

# SOEE1475 Statistics and Data Analysis

## Lecture 8: Multivariate statistics



Graeme T. Lloyd



# Today

- The third dimension, and beyond
- Ordination
- Spherical data



Today

But first...

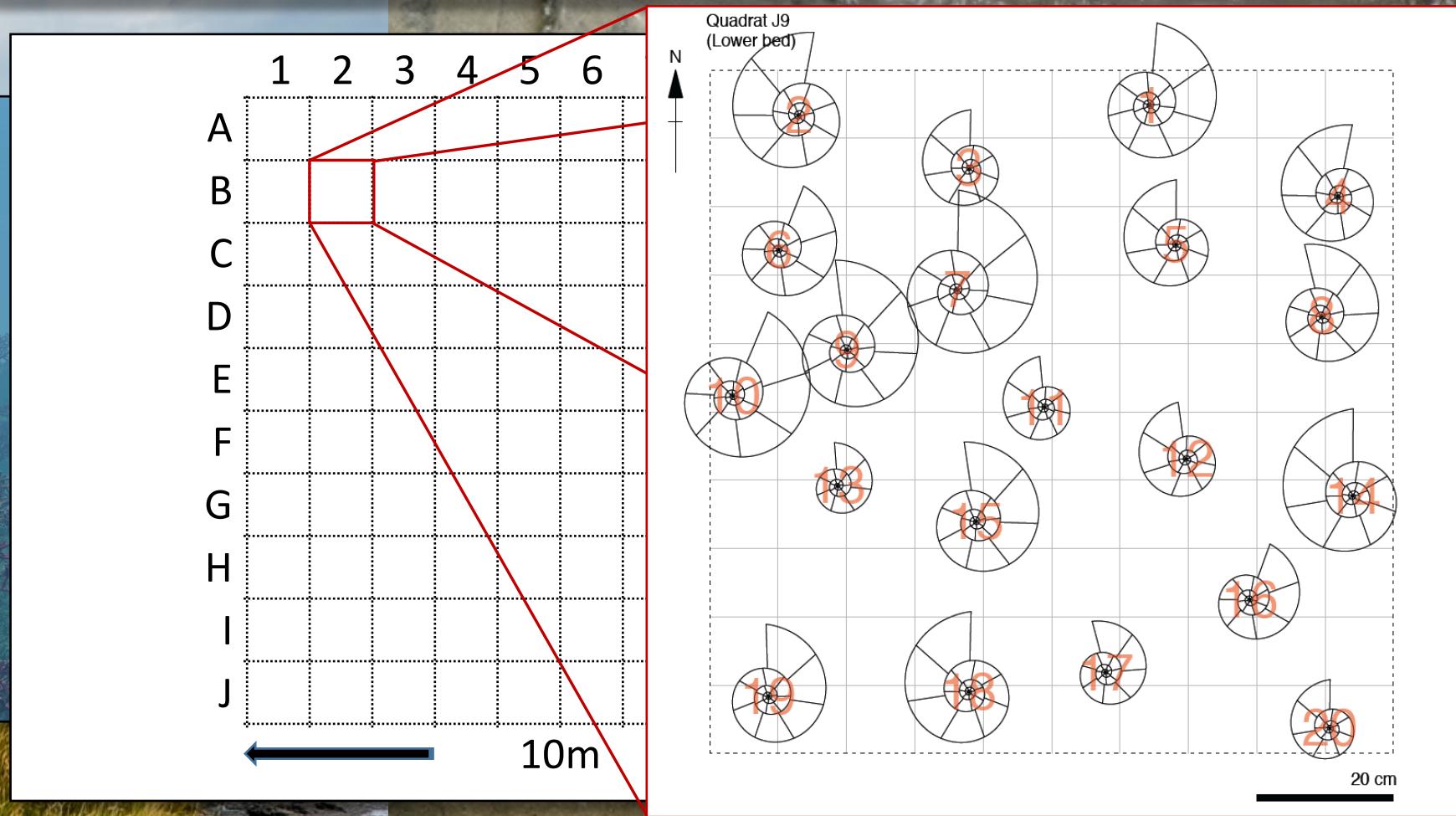
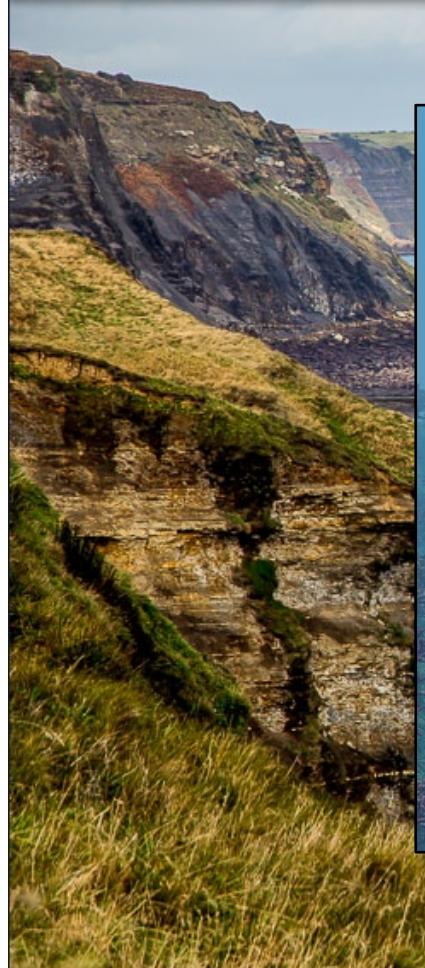


