Model Predictive Control (5LMB0)

Graded Project

Assignment 2

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Organization

- The **assignment 1 of the Graded project** work must be performed in groups of at most two students, who will submit a common report. If you are alone, try to find a partner during lectures. If you cannot find a partner, you can work together in a group of 3, as an exception, but you must submit 2 reports, one for 2 students and 1 for the additional student. The reports should be written independently, i.e., a copy of the same report for different groups of 1 or 2 students is not accepted. All students should be able to answer questions about all the parts of the assignments and reports.

- A report of **maximum 2 pages** showing the results of the project must be submitted in **PDF format** (see the report template). Appendices can be added to include additional plots and software functions.

- **The final report should be submitted accompanied by the source code in a .7z archive by e-mail to v.d.reyes.dreke@tue.nl and m.lazar@tue.nl before March 24, 23:59. The e-mail subject must begin with "Assignment 2 MPC 2022". Both source code files and e-mail subject should contain the surname (without initials) of the students that worked on the report, like MPC_Project_2022_Smith_Ploeg.7z.**

- The assessment will be made based on the quality (accuracy and critical nature of observations) of the report, correctness of the MPC design methods and of the MATLAB code, and creativity in terms of solving the assignments of the project.

- Questions regarding the assignment must be posted on the CANVAS forum or asked during the Seminars. Additional contact hours after the last lecture can be arranged upon request.

Inverted Pendulum System

Figure 1 depicts a diagram of the electrically driven inverted pendulum. This project aims to control the states of the inverted pendulum to an equilibrium point of interest. To control the states, a dc voltage (u) is used as the input of the motor that moves the mass (m). For modelling this system, the angle (q), the angular velocity ($\dot{q}$), and the current motor (i) are considered first, second and third state variables, respectively.

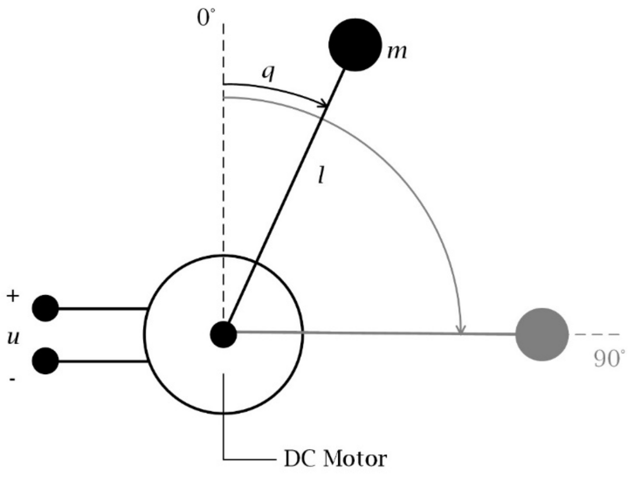

Figure 1 The electrically driven inverted pendulum.

Equations (1) and (2) describe the behavior of the system. In Eq (1), using Newton's second law, the behavior of the forces acting upon the mass is described, as follows

$$J\frac{d\dot{q}(t)}{dt} = mgl \cdot \sin\big(q(t)\big) - b \cdot \dot{q}(t) + k \cdot i(t). \tag{1}$$

In Eq (2), using Kirchoff's law, the voltage of the circuit elements is calculated, as follows

$$L\frac{di(t)}{dt} = -k \cdot \dot{q}(t) - R \cdot i(t) + u(t). \tag{2}$$

The inverted pendulum is subject to the following constraints:

$$\mathbb{X} = \{x \in \mathbb{R}^3 | |x_1| \leq 2\pi, |x_2| \leq 12, |x_3| \leq 8\}, \tag{3}$$

$$\mathbb{U} = \{u \in \mathbb{R} | \, |u| \leq 10\}. \tag{4}$$

Table 1 presents the system parameters for this project.

Table 1 System parameters.

