MA2219 Introduction to Geometry - Tutorial 1
Week 3: 28 January–1 February

1. Prove Euclid Prop. I.16: in any triangle, any exterior angle is greater than either of the interior and opposite angles.

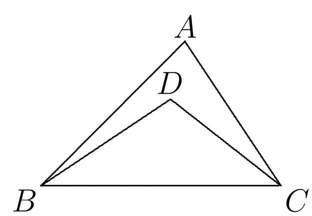
2. Prove Euclid Prop. I.17: in any triangle, the sum of any two angles is less than two right angles.

3. Prove Euclid Prop. I.18: in any triangle, the angle opposite the greater side is greater.

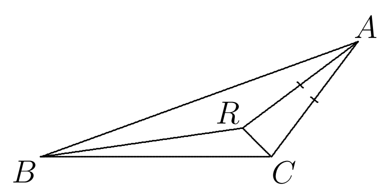
4. Prove Euclid Prop. I.19: in any triangle, the side opposite the greater angle is greater.

5. Prove the *Triangle Inequality*: given any triangle ABC, we have $|AB| + |BC| > |AC|$.

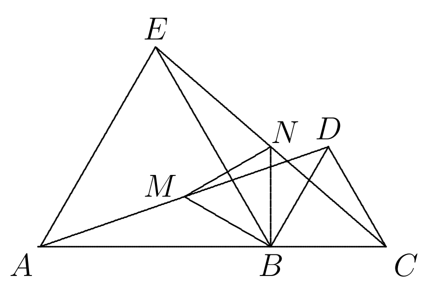
6. Let D be a point inside $\triangle ABC$. Prove that $|AB| + |AC| > |DB| + |DC|$ and $\angle BDC > \angle BAC$. (You can use Euclid Prop. I.16.)



7. Let R be a point inside $\triangle ABC$ such that $|AR| = |AC|$. Prove that $|BC| > |BR|$.



8. Let B be a point on the line segment AC. Let E and D be points on one side of AC such that $\triangle ABE$ and $\triangle BCD$ are equilateral. Let M and N be the midpoints of AD and CE respectively. Prove that $\triangle BMN$ is equilateral.



9. During the Han dynasty of China (206BC-220AD), Zhao Jun Qing gave a proof of Pythagoras' theorem in the form of a diagram in figure 1.

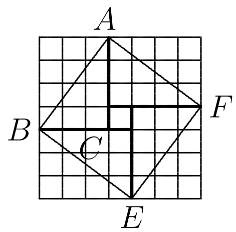


Fig. 1

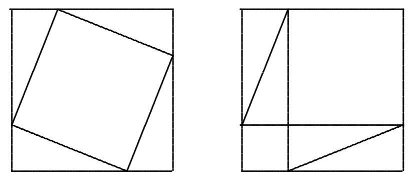


Fig 2

The diagram in figure 1 illustrates that $3^2 + 4^4 = 5^2$ (the point C chosen so that $\angle ABC$ is a right angle). Show how this proof is done.

A similar proof is given by the Indian mathematician Bhaskara-Acharya (1114AD-1185AD). This is illustrated in the 2 diagrams in figure 2. It is in fact the easiest proof of Pythagoras' theorem. Show how this proof is done.