# Ammonite quadrat redux



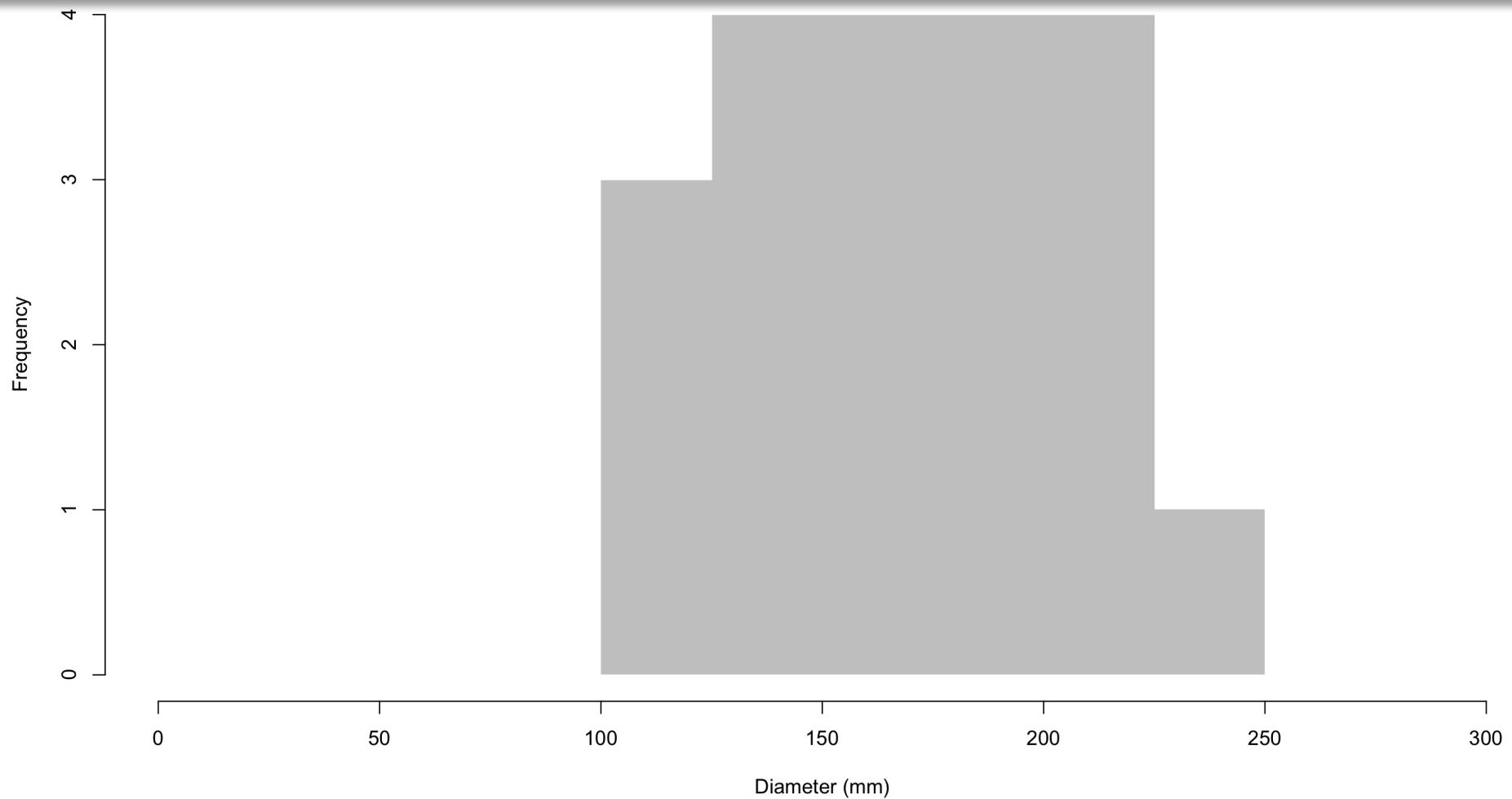


# Ammonite quadrat redux



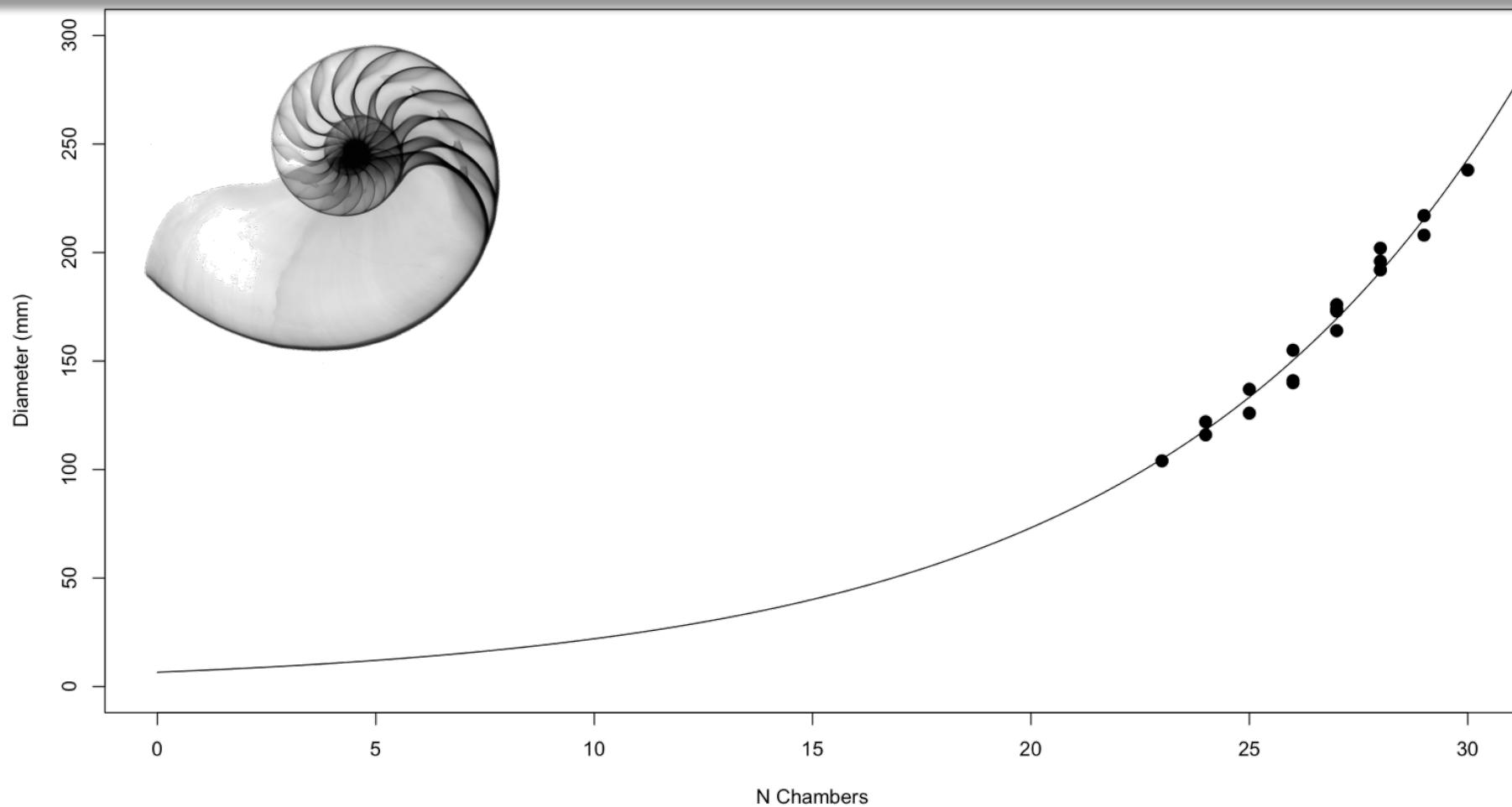


# Ammonite quadrat redux



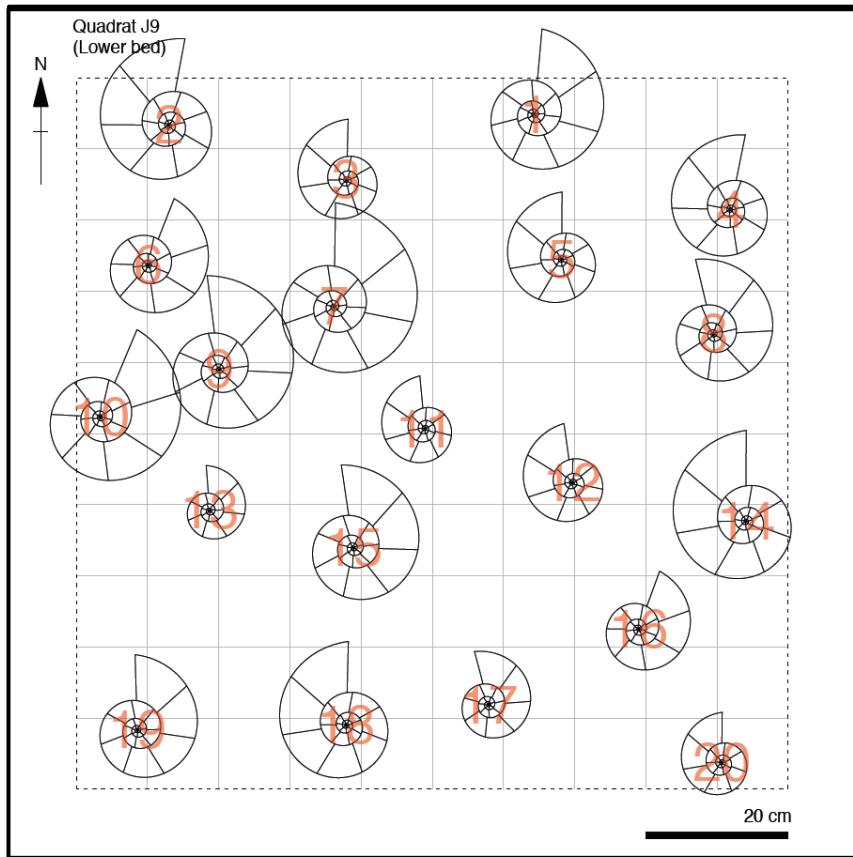


# Ammonite quadrat redux

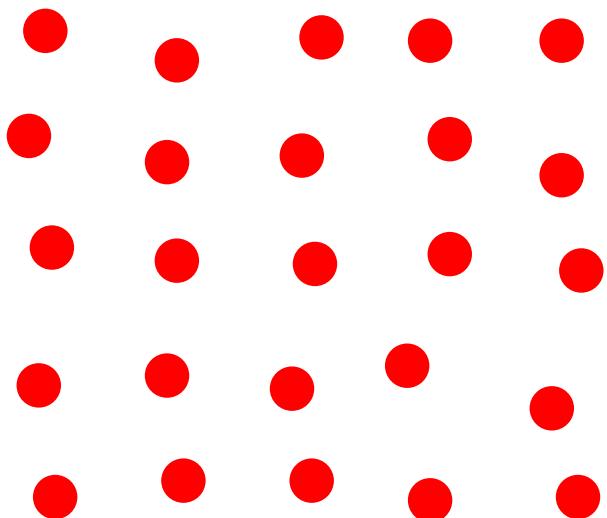




# Ammonite quadrat redux



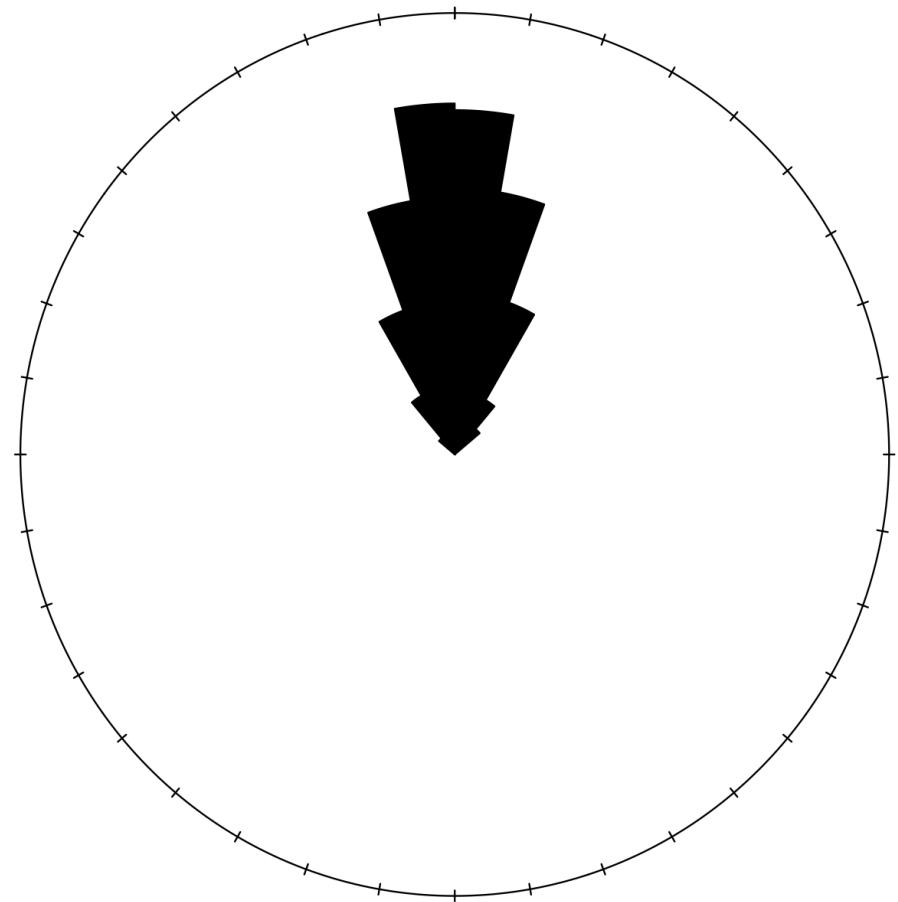
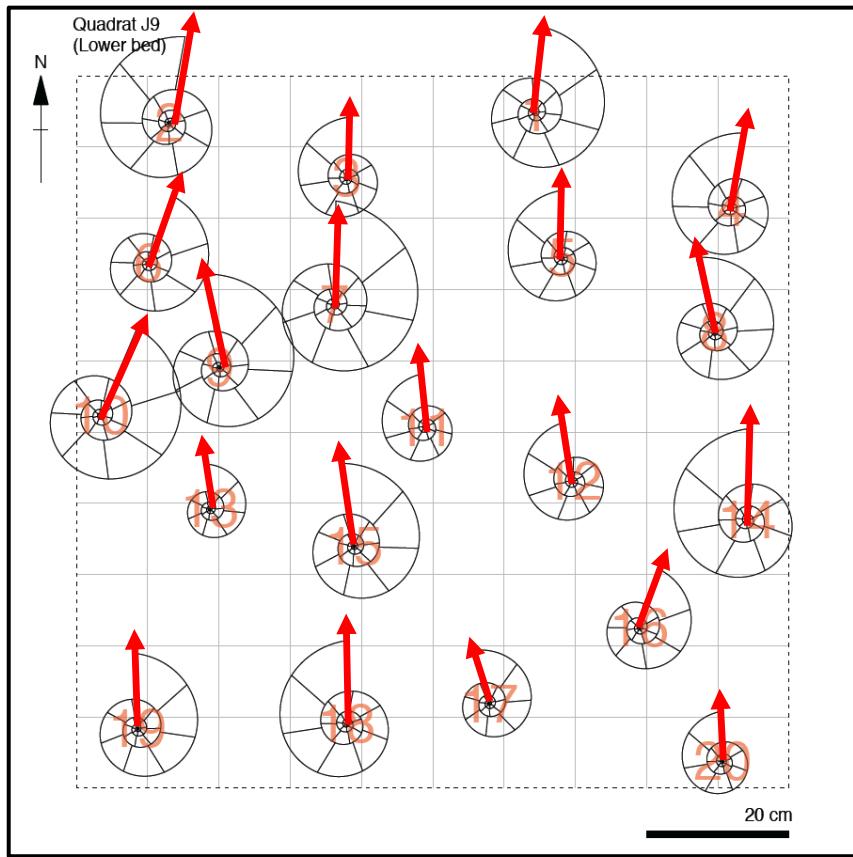
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Uniform



# Ammonite quadrat redux





# Ammonite quadrat redux





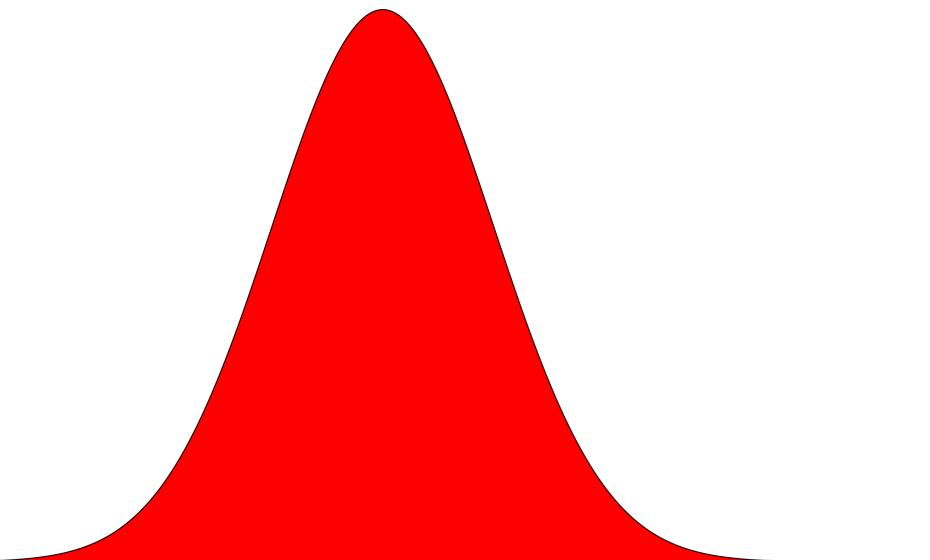
# Ammonite quadrat redux

Back to multivariate statistics...



