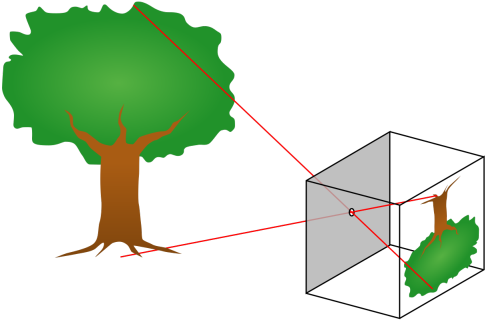
Lecture 3: Multiple view geometry

14 February, 2020

Recall: Geometry of a pinhole camera

Last time. We developed an algebraic model of a pinhole camera.

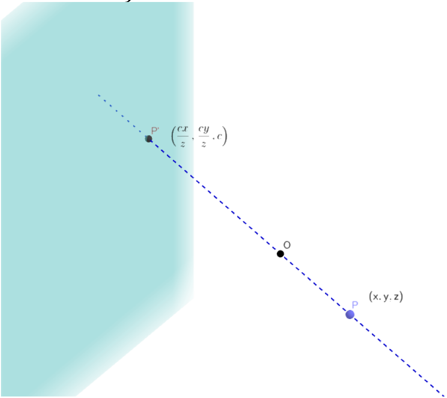
This camera projects a 3D image through a "pinhole" (or camera centre) onto a 2D plane, as in the diagram below.



Recall: Geometry of a pinhole camera

In this model we used projective coordinates in order to account for concepts such as the horizon line.

In these coordinates, a point $P = (x, y, z) \in \mathbb{R}^3$ is written as $[x : y : z : 1]$. If the camera centre is the origin, then this projects to the point $P' = \left(\frac{cx}{z}, \frac{cy}{z}, c\right)$ on the image plane $\{(x, y, z) \in \mathbb{R}^3 \,:\, z = c\}$.



Recall: Geometry of a pinhole camera

At a point (a, b, c) on this image plane $\{(x, y, z) \in \mathbb{R}^3 \,:\, z = c\}$, we use (a, b) as the coordinates. If we include the line at infinity, then we need to use projective coordinates $[a : b : 1]$.

Therefore, the point $P' = \left(\frac{cx}{z}, \frac{cy}{z}, c\right)$ on the image plane is written in projective coordinates as

$$\left(\frac{cx}{z}, \frac{cy}{z}\right) \rightsquigarrow \left[\frac{cx}{z}, \frac{cy}{z} : 1\right] = [cx : cy : z]\,.$$

Therefore our camera projects the point $P = [x : y : z : 1]$ to the point $P' = [cx : cy : z]$. If we write our points as column vectors, this can be represented by matrix multiplication

$$\begin{pmatrix} cx \\ cy \\ z \end{pmatrix} = \begin{pmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Recall: Geometry of a pinhole camera

We can rewrite this equation

$$\begin{pmatrix} cx \\ cy \\ z \end{pmatrix} = \begin{pmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

as

$$P' = MP$$

where $P' = [cx : cy : z]$ is the point on the image plane, $P = [x : y : z : 1]$ is the point in $\mathbb{R}^3$, and the matrix $M = \begin{pmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ in the above equation is called the camera matrix. It depends on the camera centre (which is the origin in this example) and the image plane (which is $\{z = c\}$ in this example).

Recall: Geometry of a pinhole camera

At the end of the last lecture we showed how to calculate the camera matrix for an arbitrary image plane and arbitrary camera centre.

Instead of rederiving all the equations for the new image plane and camera centre, we instead rotated and translated the coordinates to reduce to the above example.

This approach uses geometry to simplify the problem, rather than brute force calculations

In today's lecture. We can view a single object with more than one camera to get a better picture of the three dimensionality of the object.

