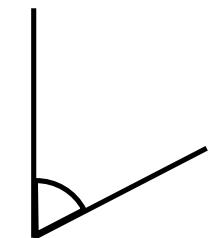
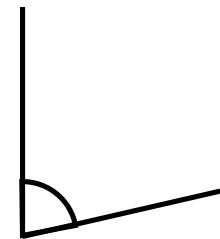
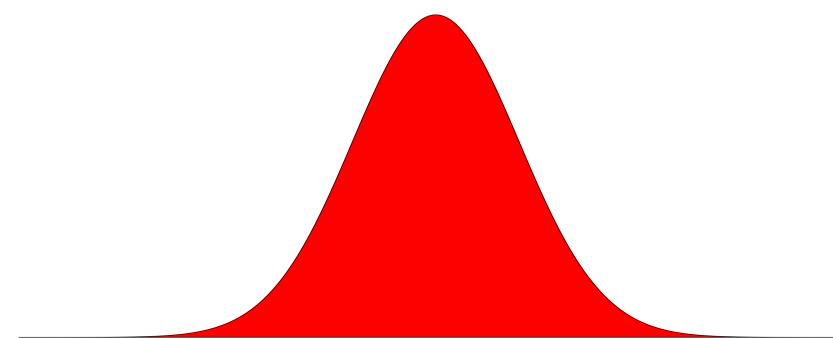
Long



Resultant

Dispersion

Mean





Resultant

Resultant length

Inverse of dispersion; scales with n

$$R = \sqrt{x_r^2 + y_r^2}$$

Mean resultant length

Rescaled, allowing comparison across samples; 0-1 range

$$\bar{R} = \frac{R}{n}$$

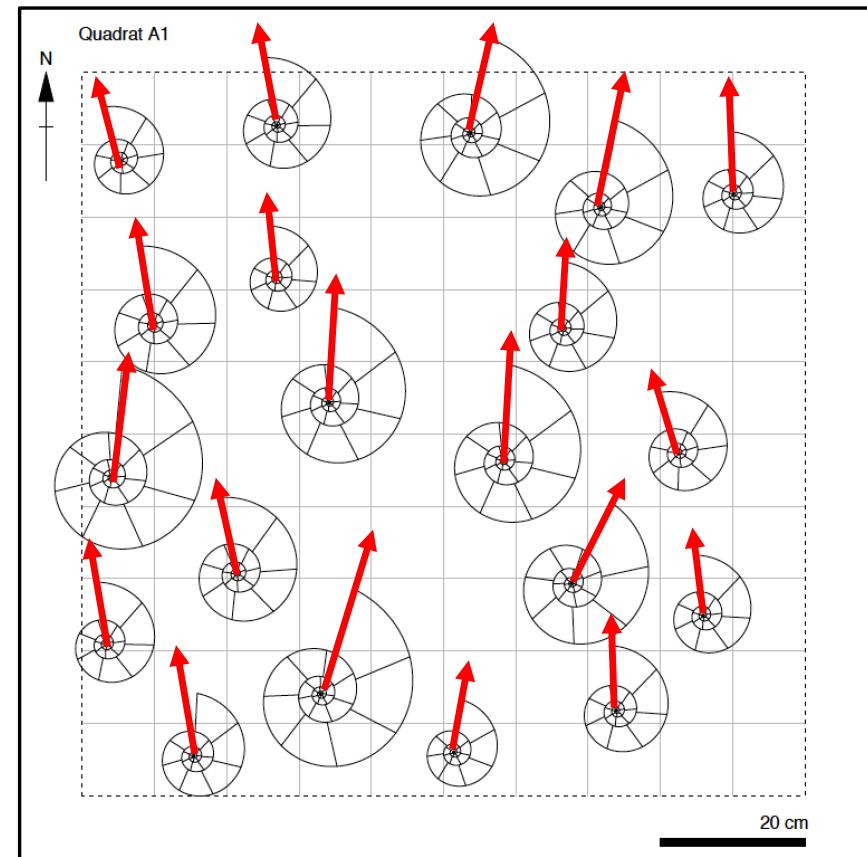
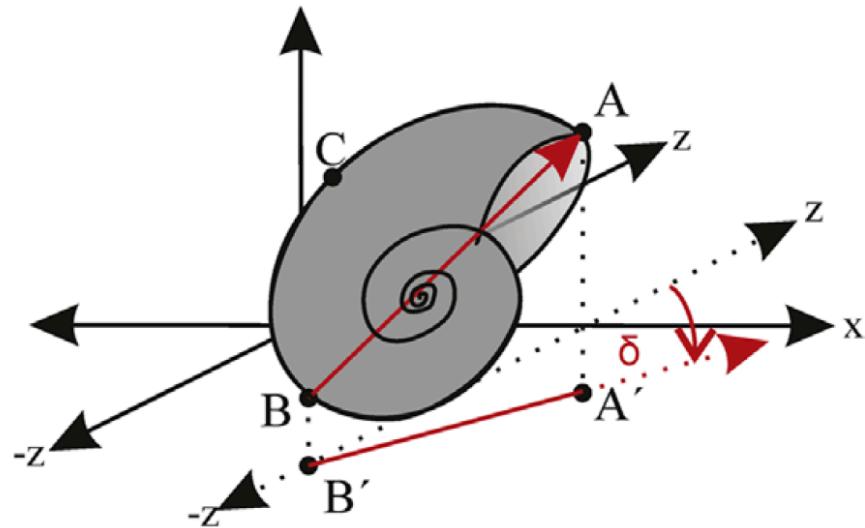
Circular variance

Varies from 0 (low) to 1 (high)

