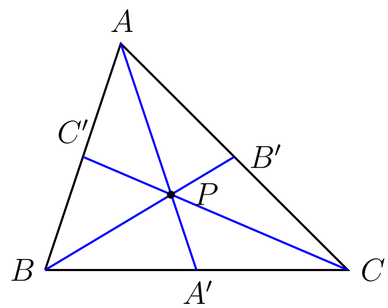
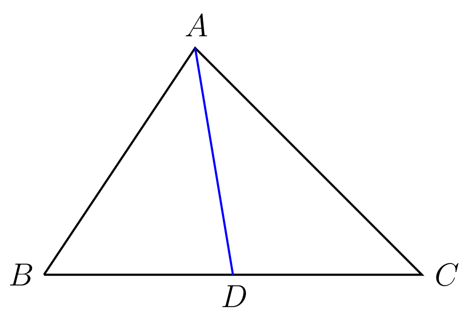
MA2219 Introduction to Geometry - Tutorial 3
Week 5: 11–15 February

1. Give a simple proof (without using Ceva's theorem) that the centroid of a triangle satisfies
$$\frac{|AP|}{|PA'|} = \frac{|BP|}{|PB'|} = \frac{|CP|}{|PC'|} = 2$$



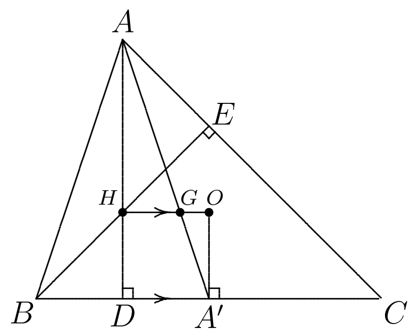
2. As suggested in Lecture 5, prove that the circumcentre of a right-angled triangle is the midpoint of the hypotenuse by "doubling" the triangle to form a rectangle.

3. Prove *Apollonius' theorem*: Given any triangle ΔABC, let D be the midpoint of BC. Then $|AB|^2 + |AC|^2 = 2|AD|^2 + 2|BD|^2$.



Hint. Use Pythagoras' theorem (you need to construct an appropriate right-angled triangle).

4. Suppose the Euler line OH of triangle ABC is parallel to BC. Prove that $|AD| = 3|OA'|$, where D is the foot of the altitude onto BC, O is the circumcentre and A' is the midpoint of BC.



5. The following “proof” shows that every triangle is isosceles. Can you find the mistake?

   Let ΔABC be any triangle. Let D be the midpoint of BC and let E be the intersection of the perpendicular bisector of BC with the angle bisector at A. Construct F on AB such that EF is perpendicular to AB and construct G on AC such that EG is perpendicular to AC.

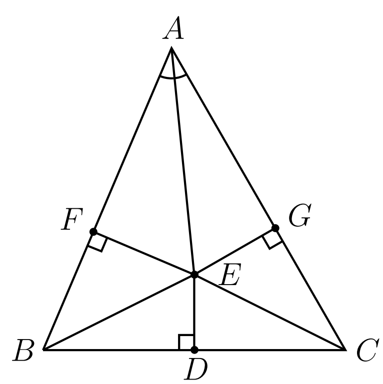
   The triangles ΔAEF and ΔAEG have one side in common and two angles equal, so they are congruent by AAS. Therefore $|AF| = |AG|$ and $|EF| = |EG|$.

   The triangles ΔBDE and ΔCDE have one side in common, $|BD| = |CD|$ and $\angle BDE = \angle CDE$. Therefore they are congruent by SAS. Therefore $|BE| = |CE|$.

   Since ΔBEF and ΔCEG are right triangles with two sides equal then they are congruent by RASS. Therefore $|BF| = |CG|$ and we can conclude

$$|AB| = |AF| + |FB| = |AG| + |GC| = |AC|$$

   and therefore ΔABC is isosceles.



6. Leonardo da Vinci (1452-1519) gives an interesting proof of Pythagoras' theorem. His proof is explained in the following diagram. Can you figure out how it works?