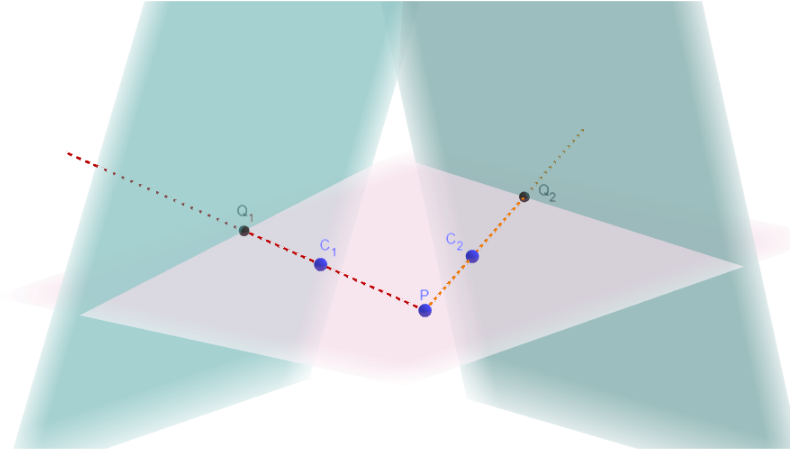
When viewing a single point in the original 3D object with multiple cameras, the projections to the image planes are related. This helps us with the reconstruction problem from the first lecture.

We will also see how using two cameras gives a sense of depth in a picture.

Consider a point P in 3D space and two cameras centred at C_1, C_2 respectively. The image planes are the two blue planes in the picture on the next slide.

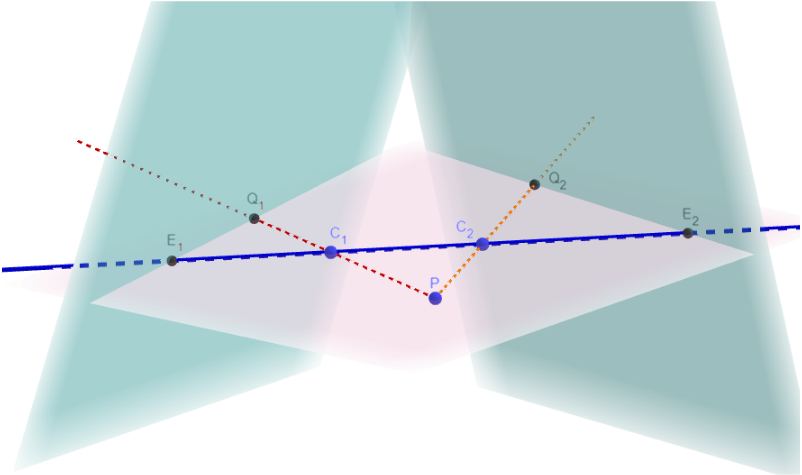
Viewing a single point with two cameras

The original point P, the two camera centres C_1, C_2 and the two image points Q_1, Q_2 must lie on the same plane.



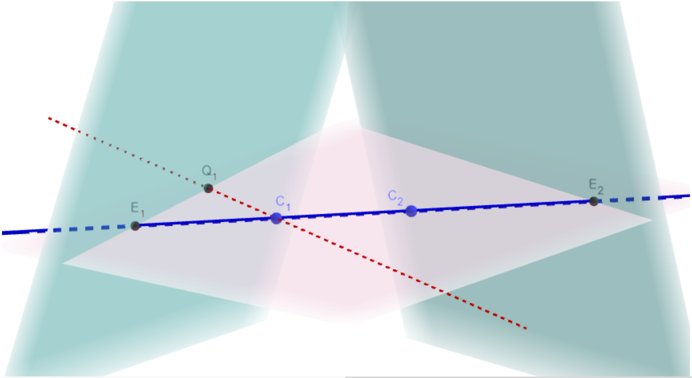
Viewing a single point with two cameras (cont.)

Therefore the line through C_1 and C_2 must also lie on this plane. This is called the camera baseline and the points of intersection E_1, E_2 with the two image planes are called the epipoles.