$$\sigma_0 = 1 - \bar{R}$$



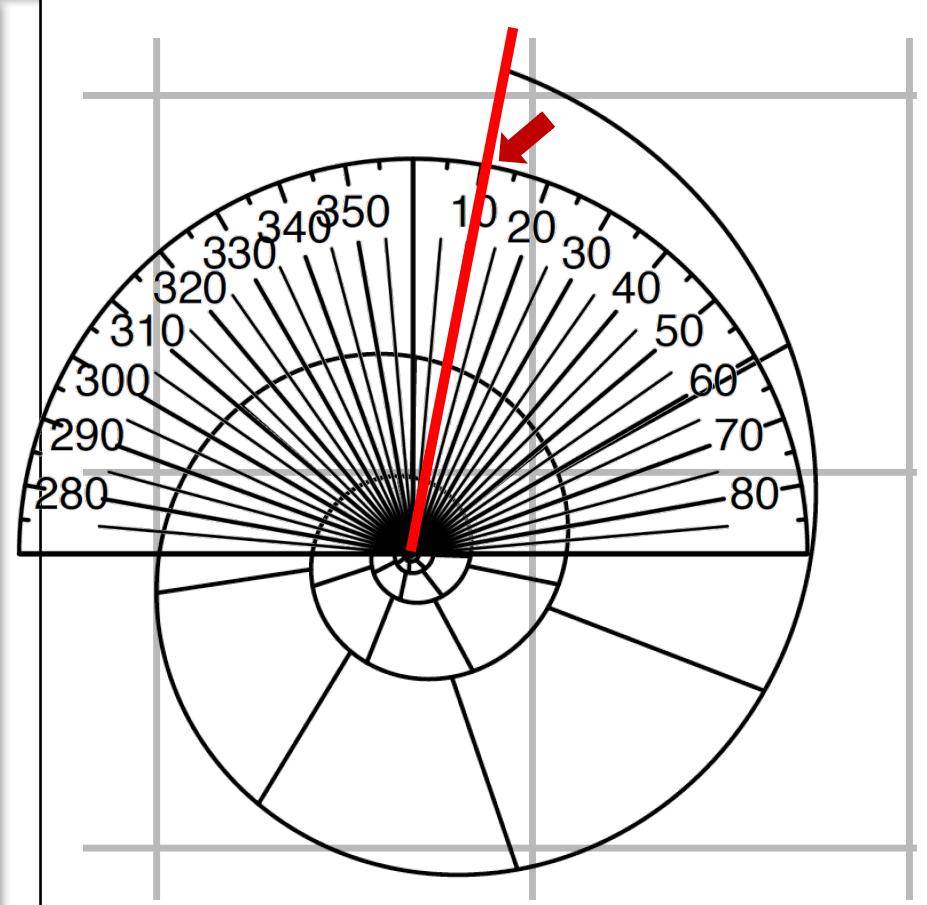
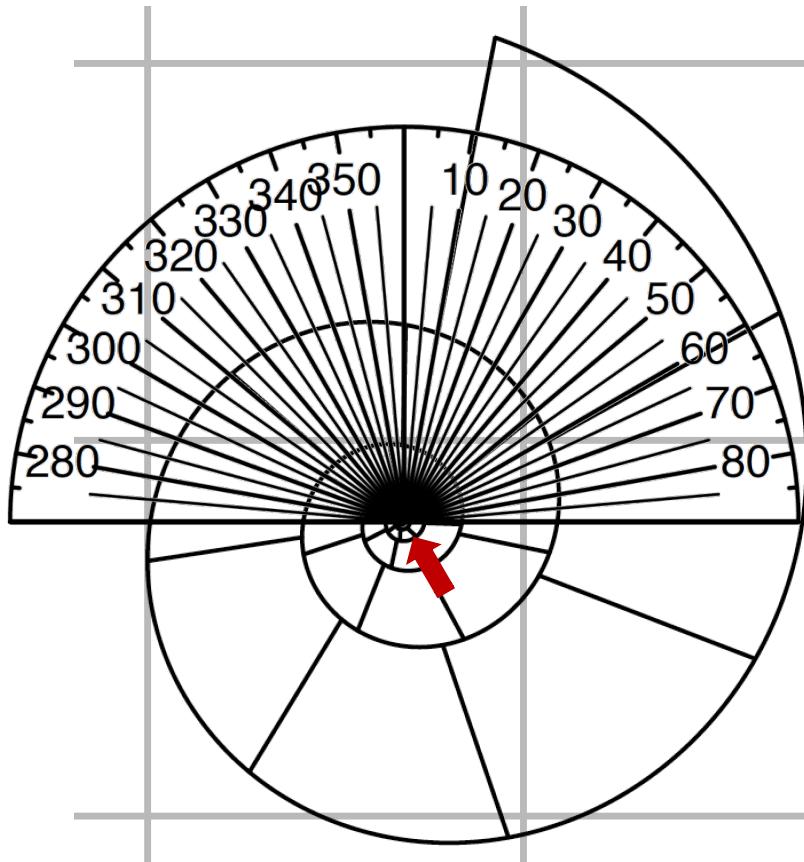
Exercise



