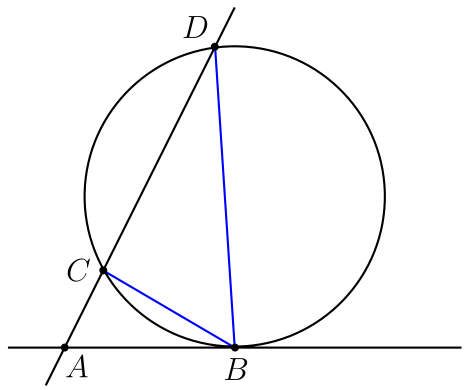
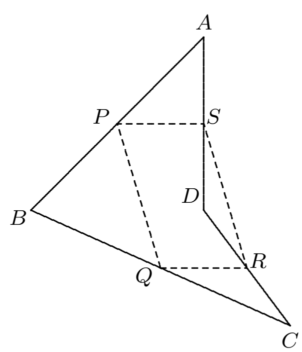
MA2219 Introduction to Geometry - Tutorial 4
Week 6: 18–22 February

1. Consider a circle as below, with A outside the circle and B, C, D on the circle such that AB is tangent to the circle and A, C, D are collinear. Prove that $|AC| \cdot |AD| = |AB|^2$ and hence $\Delta ABC \sim \Delta ADB$.

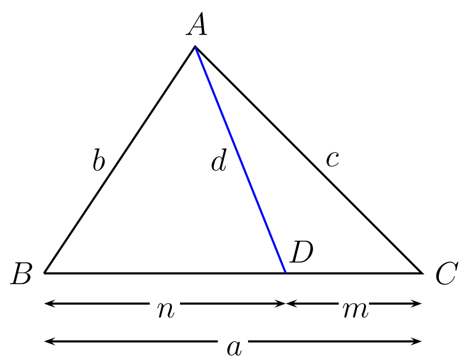


2. Let $ABCD$ be a non-convex quadrilateral in which $\angle CDA > 180°$. Let P, Q, R and S be the midpoints of the sides AB, BC, CD and DA respectively. Prove that $\text{Area}(PQRS) = \frac{1}{2}\,\text{Area}(ABCD)$.

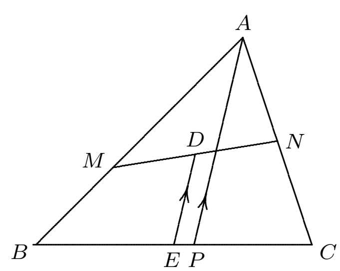
   Hint. The case of a convex quadrilateral is done on p23 of last year's lecture notes. You can modify this proof to get the non-convex case.



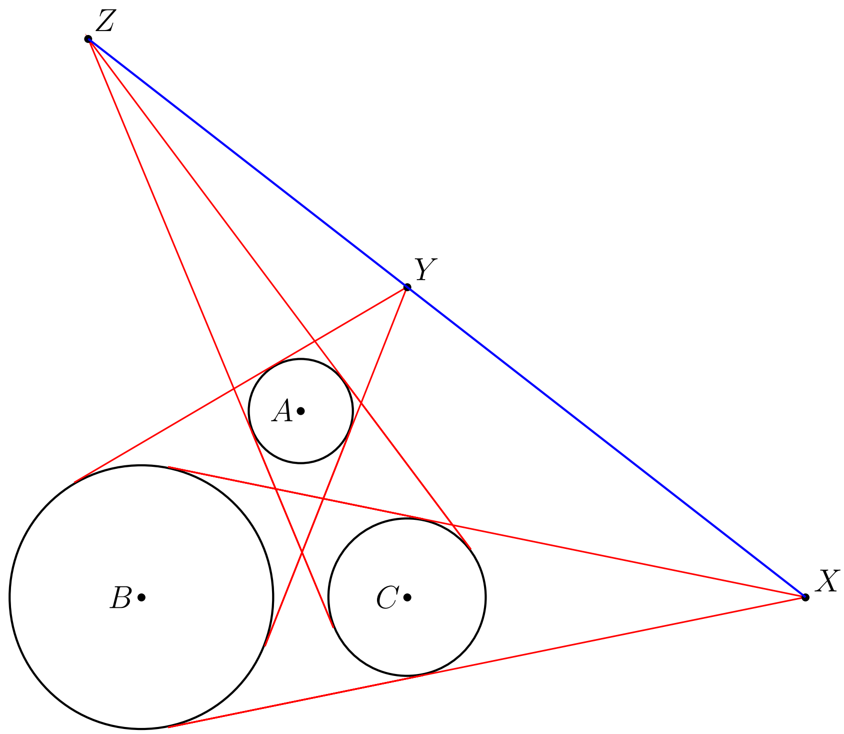
3. Prove *Stewart's theorem*: Let ΔABC be a triangle with $a = |BC|, b = |AC|$ and $c = |AB|$. Given any D on $|AB|$, let $m = |DC|$, $n = |BD|$ and let $d = |AD|$. Then $nc^2 + mb^2 = amn + ad^2$.



4. In $\triangle ABC$, M and N are points on AB and AC respectively such that $BM = CN$. Let D and E be the midpoints of MN and BC respectively. Prove that the line through A parallel to DE bisects $\angle A$.



5. Prove *Monge's theorem*. Consider three non-intersecting circles with radii r_1, r_2 and r_3. There are three pairs of common external tangents as in the diagram below. Let X, Y, Z be the points of intersection of these tangents. Prove that X, Y, Z are collinear.



Hint: Apply Menelaus' theorem to the triangle formed by the centres of the three circles.

6. The following tiling of the floor contains a proof of Pythagoras' theorem. See if you can discover how it works.