# Univariate data

Population

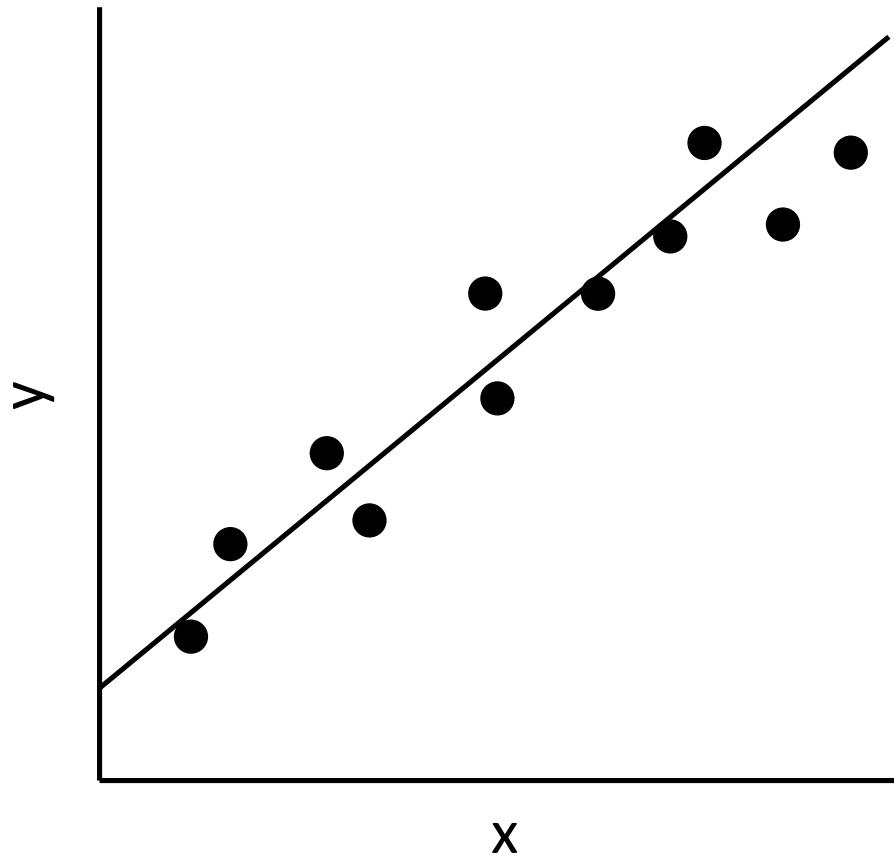


Sample



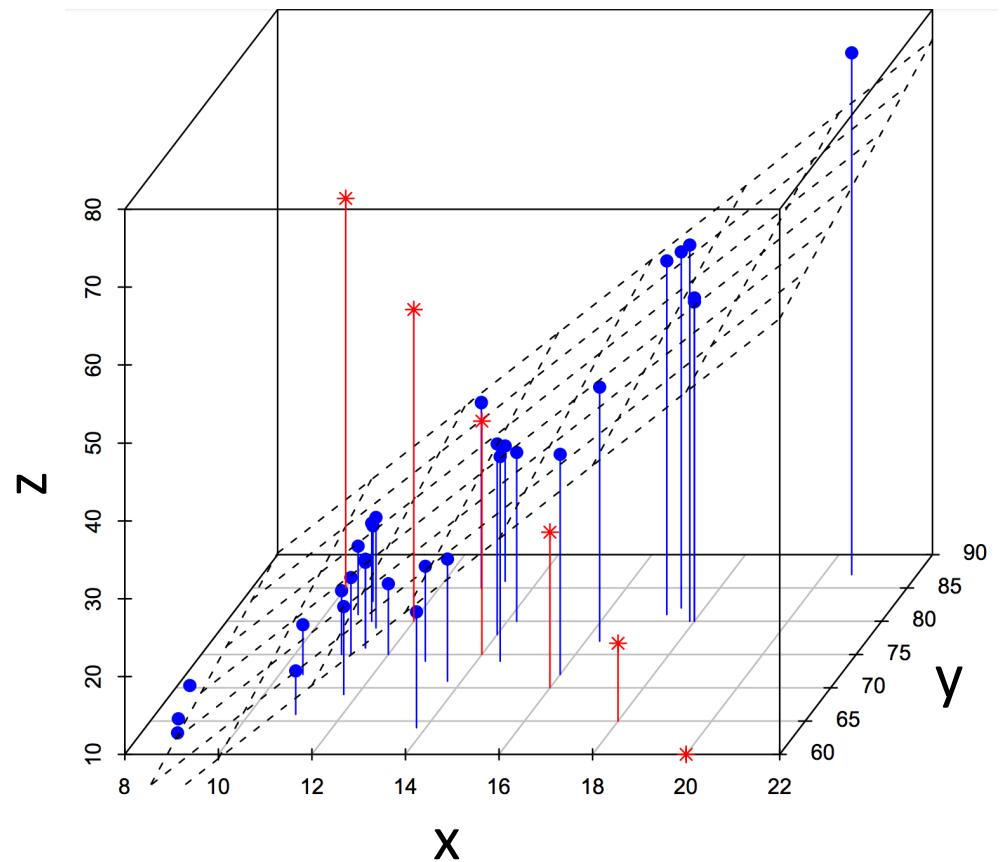


# Bivariate data



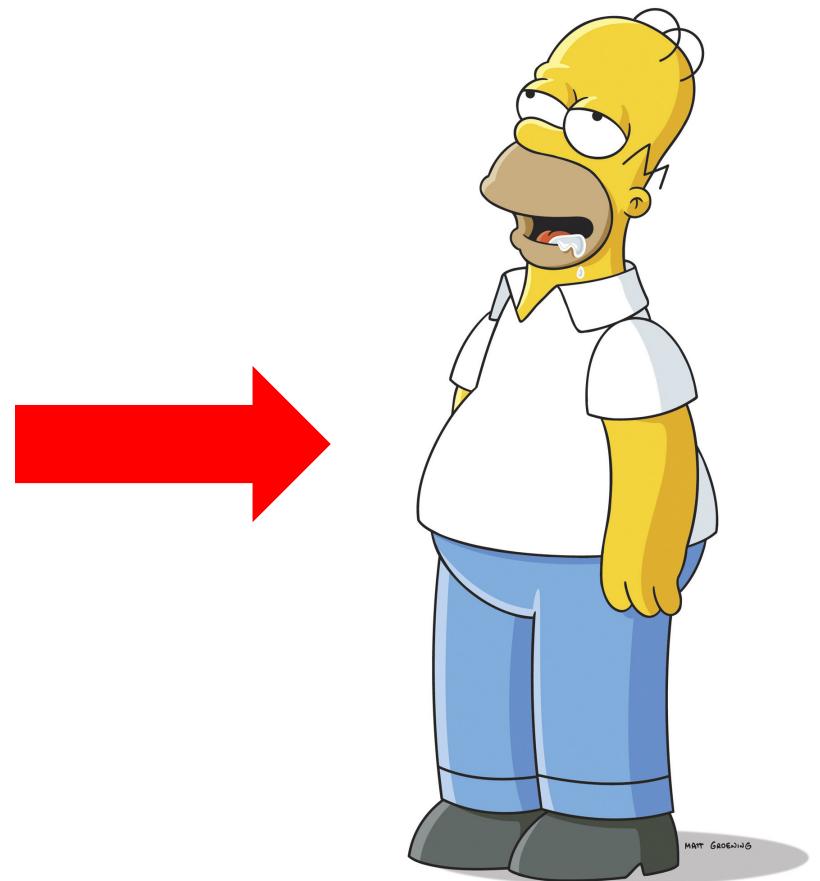
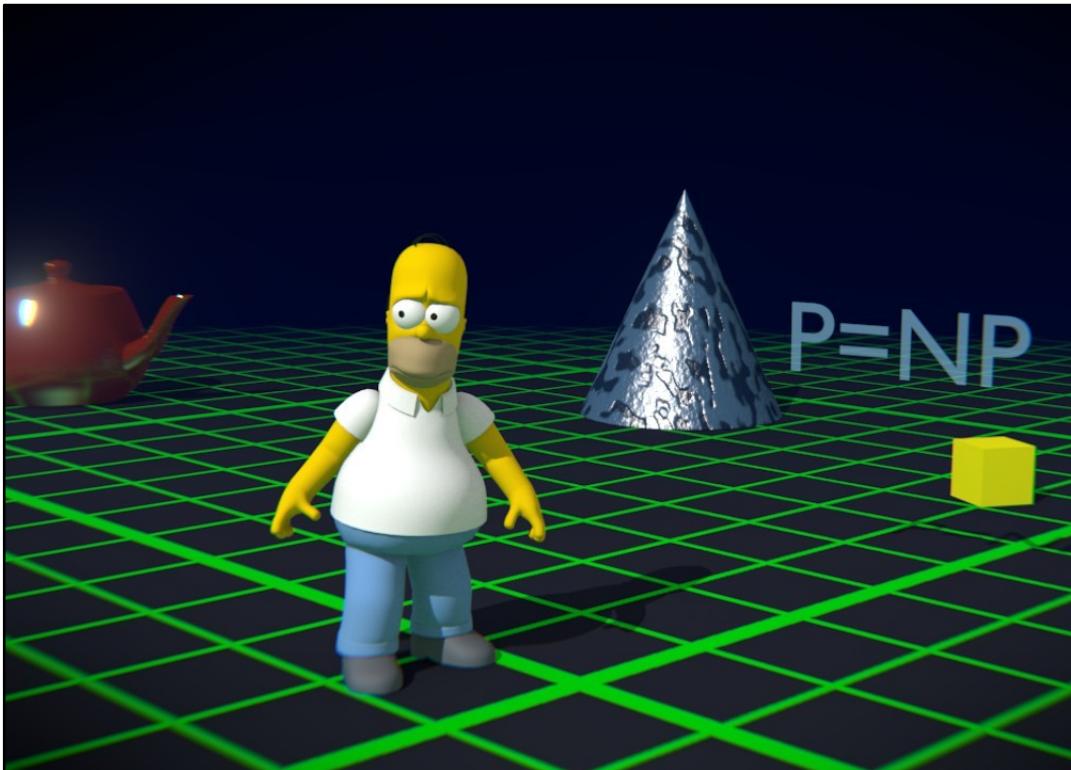