Lukeneder et al 2014 *Computers & Geosciences*



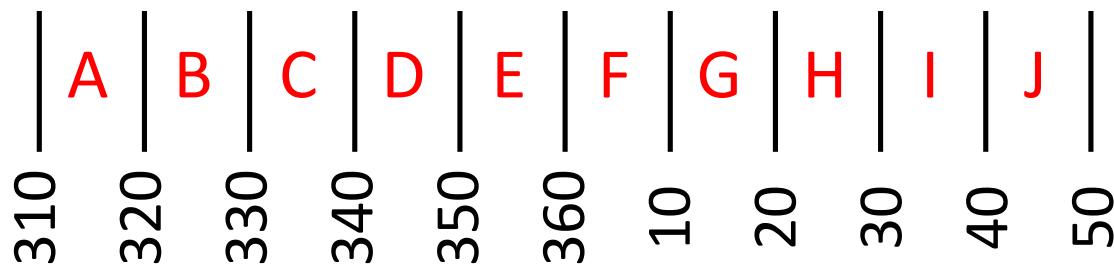
Exercise



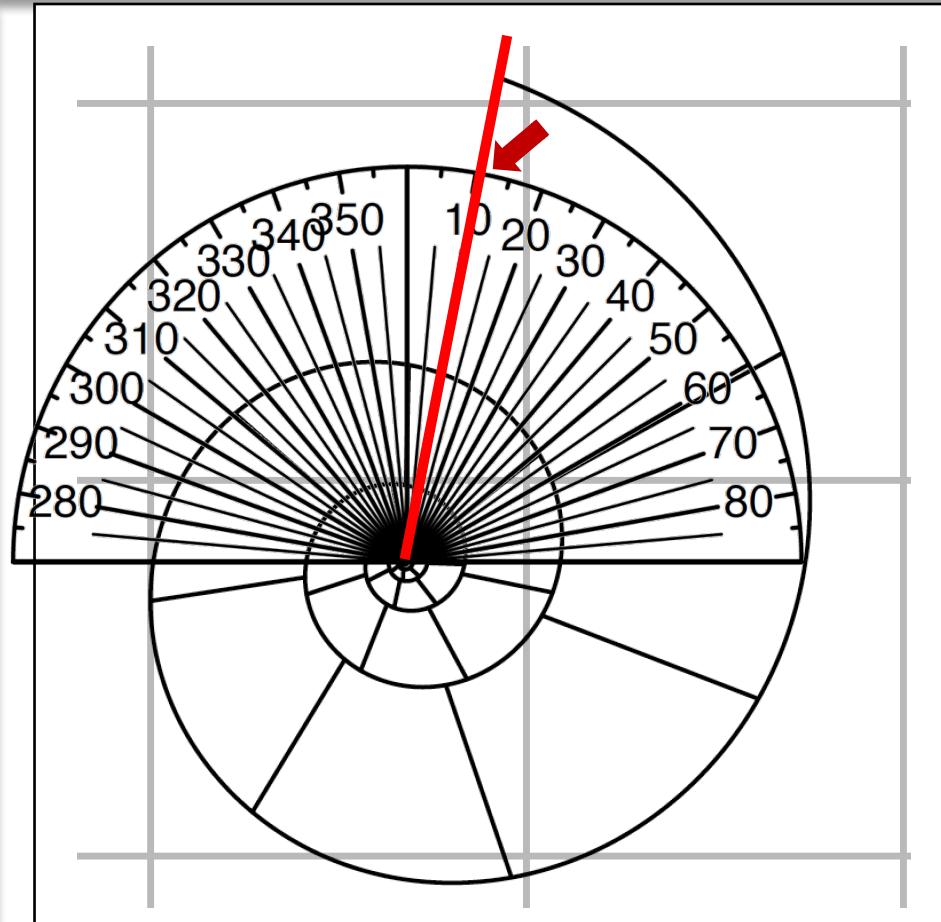


Exercise

1. Measure ammonites 1 and 2 only