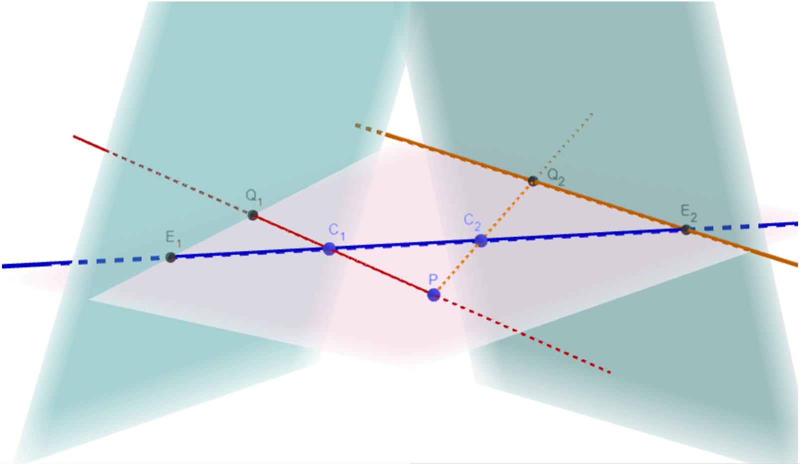
Since three non-collinear points determine a unique plane, then any three of these five points determine this plane.

Suppose we only know the two camera centres C_1, C_2 and the image point Q_1 on the first image plane. We would like to find out the relationship between (a) the original point P in 3D space, and (b) its projection Q_2 on the second image plane.



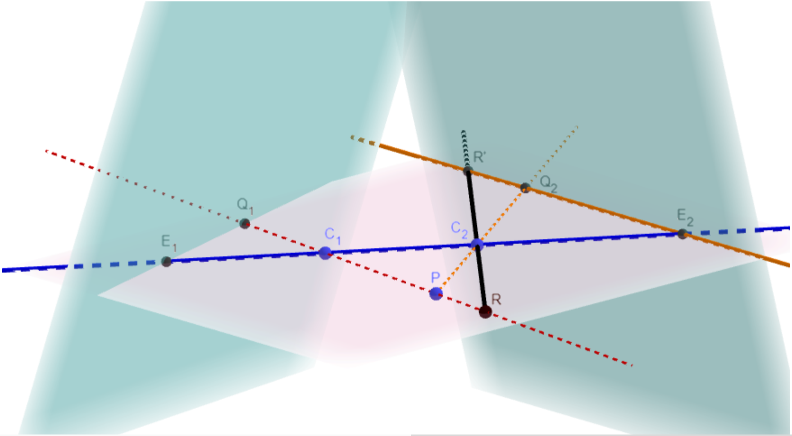
Viewing a single point with two cameras (cont.)

We know that the point P must lie on the line through Q_1 and C_1. We also know that the projection Q_2 on the second image plane must lie on the line of intersection between the second image plane and the plane through Q_1, C_1 and C_2.



Viewing a single point with two cameras (cont.)

If we move the point P along the line through Q_1 and C_1, then the image Q_2 moves along the orange line in the picture below. Note that this line passes through the epipolar point E_2. We can see that the point R projects onto the point R' which is on this orange line through E_2.



Viewing a single point with two cameras (cont.)

The position of the projection Q_2 on this line tells us the depth of the point P. You can see this in the following interactive picture.
graemewilkin.github.io/Multiple_View_Geometry/Depth_Perception.html