# Multivariate data



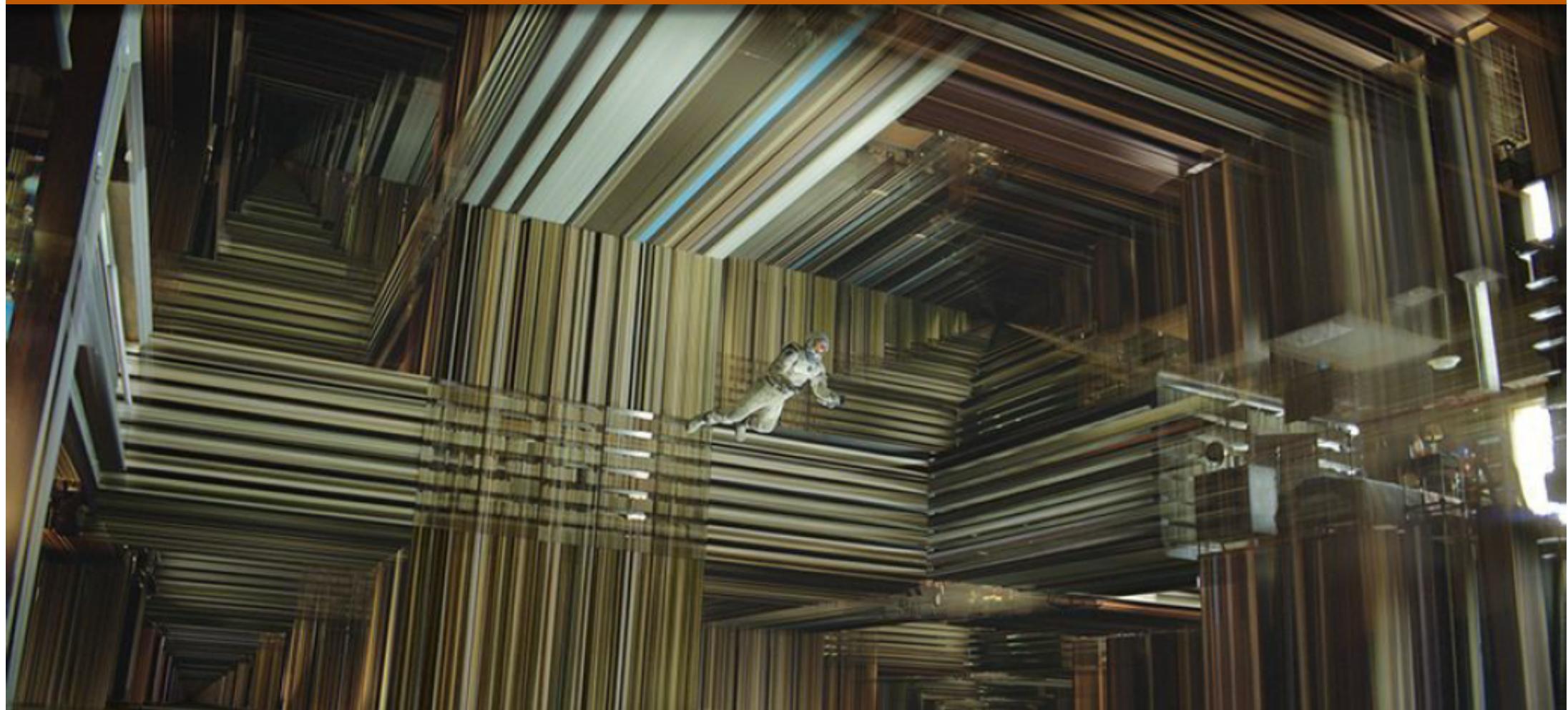


# Multivariate data



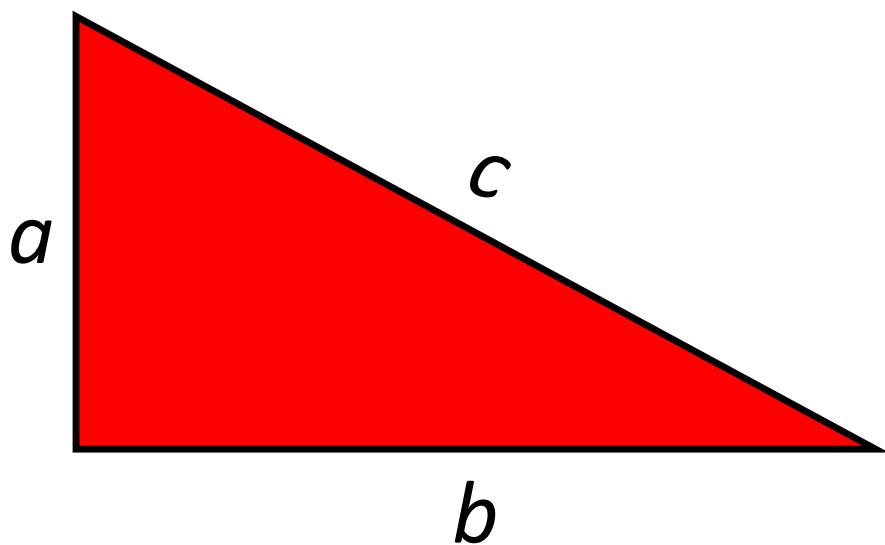


# How many dimensions?





# Triangles



$$a^2 + b^2 = c^2$$

$$c = \sqrt{(a^2 + b^2)}$$



# Triangles

$x_1, y_1$



$c$

$x_2, y_2$

$$c = \sqrt{((x_1 - x_2)^2 + (y_1 - y_2)^2)}$$

$$a^2 + b^2 = c^2$$

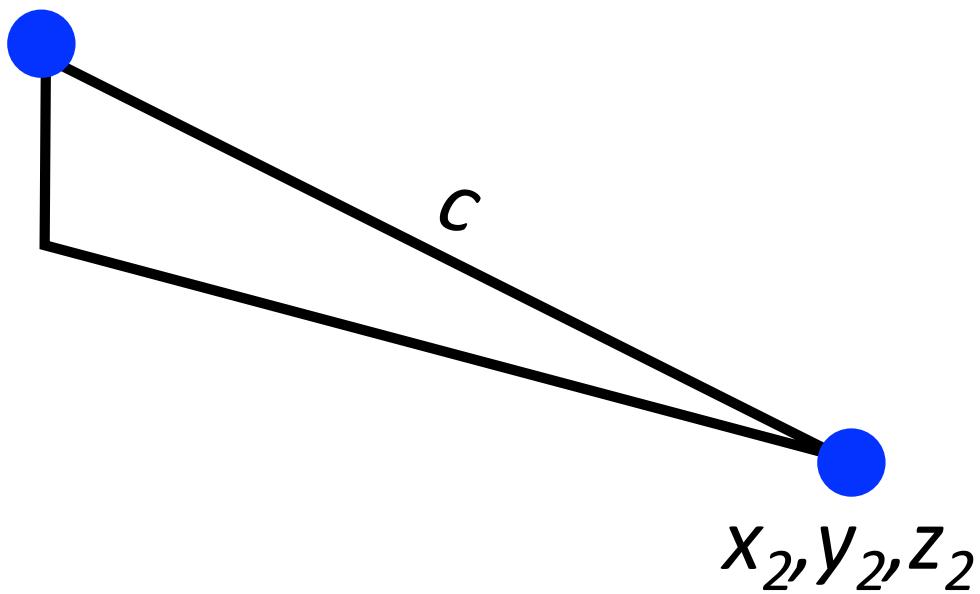
$$c = \sqrt{(a^2 + b^2)}$$



# Triangles

$$c = \sqrt{((x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2)}$$

$x_1, y_1, z_1$



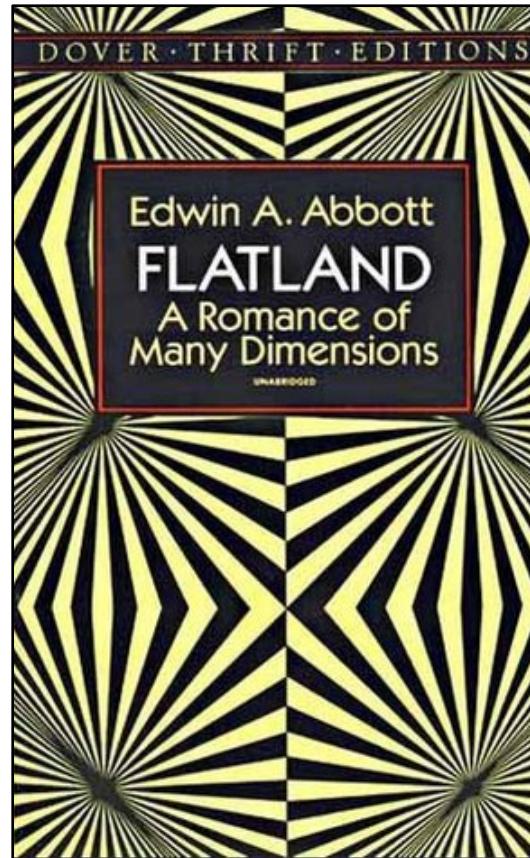


# Triangles

$$c = \sqrt{((x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 + (\alpha_1 - \alpha_2)^2)}$$

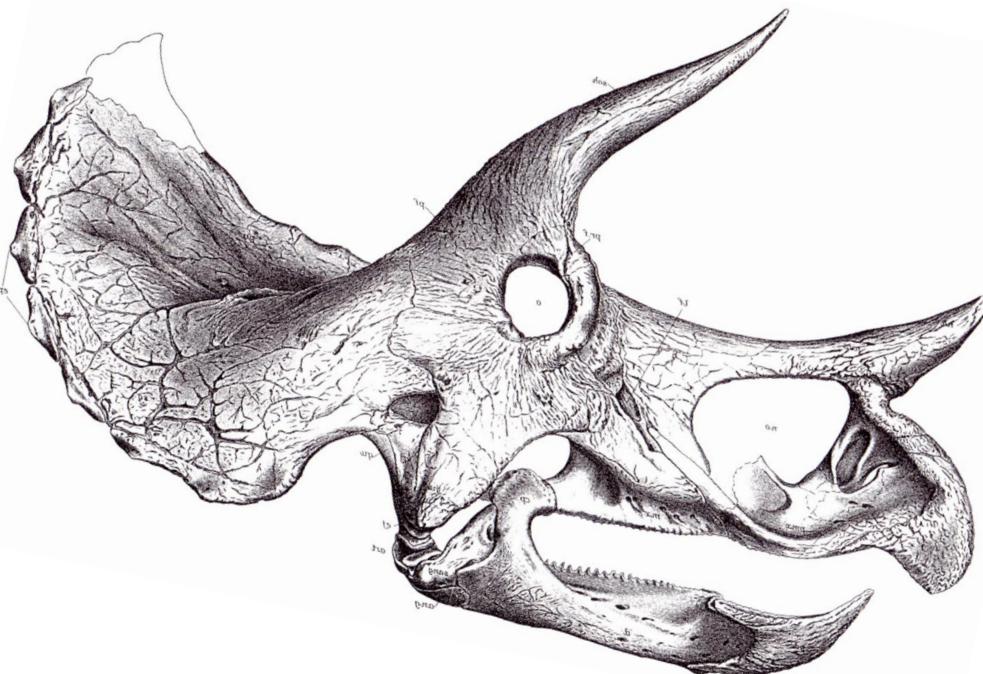


# Triangles

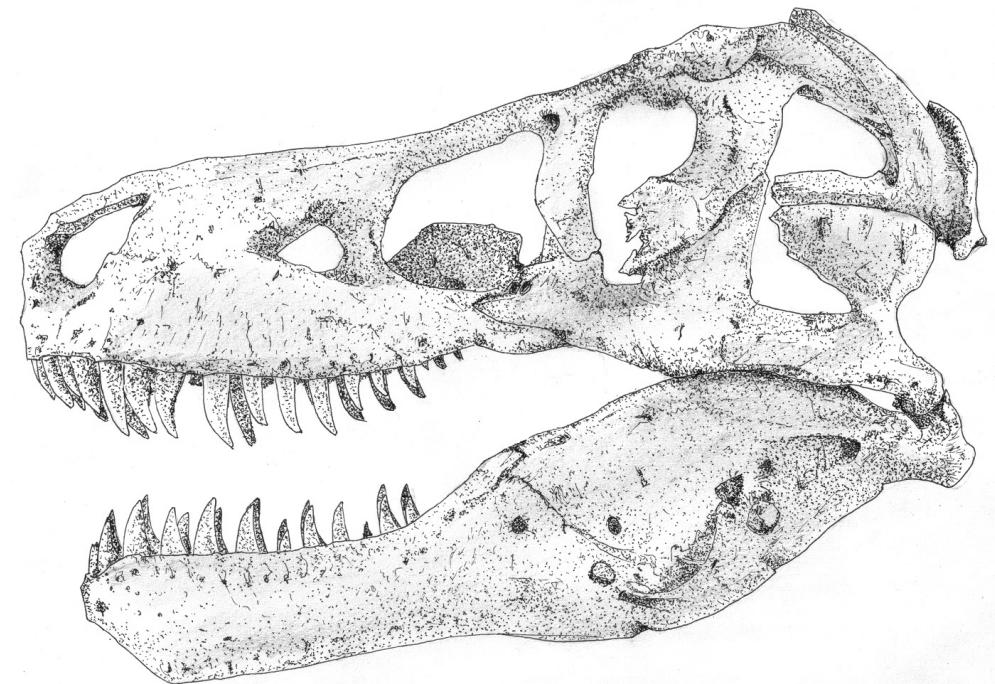




# Spot the difference



*Triceratops horridus*



*Tyrannosaurus rex*



# Spot the difference



*Triceratops horridus*

*Dilophosaurus wetherilli*

*Tyrannosaurus rex*



## Exercise – trivia time!

1. A drawer contains 12 white socks and 12 black socks. If you pick two at random what is the probability they are a matching pair? (Give your answer as a fraction)





# Exercise – trivia time!

## 2. What is the median of the integers 1 to 999?

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57  
58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107  
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678 679 680 681 682 683 684 685 686 687 688 689 690 691 692 693 694 695 696 697 698 699 700 701 702 703 704 705 706 707 708 709 710 711 712 713 714 715  
716 717 718 719 720 721 722 723 724 725 726 727 728 729 730 731 732 733 734 735 736 737 738 739 740 741 742 743 744 745 746 747 748 749 750 751 752 753  
754 755 756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778 779 780 781 782 783 784 785 786 787 788 789 790 791  
792 793 794 795 796 797 798 799 800 801 802 803 804 805 806 807 808 809 810 811 812 813 814 815 816 817 818 819 820 821 822 823 824 825 826 827 828 829  
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## Exercise – trivia time!

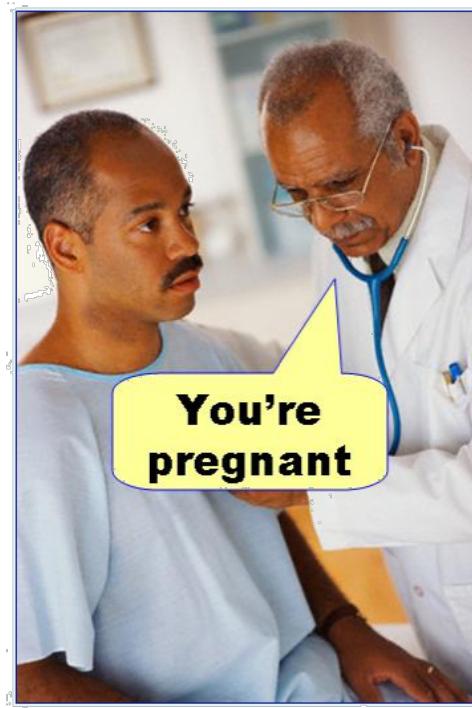
3. What problem might explain a strong negative correlation between number of pirates and global temperature?





## Exercise – trivia time!

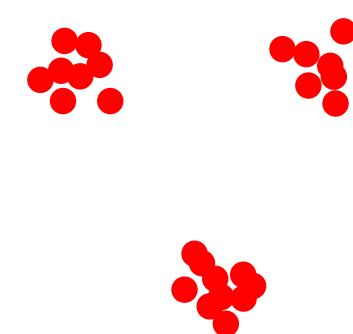
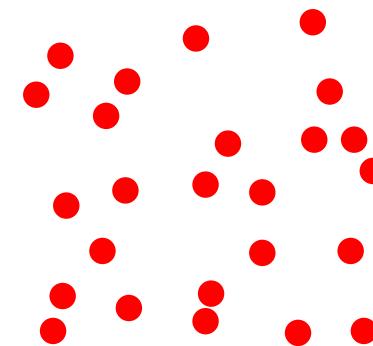
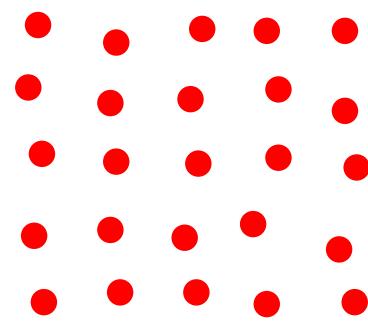
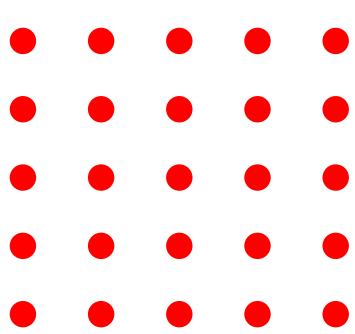
4. If you performed 100 statistical tests using an alpha of 0.02 how many times would you expect to falsely reject the null?





## Exercise – trivia time!

5. What is the maximum realizable value for the Nearest Neighbour Index ( $R$ ) in two-dimensional space?





## Exercise – trivia time!

6. Look at the following transition probability matrix. At  $t_0$  the system is in state X. What state will it be in at  $t_0 + 7$ ?

	X	Y	Z
X	1	0	0
Y	0	1	0
Z	0	0	1



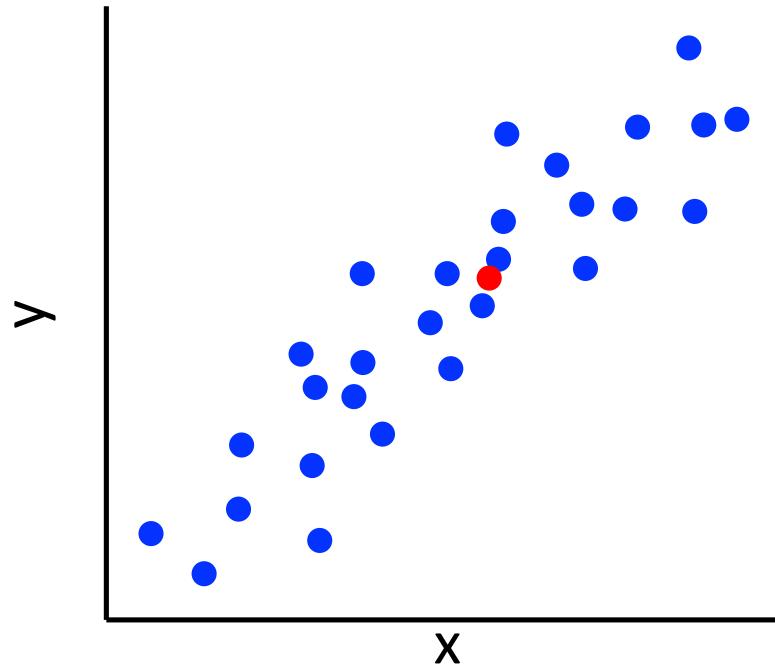
## Exercise – trivia time!