2. Units of protractor are bearings in degrees
3. Bin your data (for a histogram)

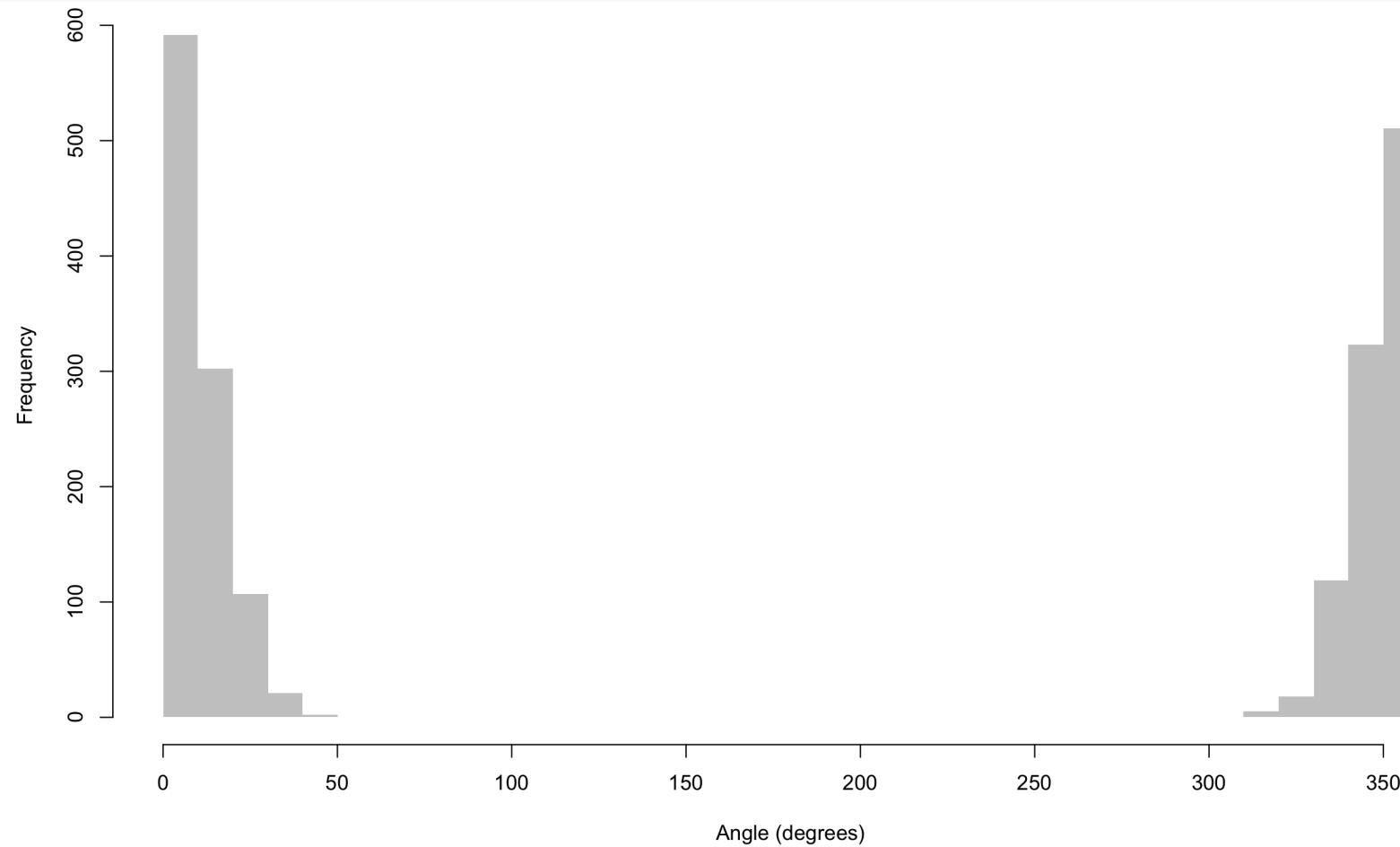


J10: 1 = 29, 2 = 354



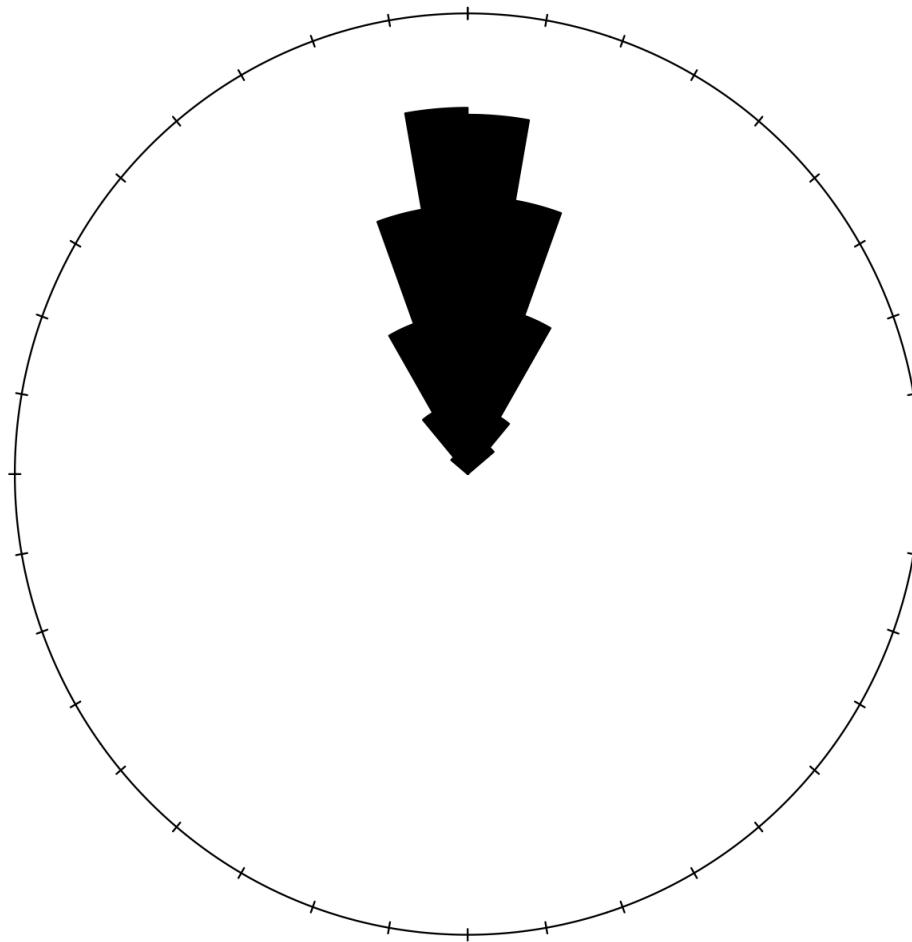