Description	Symbol	Value	Unit
Electrical resistance	R	1.0	Ω
Electrical inductance	L	$1.0\ 10^{-3}$	H
Motor constant	k	$6.0\ 10^{-2}$	NA^{-1}
Friction coefficient	b	$1.0\ 10^{-3}$	Nsm^{-1}
Pendulum mass	m	$7.0\ 10^{-2}$	kg
Pendulum length	l	$1.0\ 10^{-1}$	m
Pendulum inertia	$J = ml^2$	$7.0\ 10^{-4}$	kgm^2
Standard gravity	g	9.81	ms^{-2}

Simulink Model Description

The following Simulink model presented in Figure. 2, called "Pendulum_Nonlinear_System.slx", will be used in some of the assignments and it will be provided to the students. In addition to this Simulink file, we provide a MATLAB's script, called "MPC_Project_Pendulum_2022.m", which contains all the parameters needed to run the assignments. The first, second and third initial state are represented with the variables in the MATLAB's script "**x1_0**", "**x2_0**" and "**x3_0**", respectively. Furthermore, these variables are used as the initial conditions in the integrator blocks called "**Angle**", "**Velocity**" and "**Current**", respectively. The simulation time of this model is defined by the formula $T\_sim * T\_sample$, where "**T_sim**" is the number of time-steps of the simulation and "**T_sample**" is the sample time.