7. A von Mises with a Kappa of zero is the circular equivalent of which continuous univariate probability distribution?



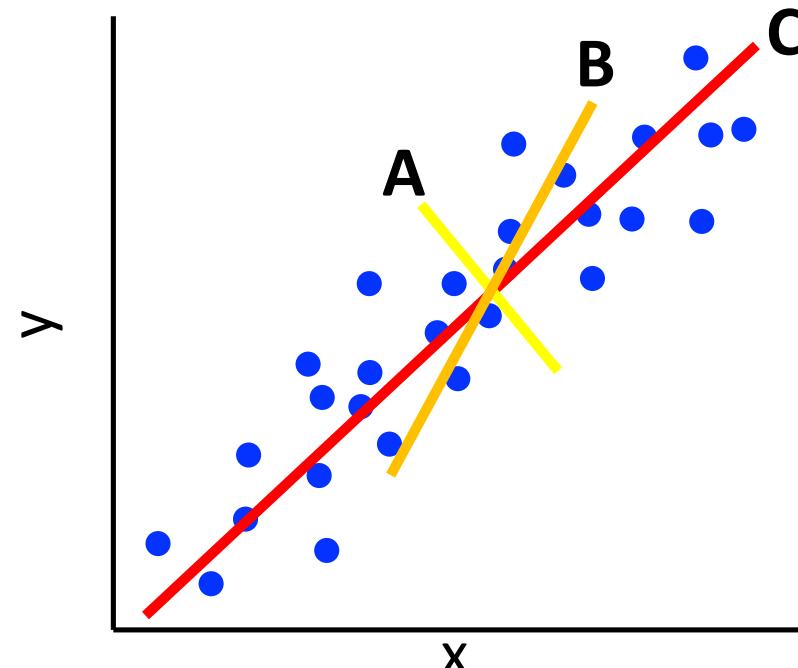
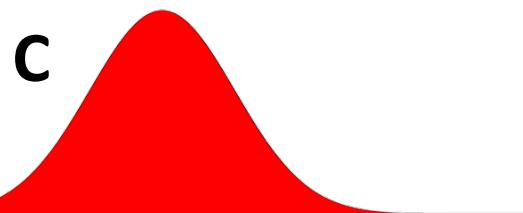
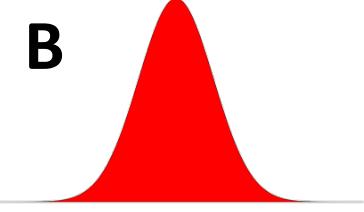
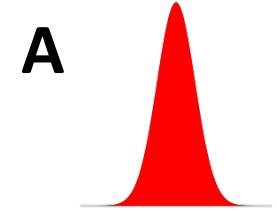


# Ordination





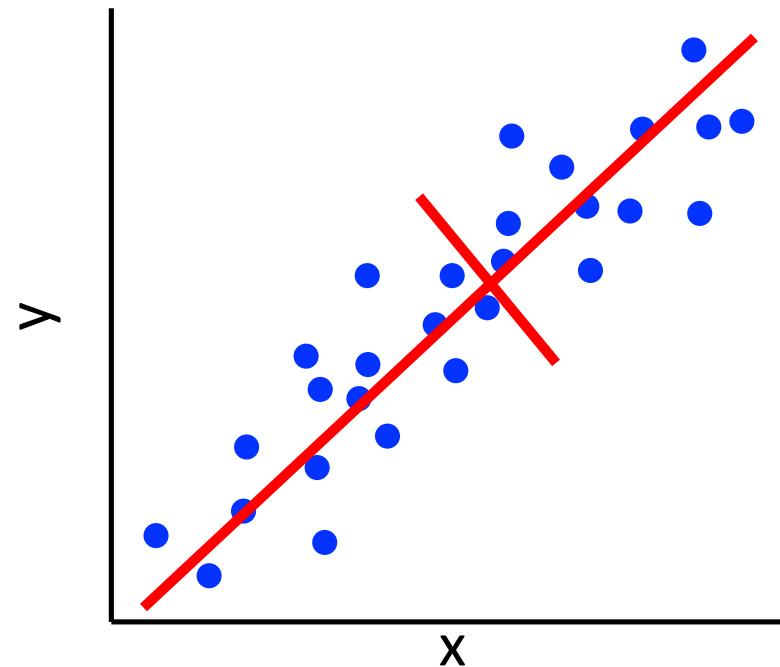
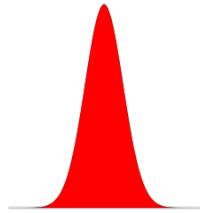
# Ordination