Exercise





Exercise

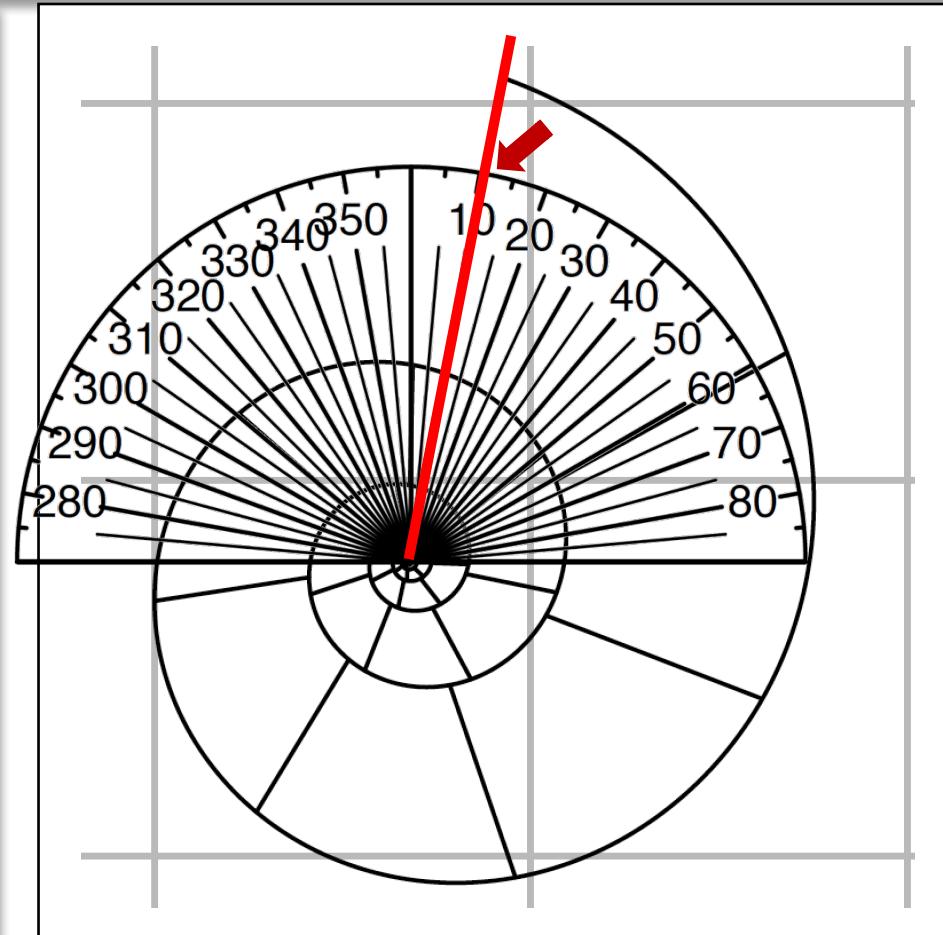




Exercise

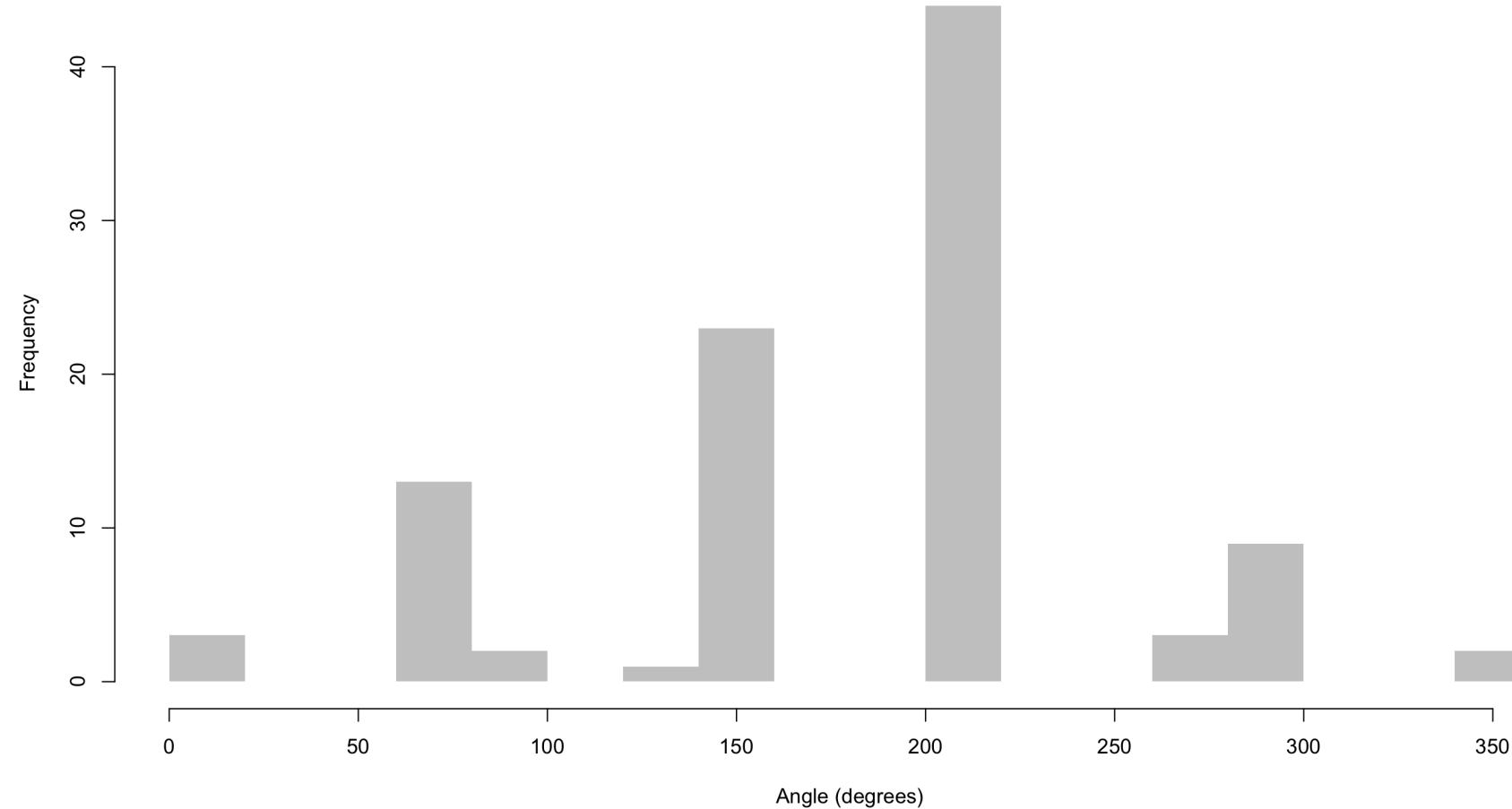
1. Measure ammonites 3-5
2. Calculate mean for all five bearings

