First Order Linearization using Taylor Series for Nonlinear system

Consider the general nonlinear dynamic control system with n states and m inputs in the matrix form as

$$\dot{x}(t) = f\big(x(t), u(t)\big) \tag{1}$$

where

$$x(t) = [x_1(t) \quad x_2(t) \quad \dots \quad x_n(t)]^T,$$

$$u(t) = [u_1(t) \quad u_2(t) \quad \dots \quad u_m(t)]^{\mathrm{T}},$$

and $f(x, u)$ are respectively, the system state-space vector, the input vector, and the vector function. The equilibrium[1] states are defined by x_{ss} and the input for holding the equilibrium states is defined by u_{ss}.

Then, the actual system dynamics in the immediate proximity of the system trajectories can be approximated by the first terms of the Taylor series. Later, the linear system is defined as in (2)

$$f\big(x(t), u(t)\big) \approx f(x_{ss}, u_{ss}) + \left.\frac{\partial f\big(x(t), u(t)\big)}{\partial x}\right|_{\substack{x_{ss}\\u_{ss}}} (x(t) - x_{ss}) + \left.\frac{\partial f\big(x(t), u(t)\big)}{\partial u}\right|_{\substack{x_{ss}\\u_{ss}}} (u(t) - u_{ss}) \tag{2}$$

where

$$\left.\frac{\partial f\big(x(t), u(t)\big)}{\partial x}\right|_{\substack{x_{ss}\\u_{ss}}} = \boldsymbol{A} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix}\right|_{\substack{x_{ss}\\u_{ss}}}, \tag{3}$$

$$\left.\frac{\partial f\big(x(t), u(t)\big)}{\partial u}\right|_{\substack{x_{ss}\\u_{ss}}} = \boldsymbol{B} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial u_1} \\ \frac{\partial f_2}{\partial u_1} \\ \frac{\partial f_3}{\partial u_1} \end{bmatrix}\right|_{\substack{x_{ss}\\u_{ss}}}. \tag{4}$$

[1] For this project, the equilibrium states are the desired final states.