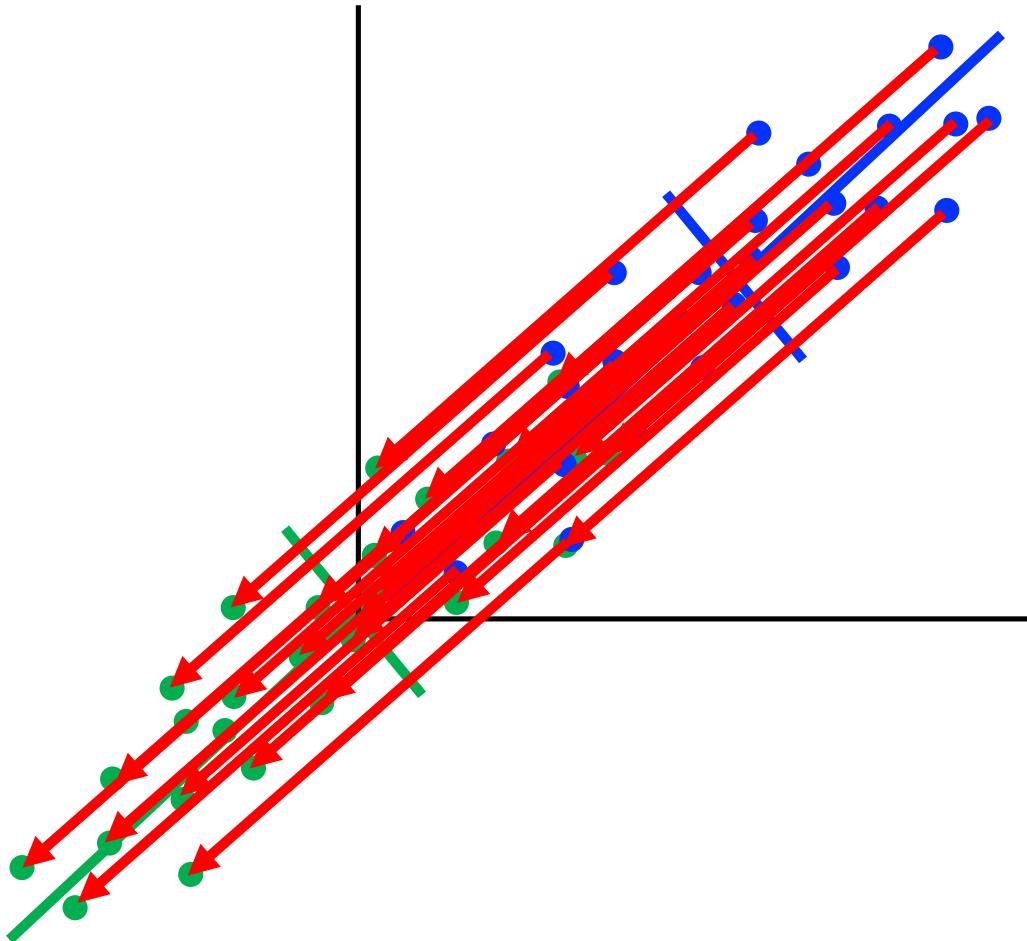
# Ordination

A



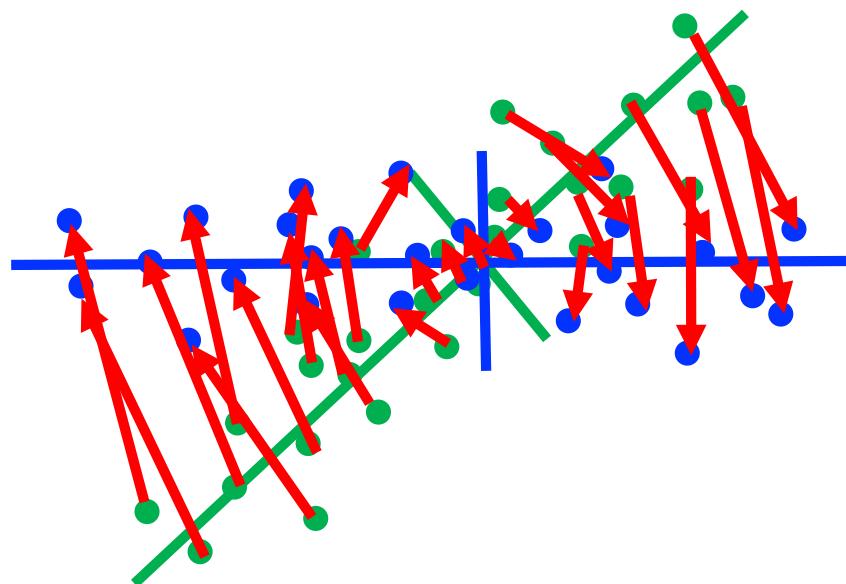


# Ordination



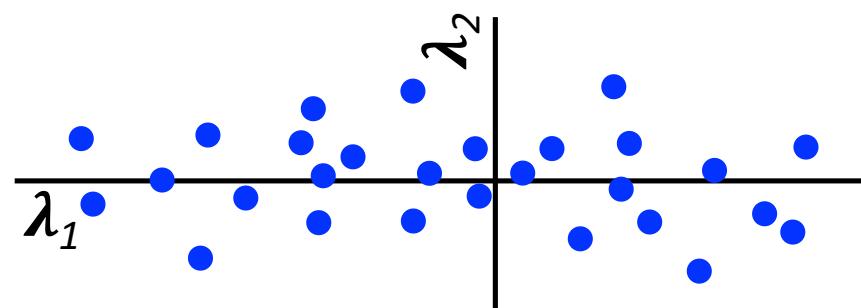


# Ordination





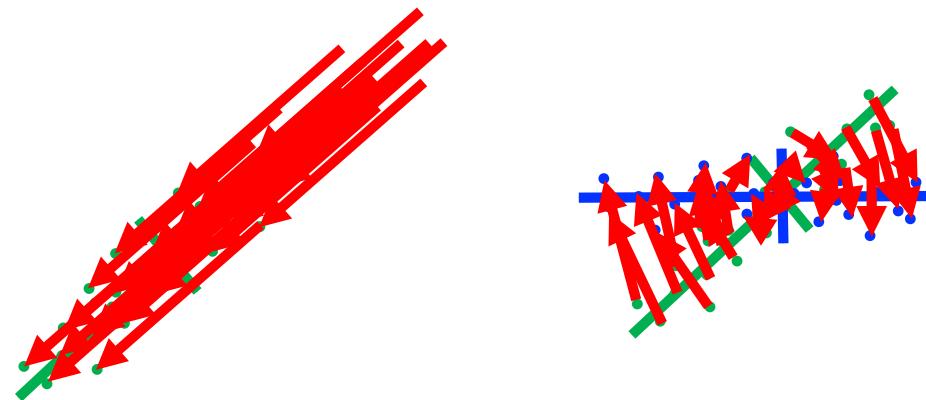
# Ordination





# Ordination

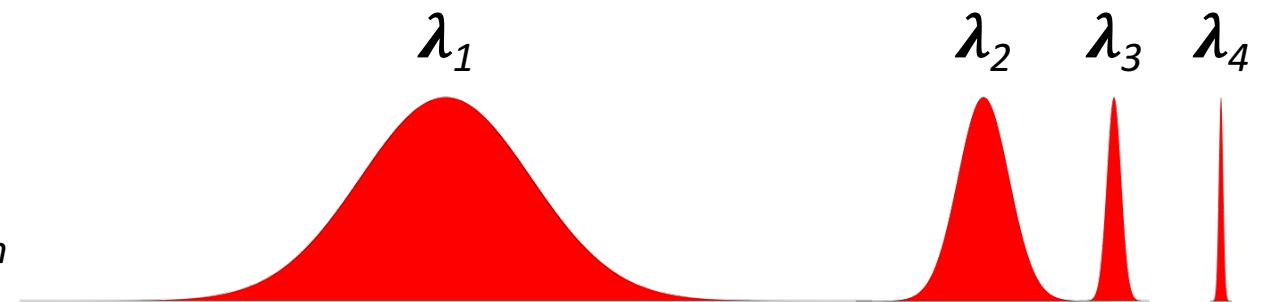
Eigenvectors



New coordinates of points in N-dimensional space

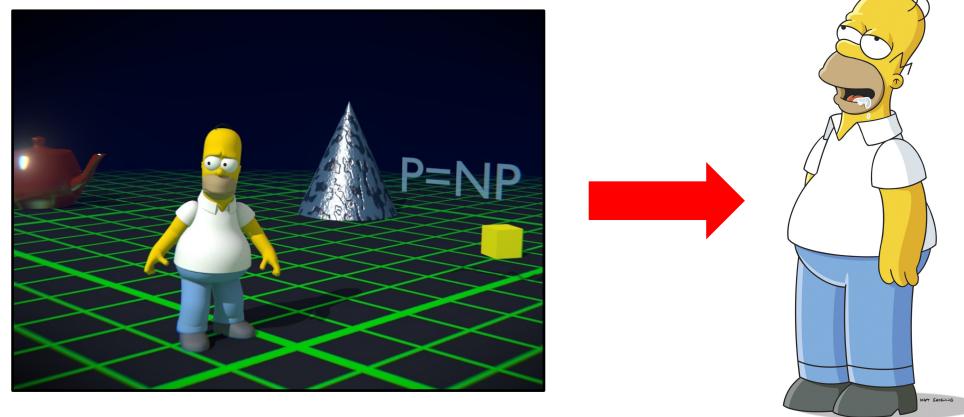
Eigenvalues

New axes;  $\lambda_1, \lambda_2, \lambda_3 \dots \lambda_n$



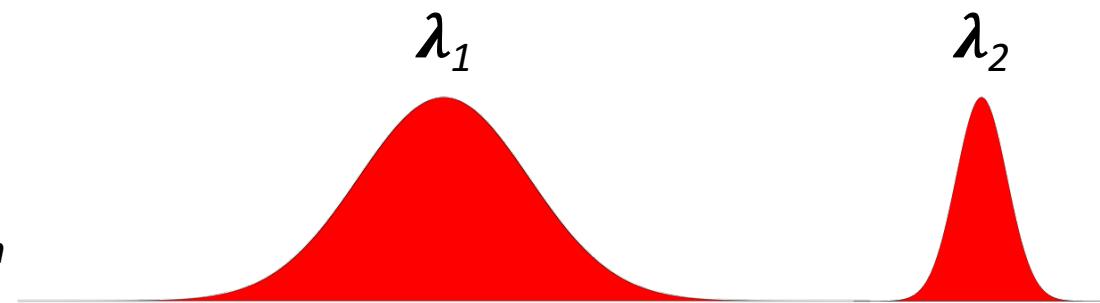


# Ordination



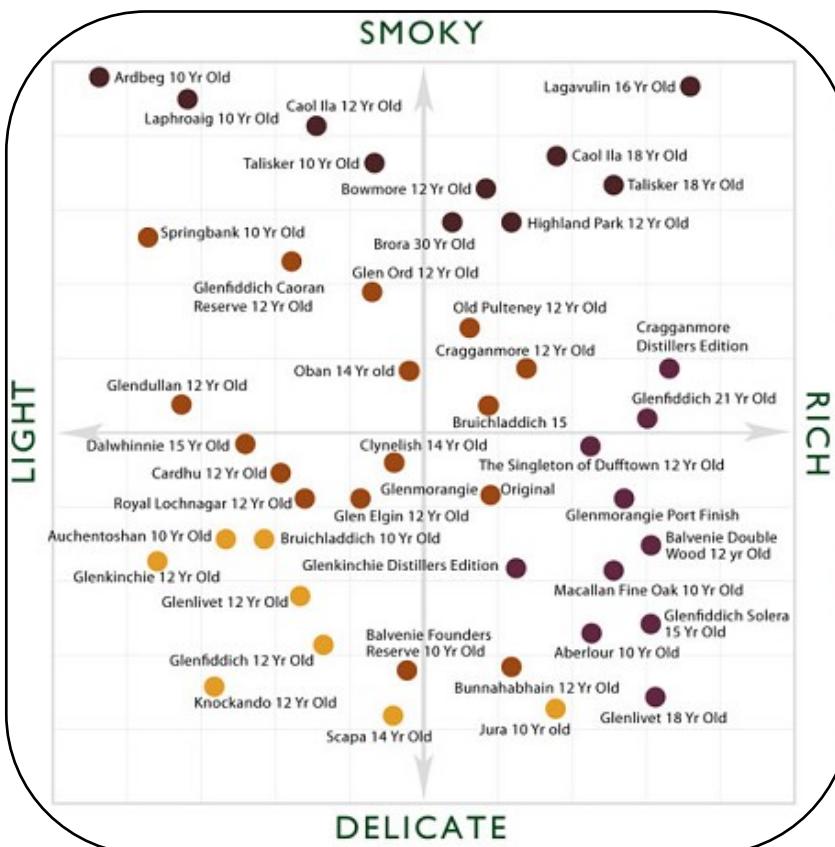
**Eigenvalues**

New axes;  $\lambda_1, \lambda_2, \lambda_3 \dots \lambda_n$

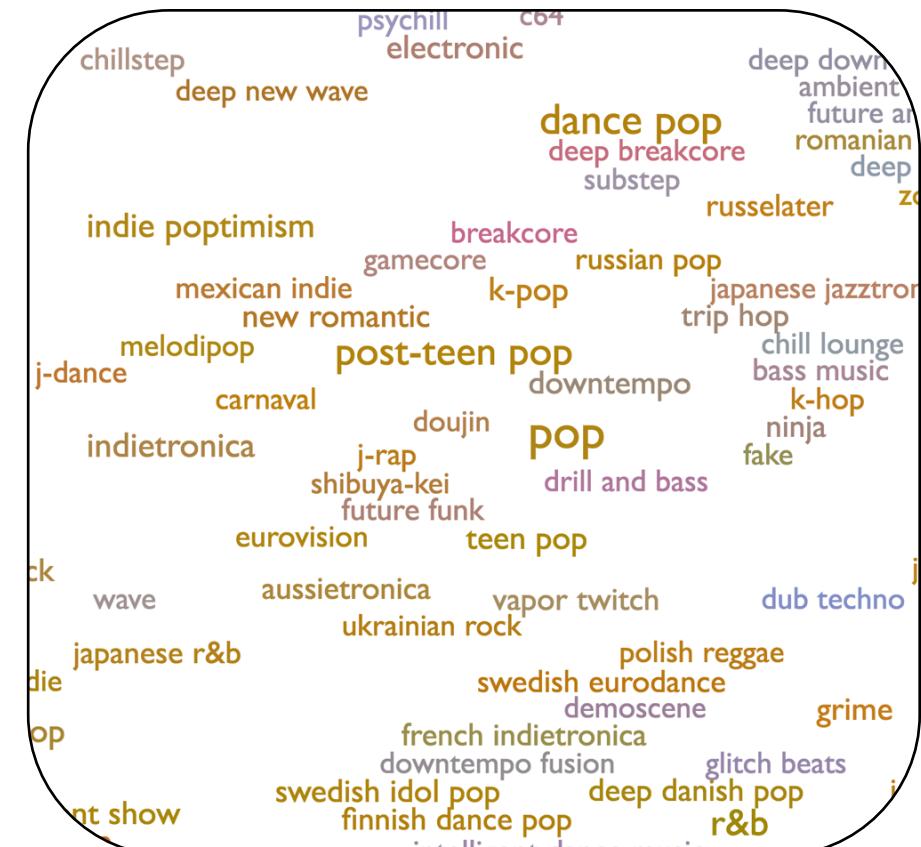




## Visualisation (-spaces)



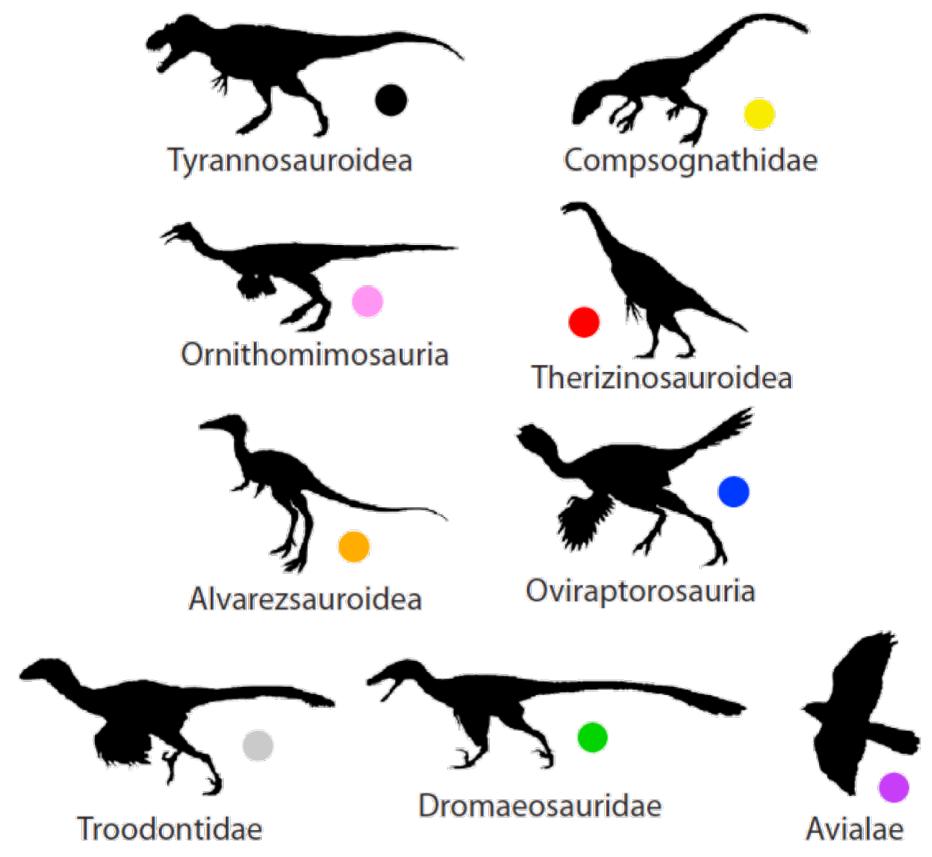
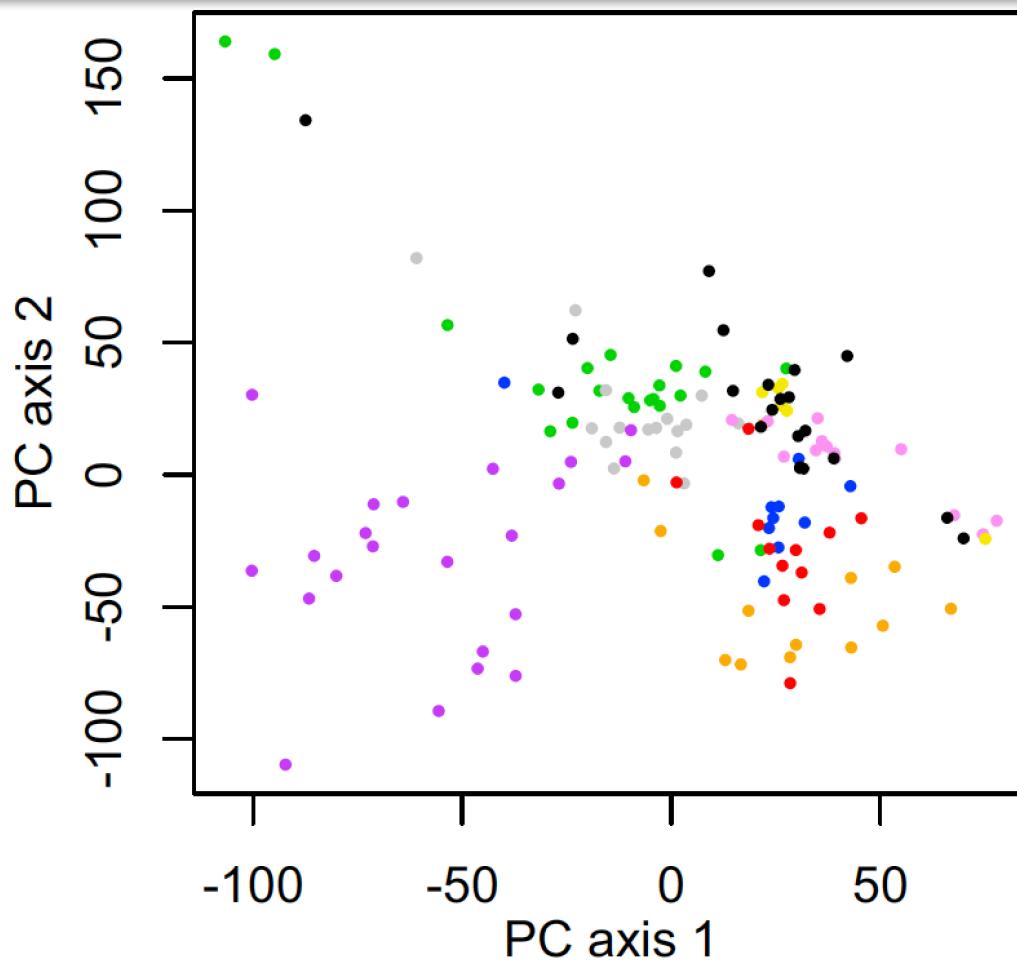
[whiskyanalysis.com](http://whiskyanalysis.com)



[everynoise.com](http://everynoise.com)