J10: 1 = 29
 2 = 354
 3 = 346
 4 = 4
 5 = 9



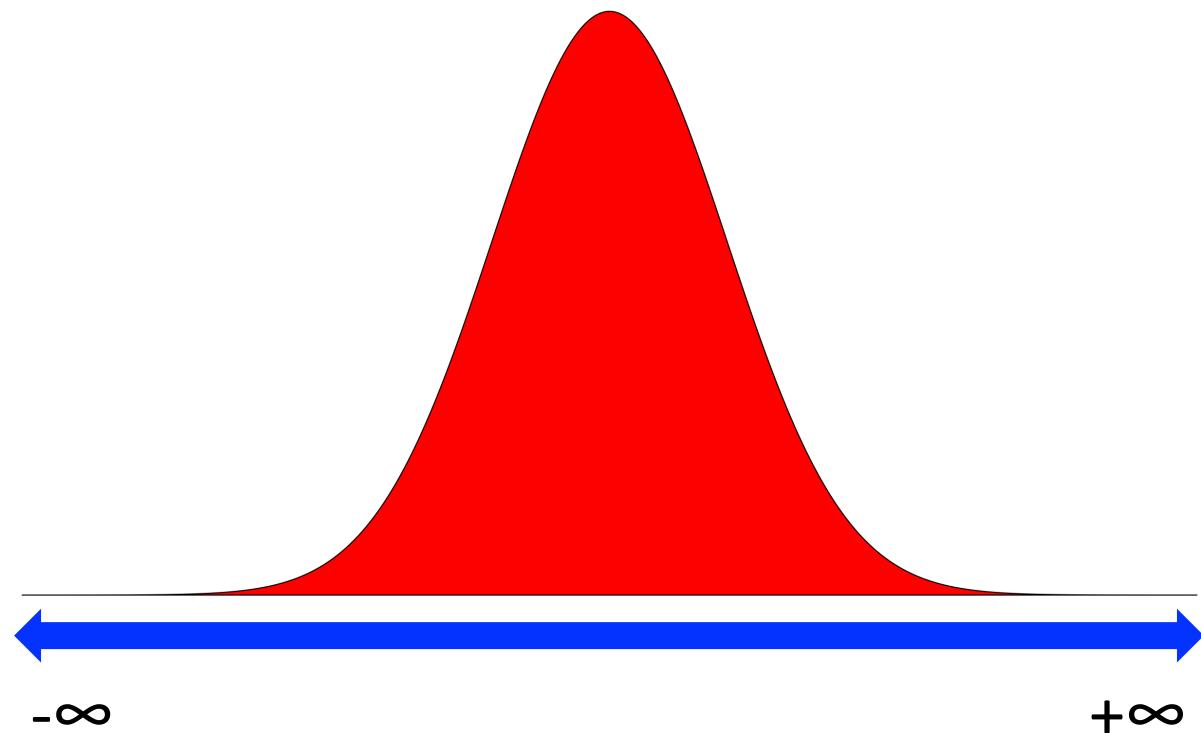


Mean bearing of first five ammonites





von Mises distribution



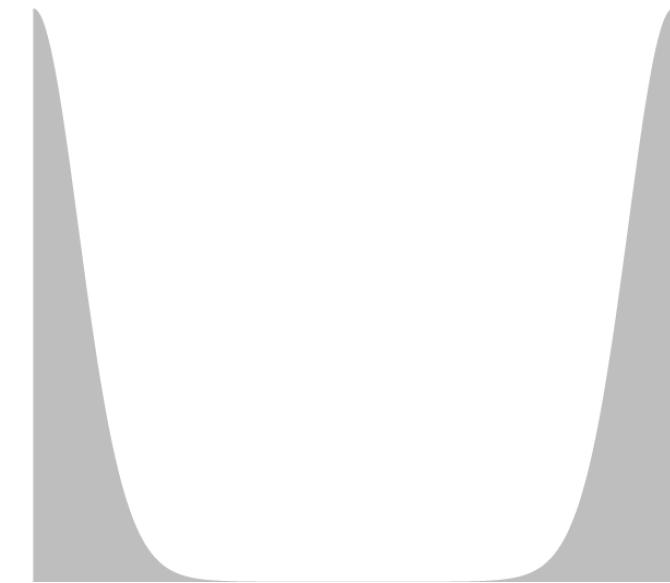
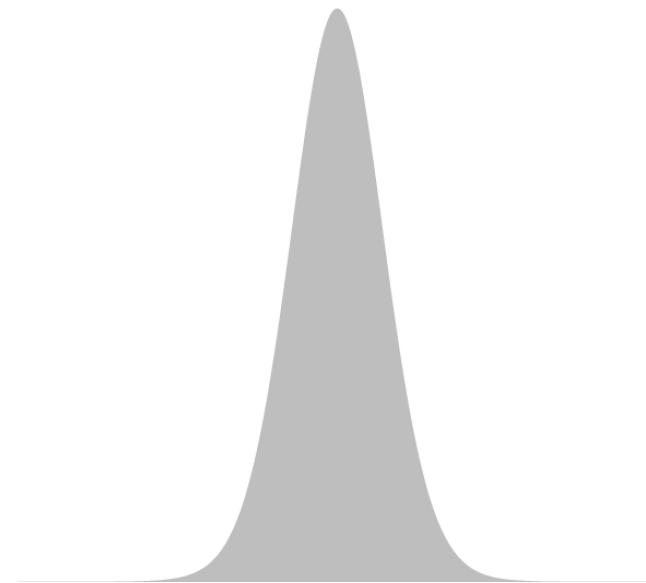
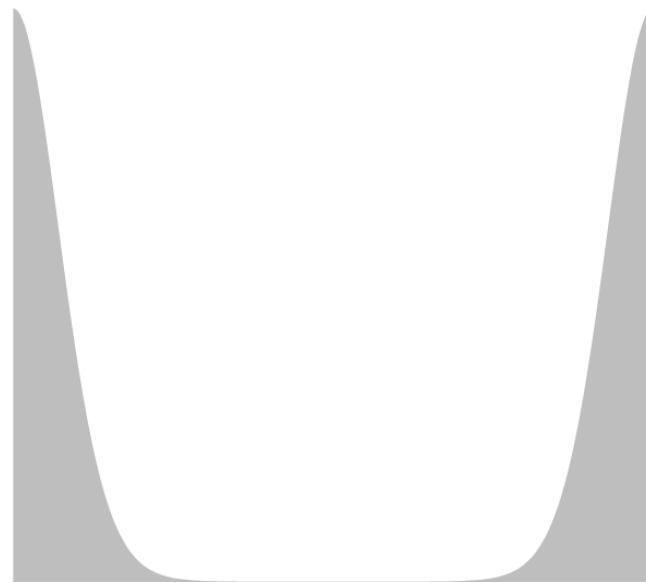