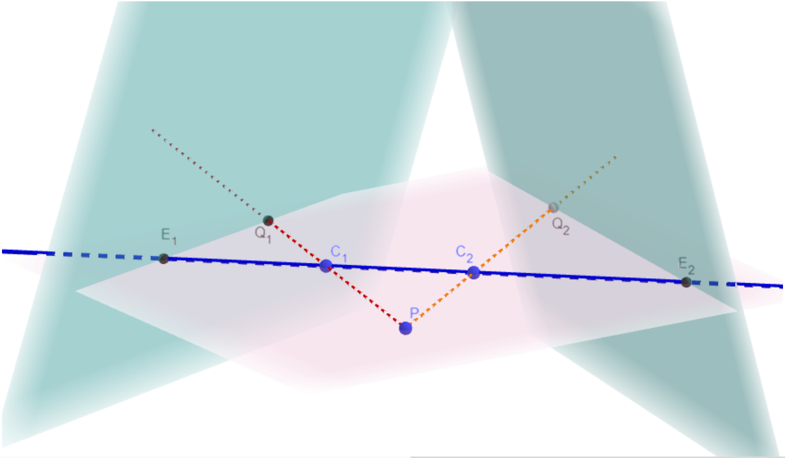
By moving the point R along the red line, the projection R' will move along the orange line. When R is far away, then the projection R' moves closer to the first image plane. When R is very close to the first image plane then the projection R' moves further away.

Our brains have an intuitive understanding of this concept, and this is how we use two images (one for each eye) to gauge the depth of an object.

A more precise mathematical description

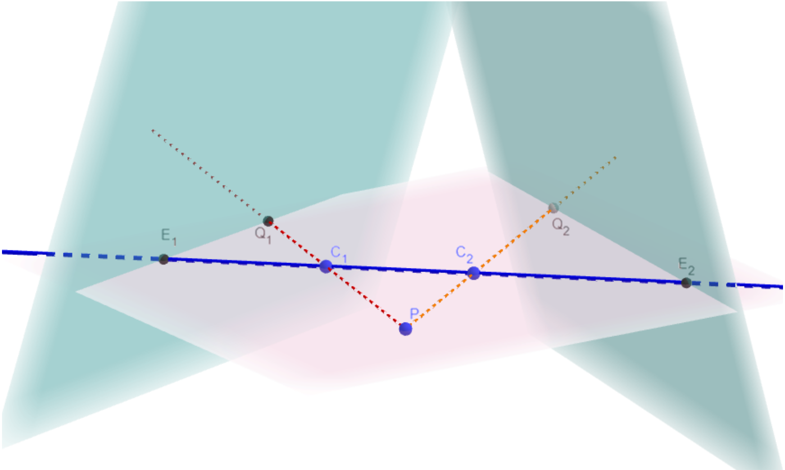
Two camera centres determine the camera baseline, defined as the line through the two camera centres.

The epipoles E_1 and E_2 are the intersection points of the camera baseline with the image planes.



A more precise mathematical description (cont.)

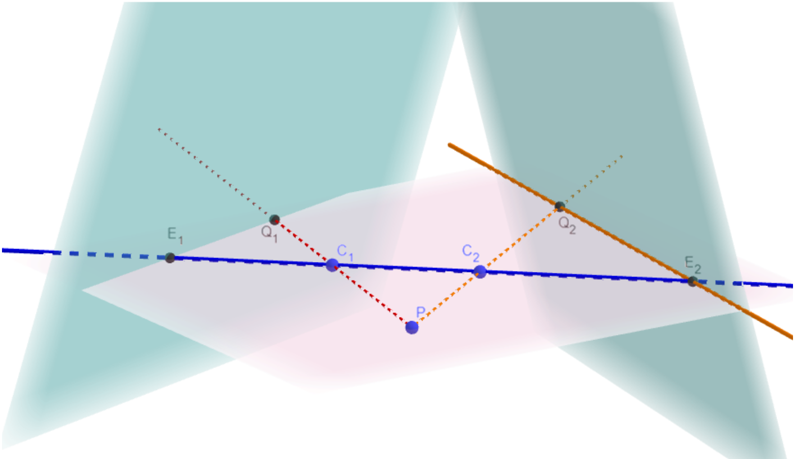
Given a point Q_1 on the first image plane, the epipolar plane of Q_1 is the plane through C_1, C_2 and Q_1. Note that this is also the plane through E_1, E_2 and Q_1.