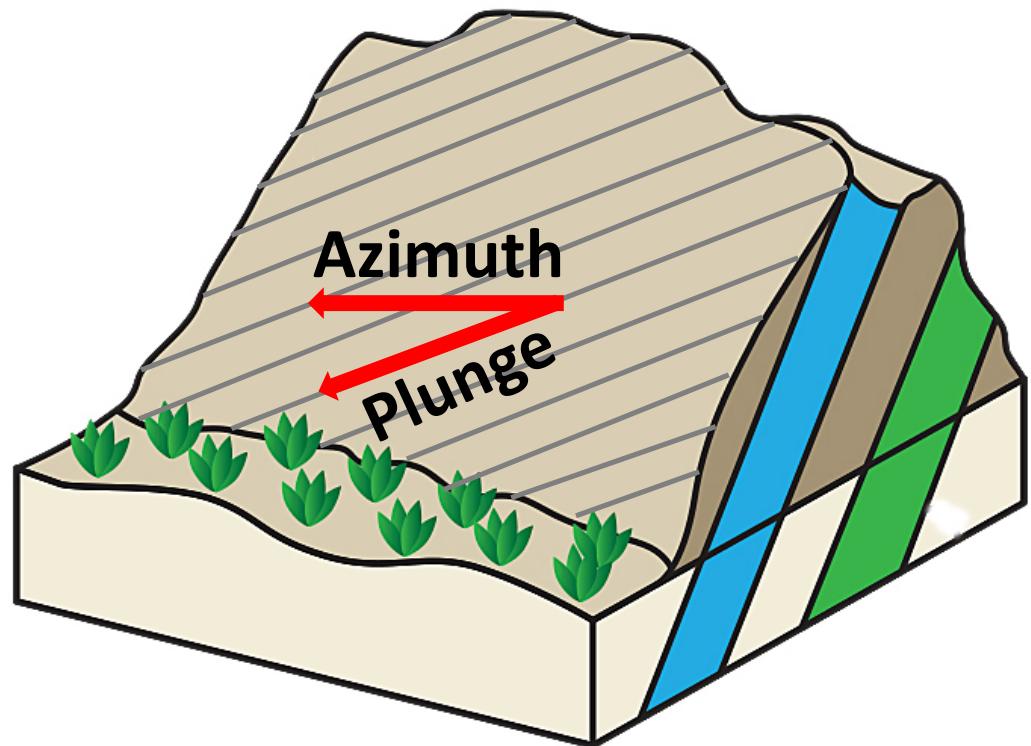
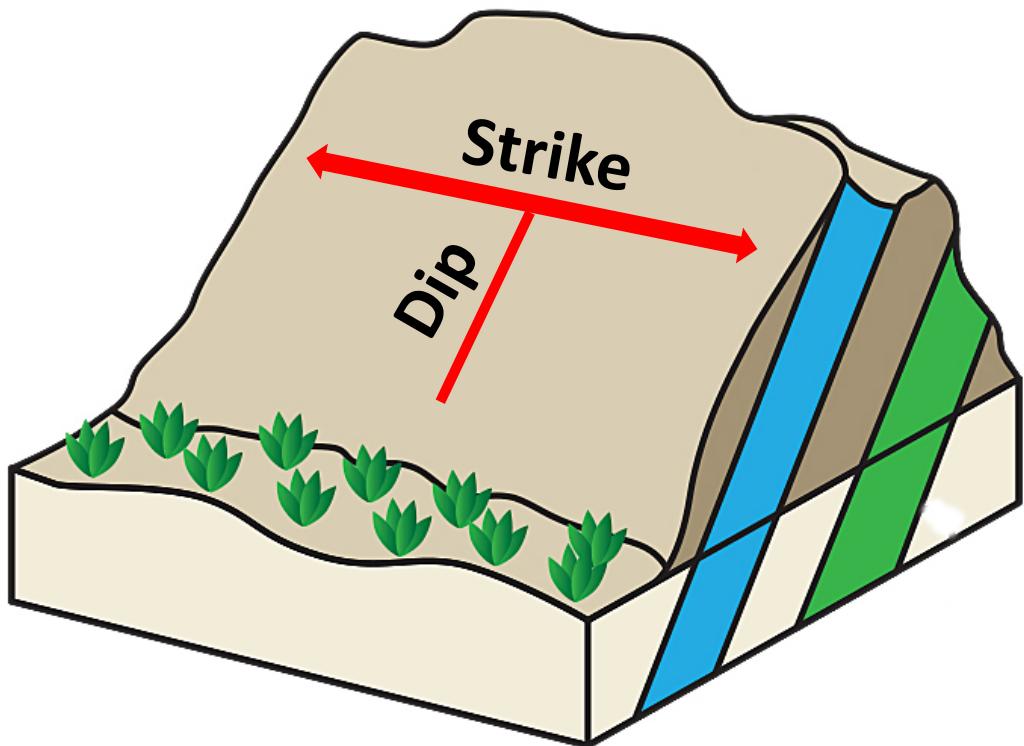
# Visualisation (-spaces)



Brusatte *et al* 2014 *Current Biology*



# Spherical statistics





# Spherical statistics

**Relative values are diagnostic of specific 3D distributions**

$\lambda_1 \approx \lambda_2 \approx \lambda_3$  = Random (i.e., uniformly distributed)

$\lambda_1 \gg \lambda_2 \approx \lambda_3$  = Clustered about preferred value

$\lambda_1 \approx \lambda_2 \gg \lambda_3$  = Evenly distributed on great circle

$\lambda_1 > \lambda_2 > \lambda_3$  = Scattered on great circle but with preferred direction



# Spherical statistics

**Point**

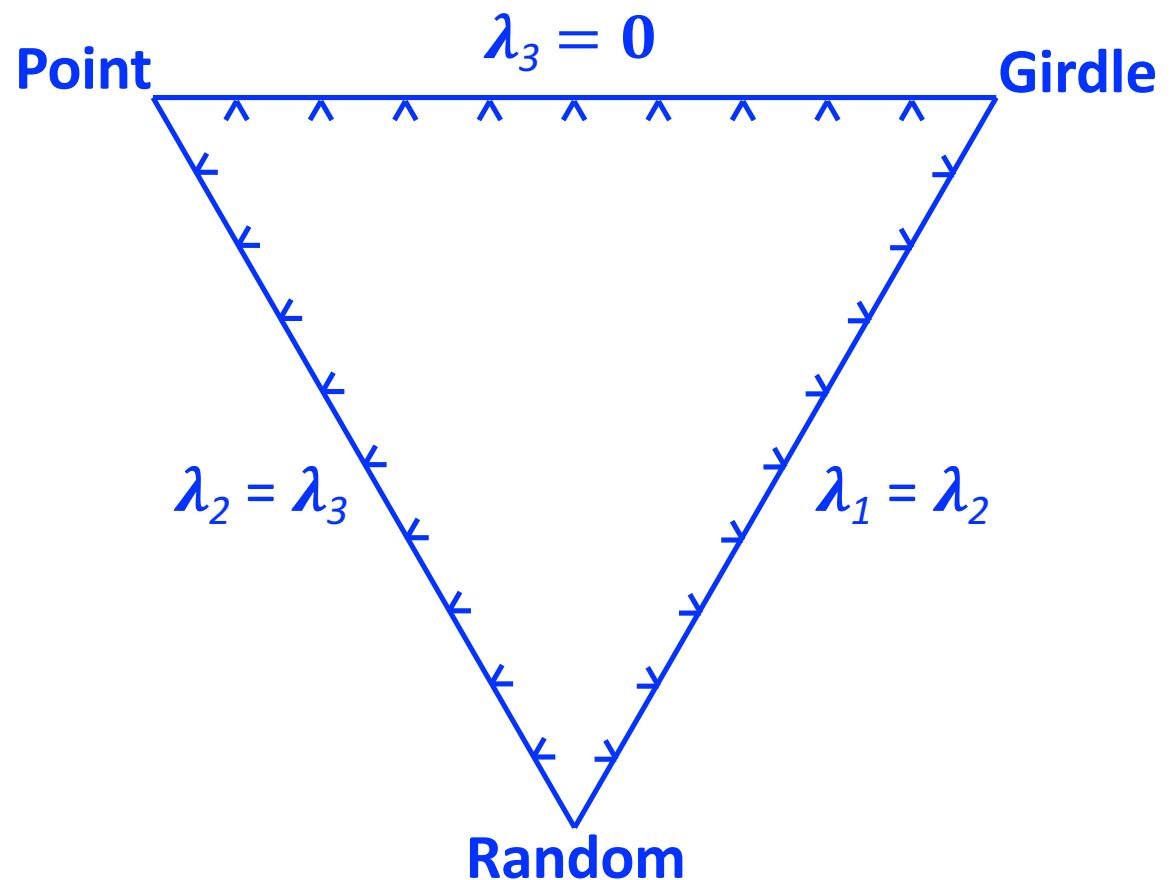
$$\text{Point} = \frac{\lambda_1 \lambda_2}{N}$$