von Mises distribution

$\mu = 0$

$\mu = 180$

$\mu = 360$





von Mises distribution

$K = 0$



$K = 1$



$K = 5$



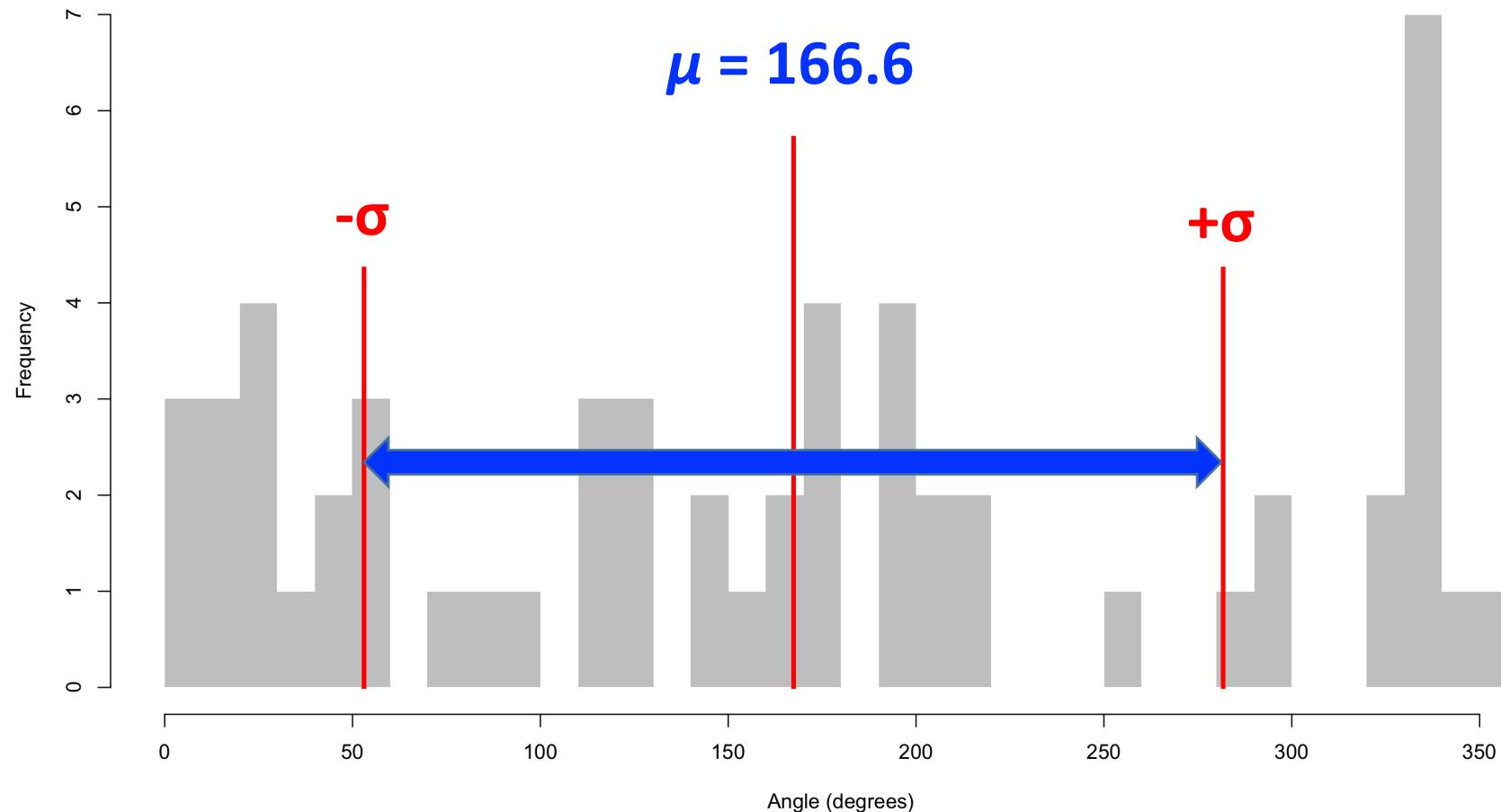


Statistical tests with directional data





Statistical tests with directional data





Statistical tests with directional data

Resultant angle

$$\bar{\theta} = \tan^{-1}(x_r/y_r) = 48.9 \text{ degrees}$$

Resultant length

$$R = \sqrt{(x_r^2 + y_r^2)} = 6.76$$

Mean resultant length

$$\bar{R} = \frac{R}{n} = 0.119$$

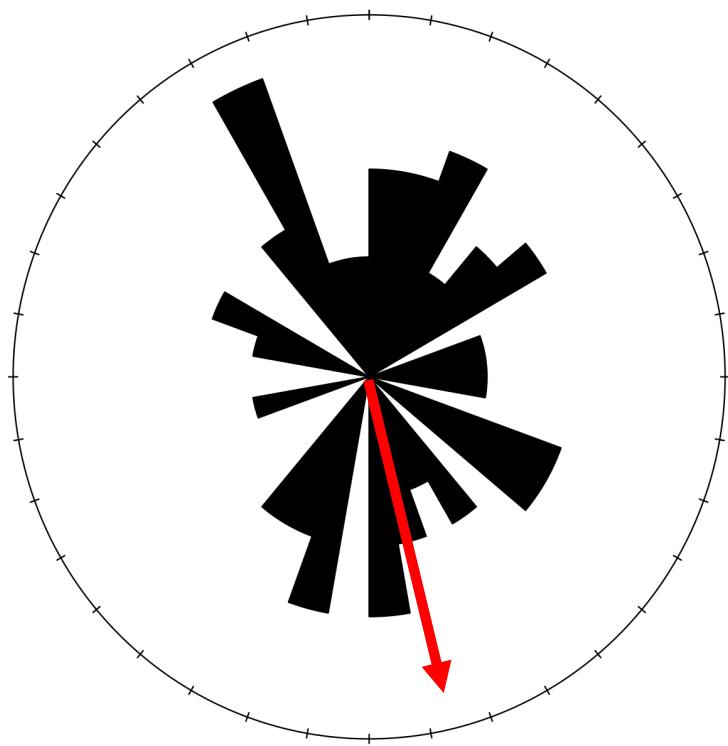
Circular variance

$$\sigma_0 = 1 - \bar{R} = 0.88$$

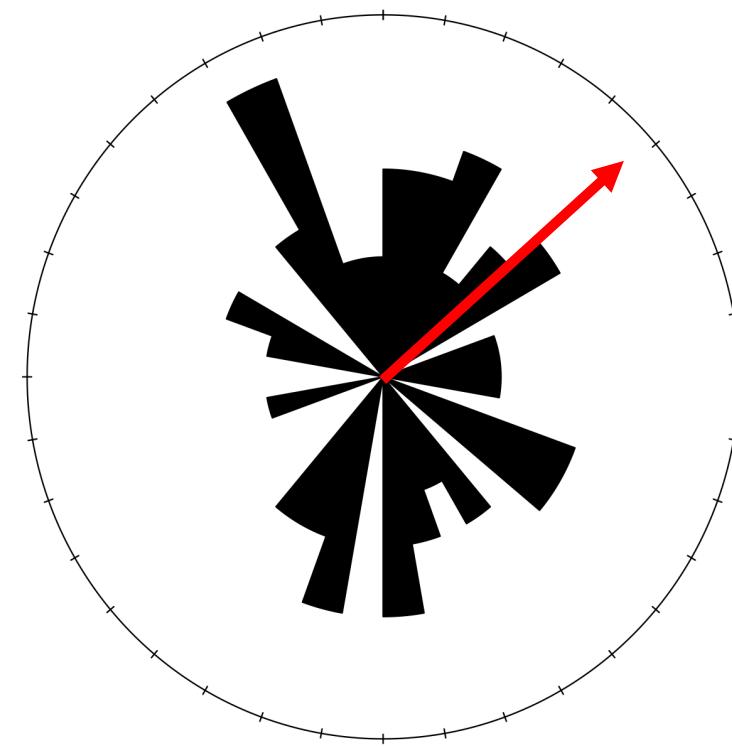


Statistical tests with directional data

Univariate mean



Circular mean





Statistical tests with directional data

Rayleigh test

Test for uniform distribution



`rayleigh.test(SkyeData)`

P-value: 0.5501

Null hypothesis

Data are uniformly distributed

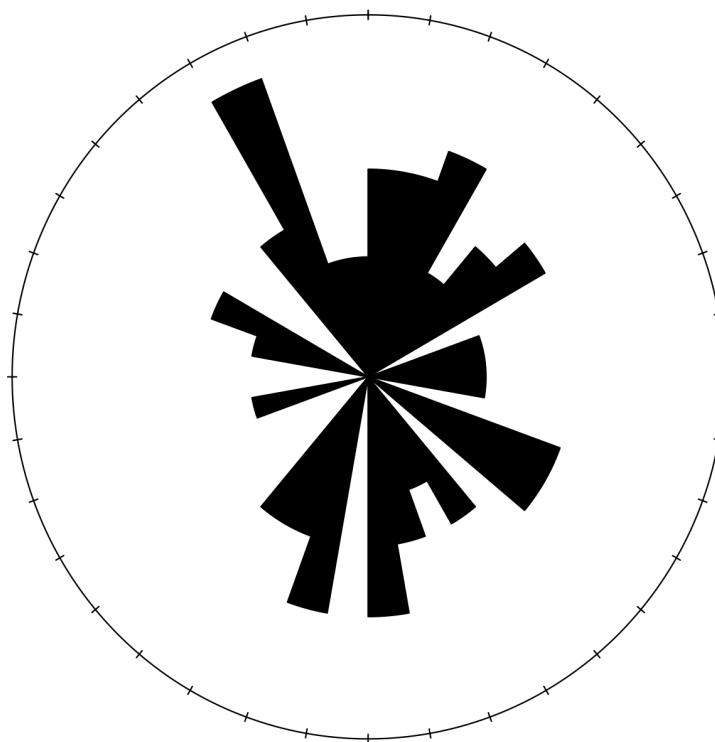
Alternative hypothesis

Data are non-uniform



Statistical tests with directional data

Cannot reject null at typical alphas (<0.5)





