MA2219 Introduction to Geometry - Tutorial 2
Week 4: 4–8 February

For questions 1 to 4, you can only use Euclid's axioms 1 to 4, the common notions and the results derived from them as well as Pasch's axiom: that if a line passes through one side AB of a triangle ABC then it must intersect one of the other sides AC or BC, or the opposite vertex C.

Preliminary Remark. Propositions 1–28 in Euclid Book I do not require Euclid's 5th axiom (the parallel postulate). This includes the following two propositions.

Euclid Prop. I.27. If a straight line falling on two straight lines makes the alternate angles equal to one another, then the straight lines are parallel to one another.

Euclid Prop. I.28. If a straight line falling on two straight lines makes the complementary angles equal to one another, then the straight lines are parallel to one another.

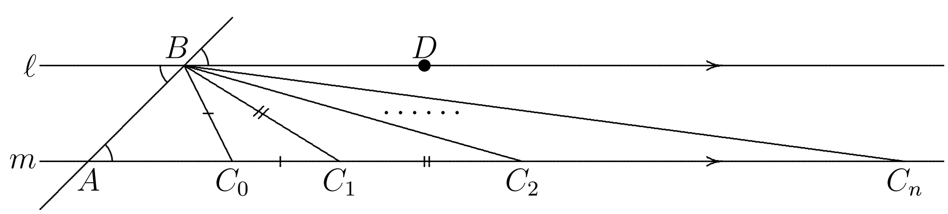
The converse of these results uses the parallel postulate.
Euclid Prop. I.29. A straight line falling on parallel straight lines makes the alternate (and complementary) angles equal to each other.

The solutions to Questions 1–4 given below use Euclid Prop. I.27 and Euclid Prop. I.28. Apart from Question 1 (which assumes Euclid's 5th axiom) you are not allowed to use Euclid Prop. I.29.

1. Playfair's Axiom states that if P is a point not on a line ℓ, then there is exactly one line through P that is parallel to ℓ. Prove that Euclid's 5th axiom implies Playfair's axiom.

2. Prove that Playfair's Axiom implies that the angle sum of a triangle is $180°$.

3. Assume that the angle sum of any triangle is $180°$. Let AB be a line intersecting two lines m and ℓ at A and B respectively such that it makes equal alternate angles with them. Let C_0 and D be points on m and ℓ respectively such that both are on the same side of AB. Suppose $C_1, C_2, \cdots, C_n$ are points on m and all on the same side of AB as C_0 such that $BC_i = C_iC_{i+1}$ for $i = 0.\ldots, n-1$. Prove that $\angle DBC_n = 2^{-n}\angle AC_0B$.



4. Prove that if the angle sum of any triangle is $180°$ then Playfair's axiom holds.

In questions 5 and 6 you are allowed to approximate distances (for example, you can set your compass to "slightly more than half-way between two points" or "slightly less than the length of the ruler"). In real life you can do this by eyeballing the measurement or by trying a few measurements until you find one that works. Your final construction should be exact, i.e. the goal of the questions is to use the approximations to make a precise construction.

5. (What to do if your ruler is too short?)

   Suppose that you have a ruler approximately 10cm long, however you would like to draw a straight line between two points which are approximately 25cm apart. Explain how to use a working compass together with your short ruler to construct the straight line.

6. (What to do if your compass is stuck?)

   Suppose that your compass has rusted so that it is stuck and can only draw circles of radius approximately 2cm. Explain how to use this compass together with a ruler to draw an equilateral triangle on a line segment AB that is approximately 5cm long.