A more precise mathematical description (cont.)

The epipolar line of Q_1 is the line on the second image plane given as the intersection of the epipolar plane of Q_1 with the second image plane. This is the orange line in the diagram below.

Note that this line always passes through the epipole E_2.



Moving the camera

Now we consider what happens to the image when we move the position of the camera centre and the image plane.

There are infinitely many ways to do this. We will focus on one type of motion, where the camera centre moves along a line, and the image plane remains parallel to the original image plane, and at the same distance from the camera centre.

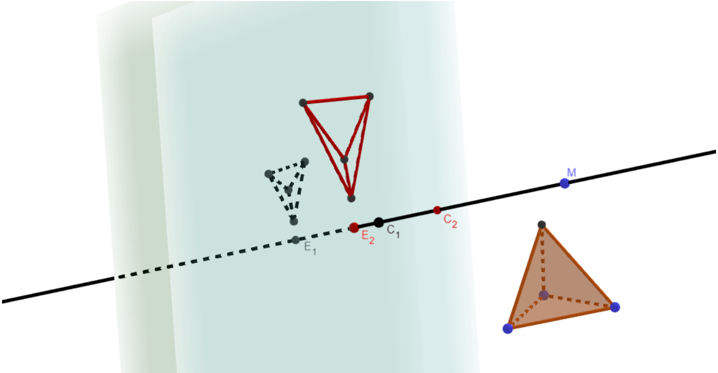
Example.
You can see a real-life example of this is in the video below of a camera zooming along a corridor.
https://www.youtube.com/watch?v=IWY21GYvzVE

Moving the camera (cont.)

How can we understand this using geometry?

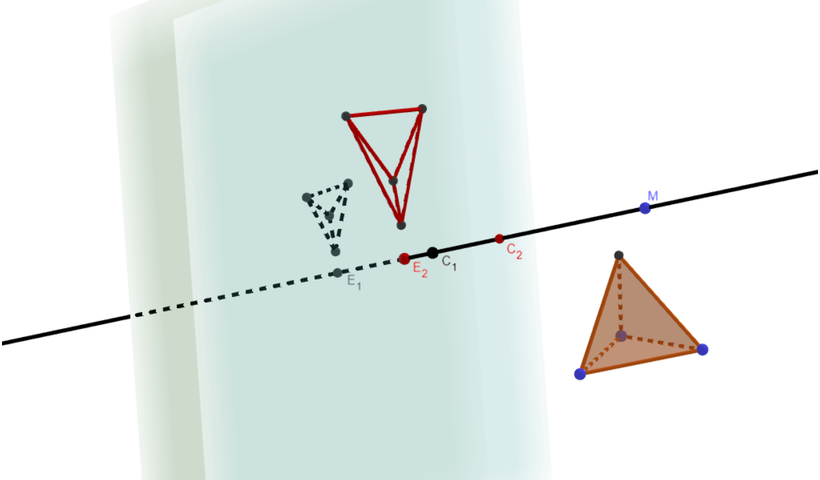
Consider the following diagram, where the 3D object is the tetrahedron, and we are projecting it to the green image plane through the camera centre C_1.

The second camera centre C_2 moves along the line C_1M in the diagram below, and we project the tetrahedron through C_2 onto the blue image plane.



Moving the camera (cont.)

