Heat diffusion

Project for the course on Model Reduction (5MB10)

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Introduction

In this project we consider how a temperature distribution on a locally heated surface evolves over time. We find this theme in many technical applications ranging from fused deposit modeling in 3D printing, thermal studies of composite materials, medical applications such as hyperthermia treatments and in optical lithography where thermal loads on silicon wafers and reflective materials are the result of deep or extreme ultraviolet light sources that are applied locally on materials.

To expand on the latter application, optical lithography is a key step in the manufacturing of integrated circuits. The integrated circuit is created by accurately exposing light with a specific intensity, on a specific location and with a specific duration on a wafer so as to project a well defined pattern of circuitry as an image on the light-sensitive photoresist material on the wafer. It is the wavelength of the light source that largely determines the smallest feature size that can be printed.

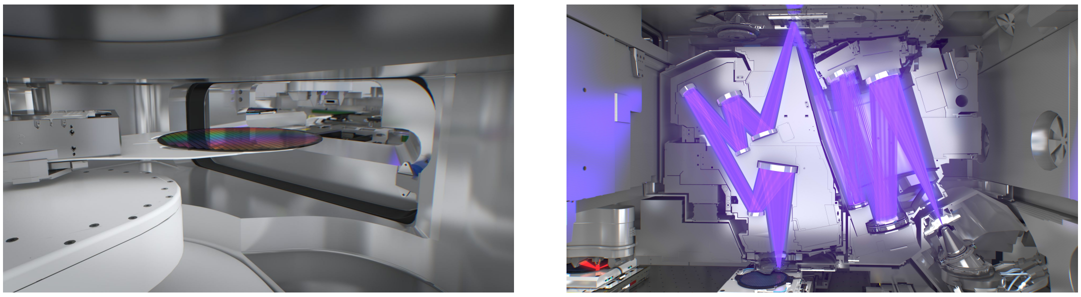

Figure 1: Photolithography processes and wafer heating

Today's modern lithography machines for the mass production of microchips operate with extreme ultraviolet light sources of 13.5 nm wavelength. The performance of these machines is largely determined in terms of overlay and focus errors at the level of the wafer. These errors have various sources, including contamination, wear of materials, heating of components and mechanical stresses. Deformations of the wafer that are thermally induced by the supplied heat of the light source are particularly important and are compensated so as to keep the wafer surface flat and prevent from focus errors. To do this, a thorough understanding of the heat distribution mechanisms of the light source on the wafer is necessary.

The model

In this project we consider a model that describes the temperature evolution on a surface. We consider a rectangular area to reflect a modern wafer of dimension $L_x \times L_y$ and aim to describe the temperature variation at the surface as a function of time. See Figure 2.

The temperature at surface position (x, y) and at time t is given by $T_{\text{amb}} + T(x, y, t)$, where T_{amb} is the ambient temperature which is assumed to be constant. The model features two time-dependent heat fluxes $u_i(t)$, $i = 1, 2$, that are applied at disjoint areas centered at the points (X_i, Y_i) on the surface.

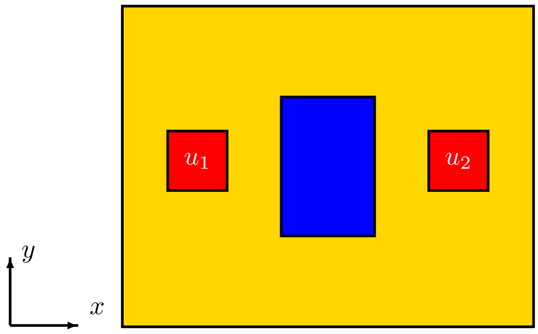

Figure 2: Configuration of surface with heat flux locations

Taking the locations of the sources into account, this means that the total heat flux that is applied at an arbitrary position (x, y) on the surface at time t is given by

$$u(x,y,t) = \begin{cases} u_1(t) & \text{if } |x - X_1| \leq \frac{W}{2}, \quad |y - Y_1| \leq \frac{W}{2} \\ u_2(t) & \text{if } |x - X_2| \leq \frac{W}{2}, \quad |y - Y_2| \leq \frac{W}{2} \\ 0 & \text{elsewhere} \end{cases}. \tag{1}$$

Hence, heat is supplied at two square-shaped areas of width W at midpoint positions (X_i, Y_i). The surface is assumed to be a cross section of possibly different composite materials that are indicated as the yellow and the blue area in Figure 2. Both of these areas are assumed to have a uniform material density ρ [kg/m^3] and a uniform heat capacity c [J/kg K]. The blue area is rectangular, centered in the middle and has dimension $\ell_x \times \ell_y$ with $\ell_x < L_x$ and $\ell_y < L_y$.

The temperature variation on the surface is described by the parabolic partial differential equation

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (K \nabla T) + u \tag{2}$$

where ρ and c are material density and heat capacity. In general, K is a positive definite matrix of dimension 2×2 representing the thermal conductivity, the input u is defined in (1) and ∇ denotes the usual gradient operator.

In Euclidean surface coordinates with $x \in [0, L_x]$ and $y \in [0, L_y]$, the model (2) assumes the form

$$\rho(x,y)c(x,y)\frac{\partial T}{\partial t}(x,y,t) = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} K(x,y) \begin{bmatrix} \frac{\partial T(x,y)}{\partial x} \\ \frac{\partial T(x,y)}{\partial y} \end{bmatrix} + u(x,y,t) \tag{3}$$

where $\rho(x, y)$, $c(x, y)$ are the location dependent density and heat capacity, $K(x, y) \succ 0$ is the location dependent thermal conductivity matrix.

Some terminology:

- The model (3) is called *homogeneous* if ρ, c and K do not depend on (x, y).
- The model is called *isotropic* if $K = \kappa I$ with I the 2×2 identity matrix. In that case, also κ is called the thermal conductivity and has [W/m K] as its unit.

The surface is assumed to be insulated at its boundaries in the sense that

$$\frac{\partial T}{\partial x}(0,y,t) = 0; \qquad \frac{\partial T}{\partial x}(L_x,y,t) = 0 \tag{4}$$

$$\frac{\partial T}{\partial y}(x,0,t) = 0; \qquad \frac{\partial T}{\partial y}(x,L_y,t) = 0 \tag{5}$$

for all $t \geq 0$ and all coordinates $0 \leq x \leq L_x$ and $0 \leq y \leq L_y$.

Purpose

We will be interested in deriving a fast and simple model that accurately describes the temperature evolution of this system. In particular, we will be interested in the heat diffusion process that takes place as a result of individual heat loads u_1 and u_2 applied at the surface.