**Girdle**

$$\text{Girdle} = \frac{3(\lambda_2 \lambda_3)}{N}$$

**Random**

$$\text{Random} = \frac{3(\lambda_3)}{N}$$





# Spherical statistics

Eigenvalues

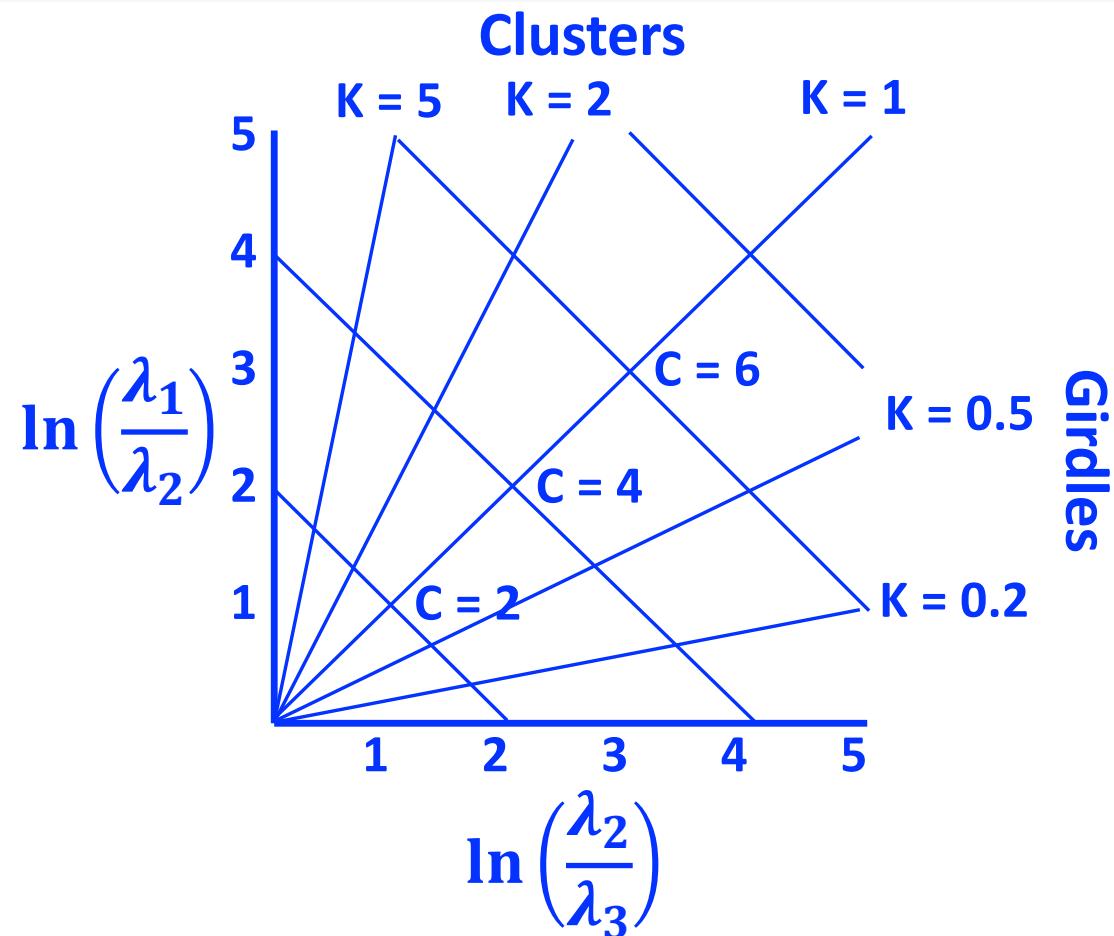
$$\lambda_1, \lambda_2, \lambda_3$$

Shape parameter

$$K$$

Strength parameter

$$C$$



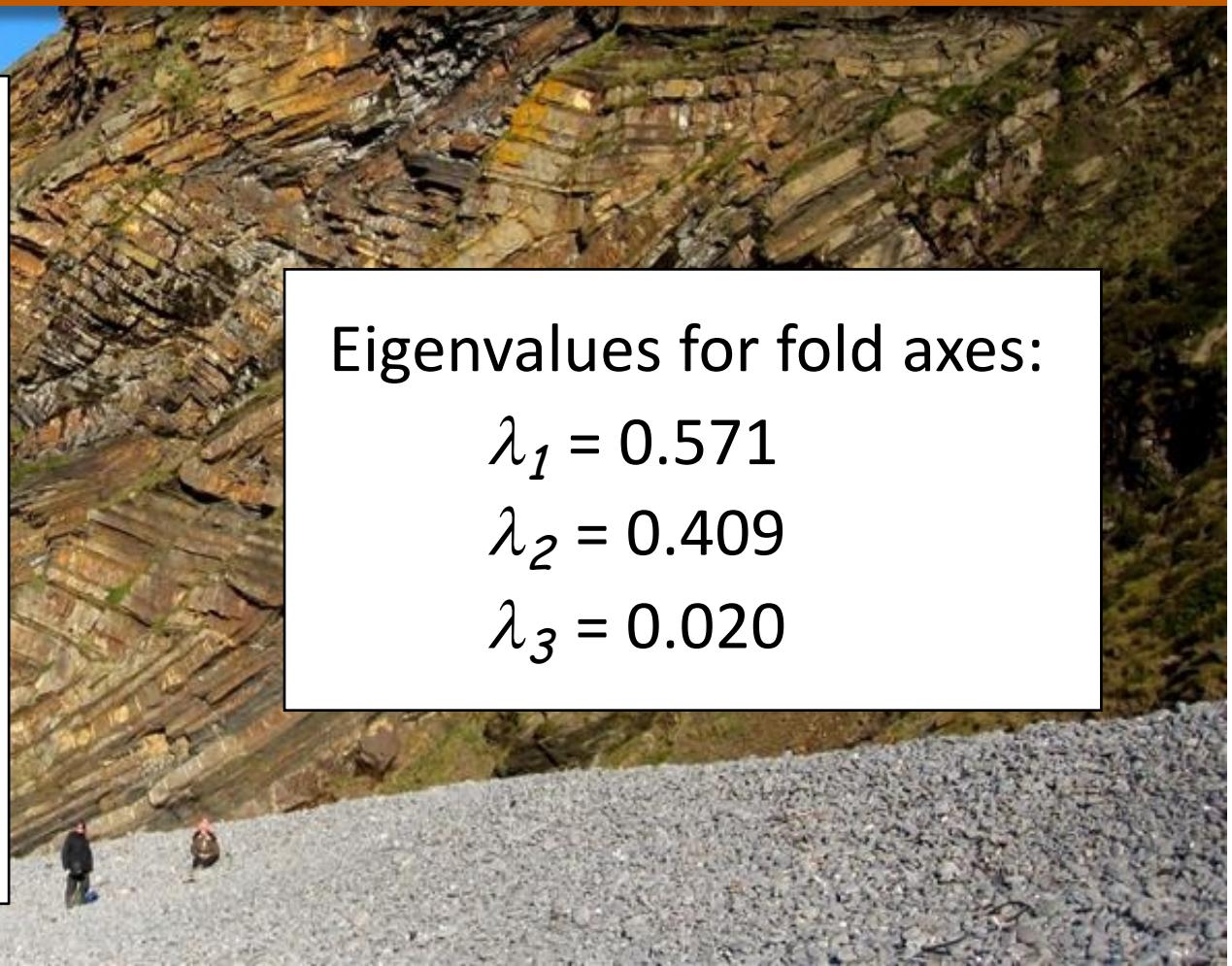
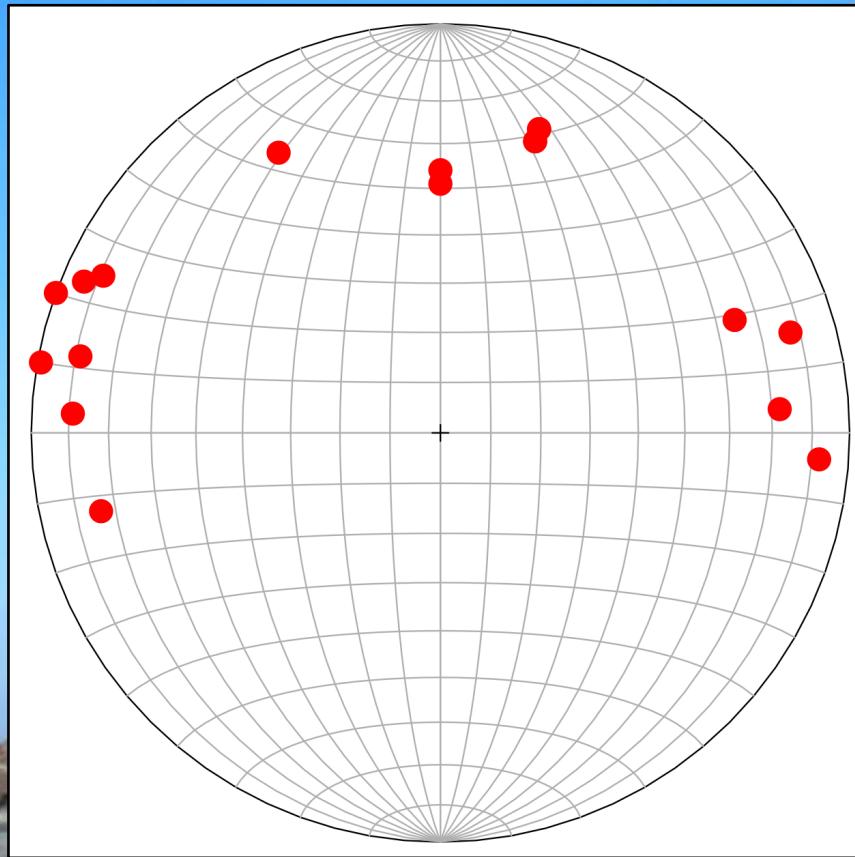


# Spherical statistics





# Spherical statistics



Eigenvalues for fold axes:

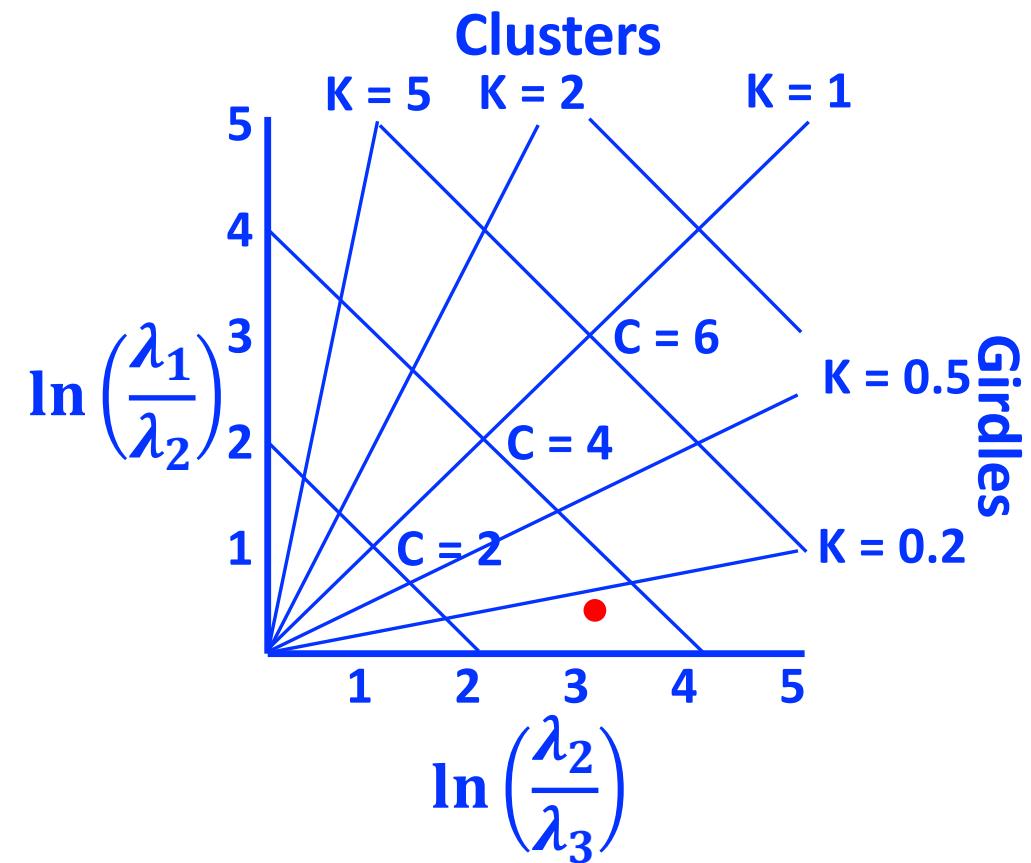
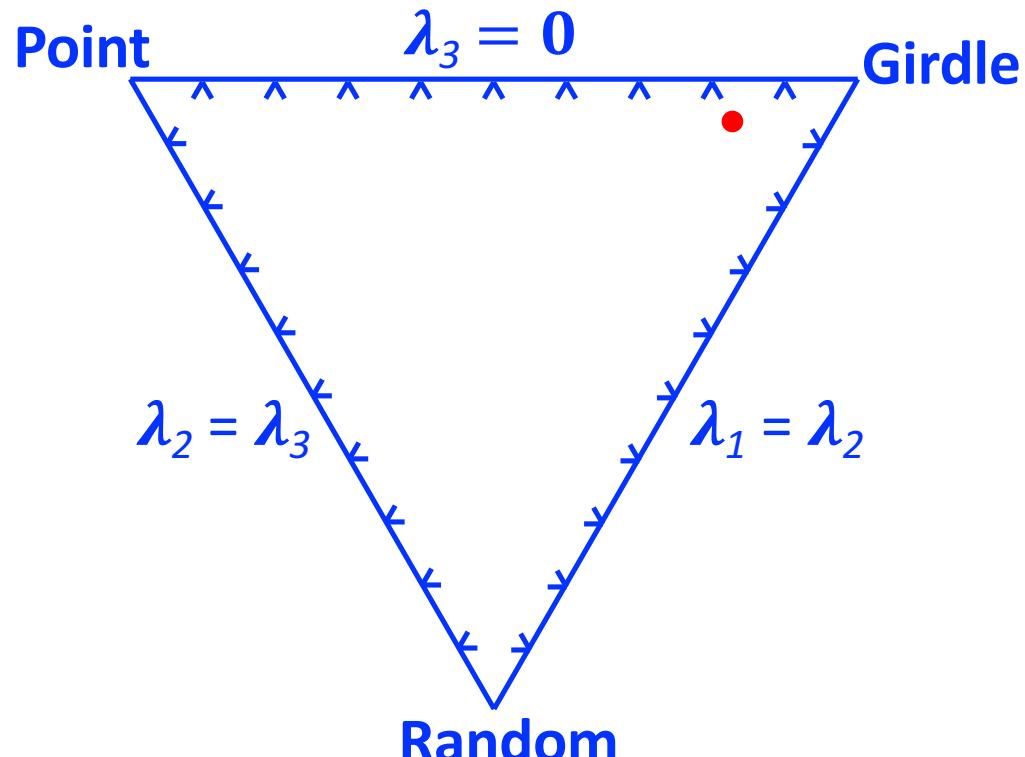
$$\lambda_1 = 0.571$$

$$\lambda_2 = 0.409$$

$$\lambda_3 = 0.020$$

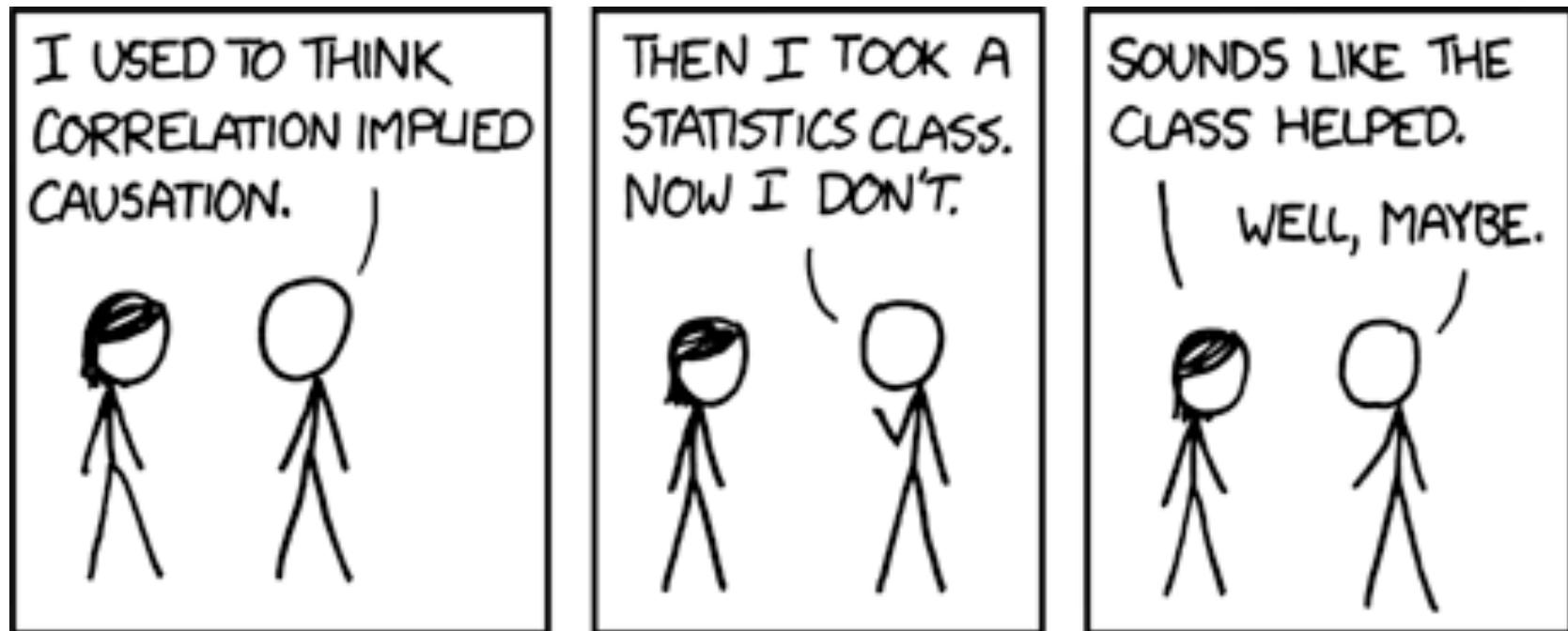


# Spherical statistics





# Almost done



Source: xkcd