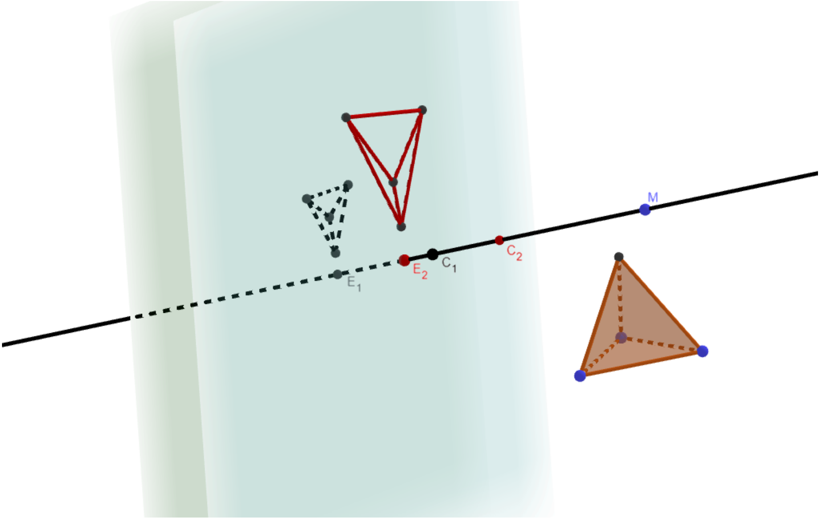
Since the camera centres move along a line, then the epipole E_1 is at the intersection of this line with the original image plane, and the epipole E_2 is the intersection of this line with the moving image plane.



Moving the camera (cont.)

You can click on the following link for an interactive picture

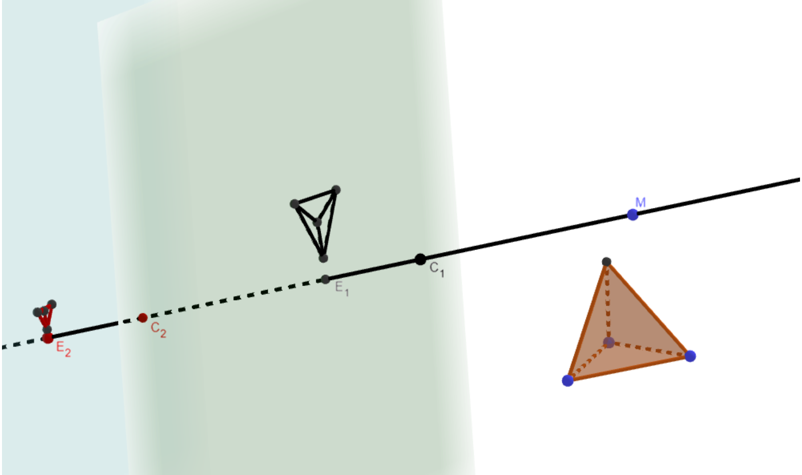
graemewilkin.github.io/Multiple_View_Geometry/Moving_Camera_Tetrahedron.html



Moving the camera (cont.)

If C_2 is very far away from the original image, the the projection is very close to the epipole E_2.

Therefore, as we move the camera centre C_2 along the line, the image points move along lines radiating out from the epipole E_2.



Moving the camera (cont.)

We can see this in the following way (diagram on the next slide). Consider the simple case of a single point P projecting onto points Q_1 and Q_2 on the two image planes.

Draw a plane through P, C_2 and E_2 (the orange plane in the diagram). Then the line PC_2 lies on this plane, and so the point Q_2 must lie on the intersection of this plane with the green image plane.

The intersection of these two planes is the blue line in the diagram.