Statistical tests with directional data

von Mises distribution

Effectively circular normal

Kappa value

Estimation is effectively a separate test for uniform

$K=0.24175$

Low value, indicating well dispersed data



Statistical tests with directional data

Chi-squared test

Can apply here to test for uniform; 20 degree bins

Chi-squared = 35.52

Degrees of freedom = $18 - 1 = 17$

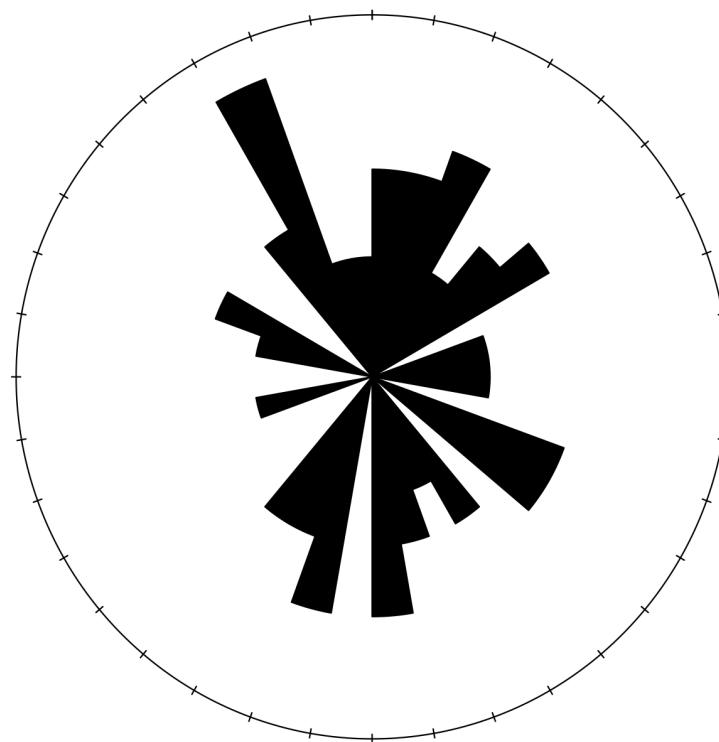
Alpha = 0.05

Critical value 8.67, therefore reject null; data are non-uniform



Statistical tests with directional data

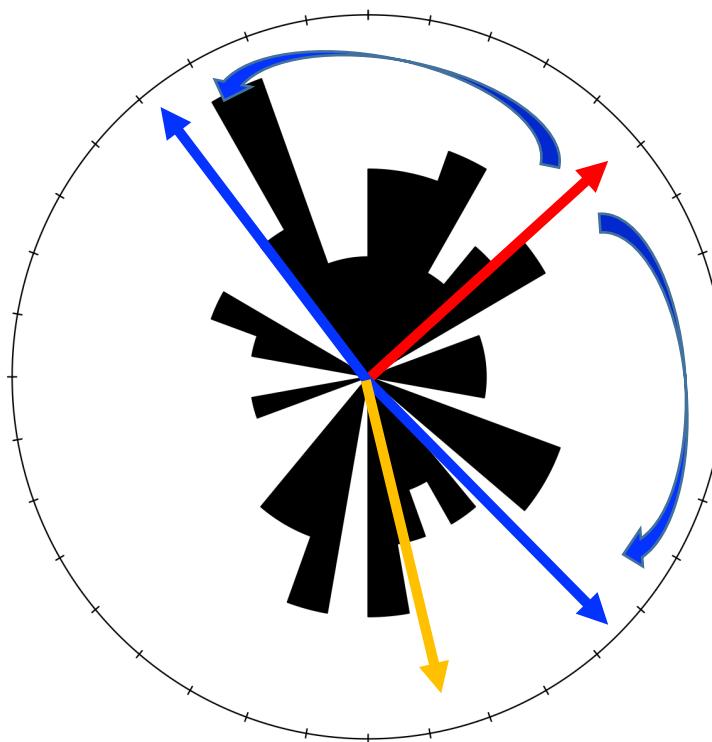
Issue may be polymodality





Other tests

One sample t-test equivalent (alpha = 0.05)





Other tests

Two-sample t-test equivalent

R_1, R_2

Compares resultants of two samples

R_1 and R_2 are resultants of each sample

R_T

R_T is resultant from combined data

Close means indicate approximately equal values

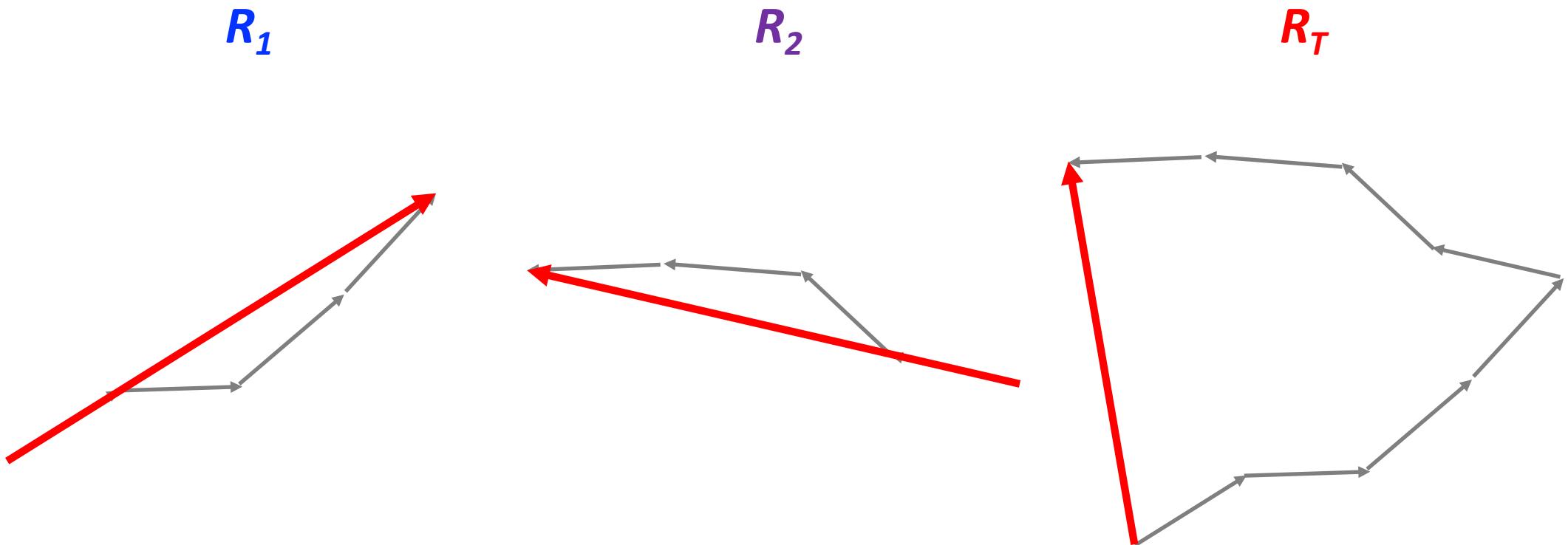
$R_1 + R_2 \approx R_T$

Distant means indicate smaller combined resultant

$R_1 + R_2 \gg R_T$



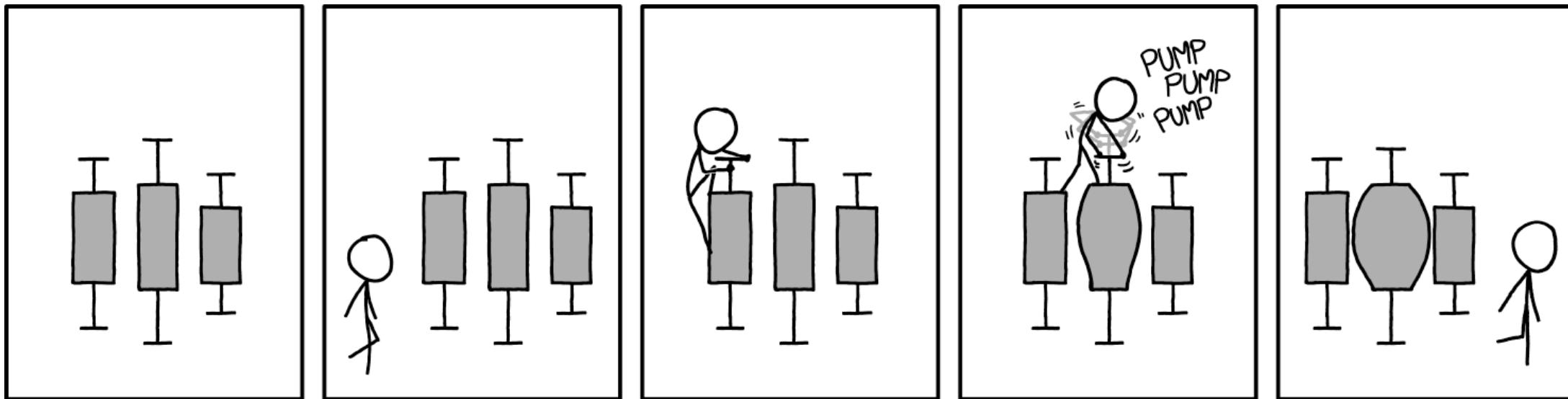
Other tests



Modified F-statistic allows null hypothesis statistical testing



Boom goes the dynamite



Source: xkcd