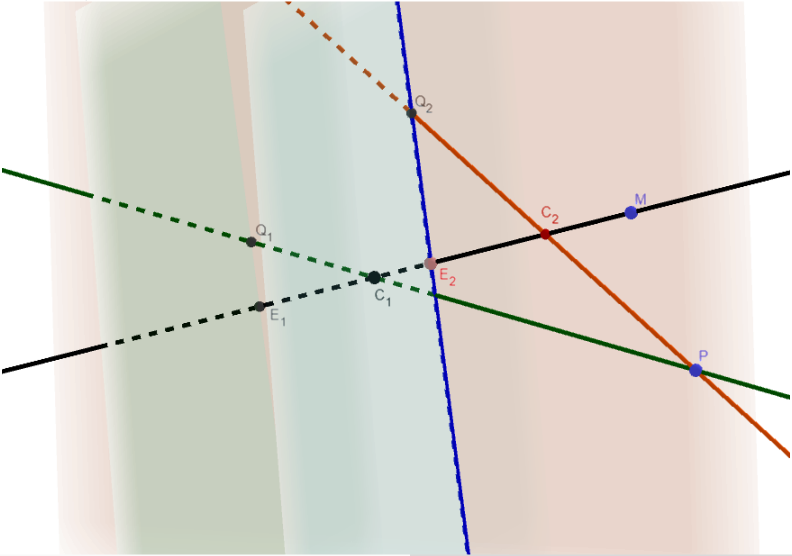
Since E_2 is on both planes, then the blue line must contain E_2.

Therefore, the image Q_2 moves along a line through E_2.

You can see how this works in the following interactive diagram
graemewilkin.github.io/Multiple_View_Geometry/Moving_Camera_Image_Point.html

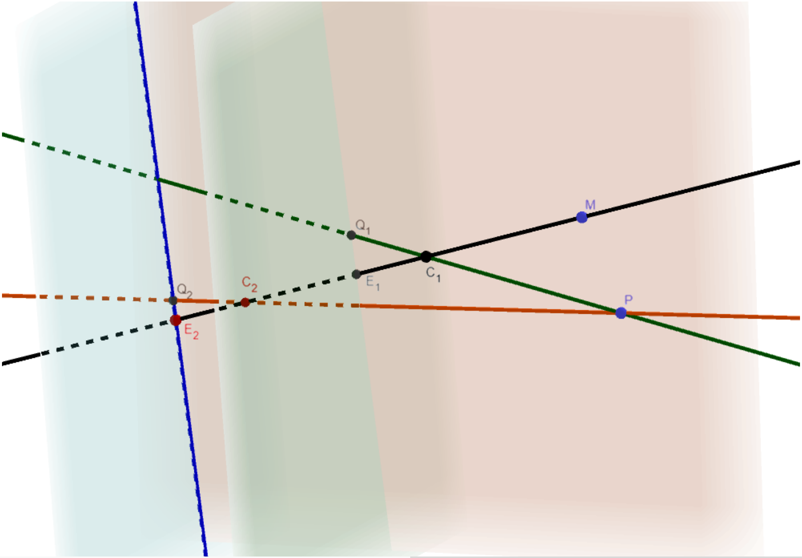
Moving the camera (cont.)

The point P projects to Q_2 on the blue image plane. This point Q_2 is always on the intersection of the red plane and the blue plane.



Moving the camera (cont.)

The camera centre C_2 is now far away from the original object P, and so Q_2 is close to E_2.



Summary

In these lectures we have seen a number of new things.

1. Modelling a pinhole camera by projecting through the camera centre onto the image plane.

2. Using projective coordinates as the most general way to describe these projection.

3. Using linear algebra to calculate the coordinates of the projection onto the image plane.

4. Using two cameras to see depth and the “three dimensionality” of an object.

5. Using geometry to understand more about the two camera configuration.

Projects

Here are a number of projects to work on that extend the ideas from these lectures. Some are theoretical, and some are more oriented towards applications.

Projects.

1. Give an overview of the theory of projective geometry.
2. Explain the fundamental matrix of a pair of pinhole cameras (see the chapter "Epipolar geometry and the fundamental matrix" by Hartley and Zisserman on Moodle).
3. Investigate the errors that can occur when computing the camera matrix for real-world examples.
4. Give a general overview of how projective geometry is used in computer vision.

Projects

Things to keep in mind.
(a) Pick one idea from the topic that you find really interesting. The rest of the project should build up to that idea.
(b) Your project should have content from pure mathematics. For example, if you are doing project 3 or 4, it is important to include something explaining how the theory is developed from pure mathematics in addition to the real world issues and/or history.

Projects.

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