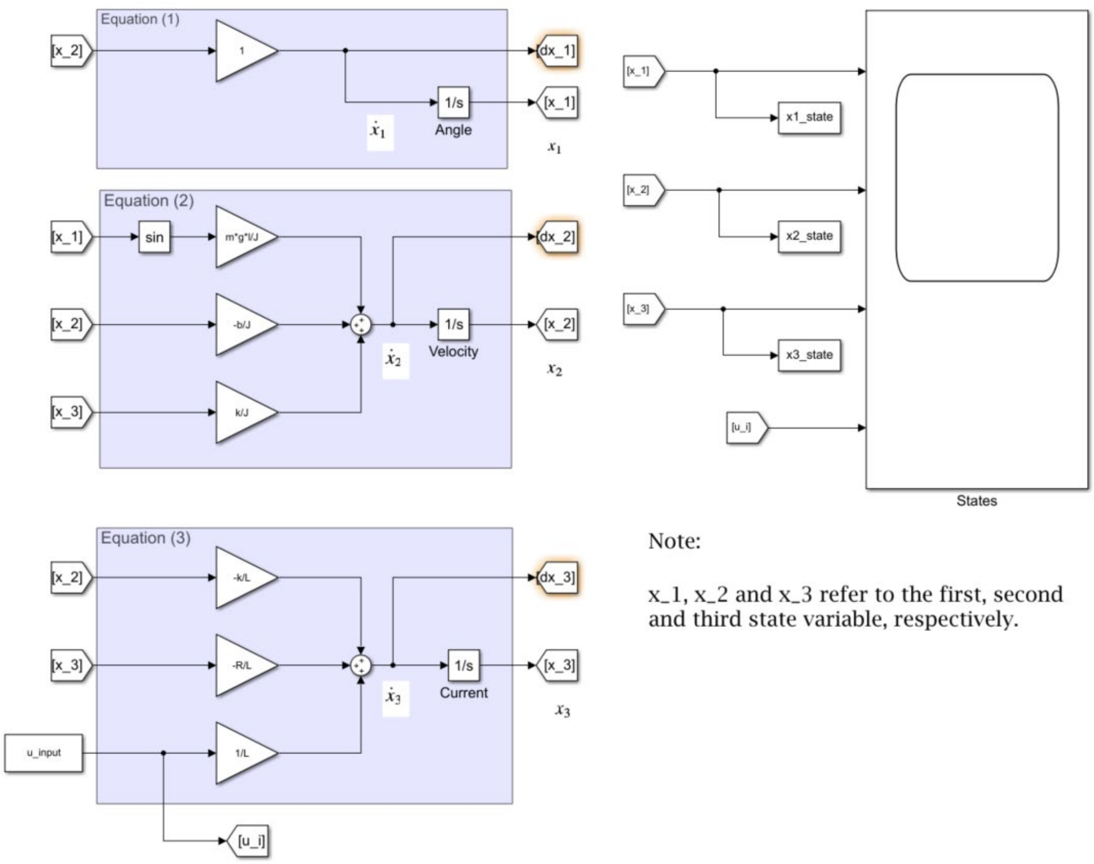

Figure 2 Simulink Model of the Pendulum System

Graded Assignment 2

For grading, there are **10 points for the assignment** in total. Each task of the different parts of the assignment is graded depending on its complexity. The value of each task is highlighted with a blue font. The final grade of the homework project is obtained as the average of the individual assignments. Every task is graded based on the correctness of the software and the critical discussion of the results in the report.

The necessary details expected for each assignment in the report are described in Appendix A.

Part 1. Design of Linear MPC with stability and recursive feasibilities guarantees.

a) Stabilizing MPC based on LQR local feedback (6 points):

Using the discrete-time linearized model of the system from (1) and (2) obtained in the Graded Assignment 1, design linear MPC with a control law equal to the LQR for any N when constraints are not active that guarantees stability and recursive feasibility given the following conditions:

$$N = 10, \qquad Q = \begin{bmatrix} 120 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \qquad R = 10$$

where N is the prediction horizon, Q is the cost matrix for states, R is the cost for the input.

Recall that the discrete-time linear model, i.e.,

$$x(k+1) = \mathbf{A}x(k) + \mathbf{B}u(k).$$

is obtained by linearizing (1) and (2) at the equilibrium state $\boldsymbol{x_{ss}} = [\boldsymbol{\pi/4} \quad \mathbf{0} \quad \mathbf{-0.8093}\,]^{\boldsymbol{T}}$ and the corresponding equilibrium input $u_{ss} = \mathbf{-0.8093\ V}$ and then, discretizing the resulting linear model using the MATLAB function "c2d" and a sample time (**Ts**) of 4 ms.

Given initial states $\boldsymbol{x(0)} = [\mathbf{2.5\pi/2} \quad \mathbf{10} \quad \mathbf{-1.549}]^{\boldsymbol{T}}$, simulate the linear MPC controller in closed-loop with the **linear discrete-time system**. Using the same initial conditions, simulate the linear MPC controller in closed-loop with the **Simulink model**.

The system should stabilize in $\boldsymbol{x_1^{ss} = \pi/4\,rad}$, $\boldsymbol{x_2^{ss} = 0\,rad/s}$, $\boldsymbol{x_3^{ss} = -0.8093}$ A and $\boldsymbol{u_{ss} = -0.8093}$ V.

b) Stabilizing MPC using LMI and the Lyapunov inequalities (4 points):

Given the matrices Q and R from 1a), the initial states $\boldsymbol{x(0)} = [\mathbf{1.1\pi/4} \quad \mathbf{6} \quad \mathbf{-1}]^{\boldsymbol{T}}$, and knowing that the prediction horizon of the MPC is limited to $N = 10$, calculate the terminal cost P and the gain K for a stabilizing control law using the LMI approach such the MPC is stable and recursive feasible.

Simulate the resulting linear MPC controller in closed-loop with the **linear discrete-time system**. Using the same initial conditions, simulate the linear MPC controller in closed-loop with the **Simulink model**.

Plot the 2D projection of the feasible set of states corresponding to the angle (x_1) and the angular velocity (x_2). Compare the resulting plot with the feasible set obtained in Part 1a).

Notice: For avoiding numerical errors during the simulations of the MPC, please formulate the equilibrium point as follows

- $\boldsymbol{x_{ss}} = \left[x_1^{ss} \quad 0 \quad -\frac{(m*g*l*(sin(x_{1_{ss}})))}{k}\right]^T$
- $\boldsymbol{u_{ss}} = [x_3^{ss}]$

APPENDIX A: Report requirements

The report is limited in size, so please be accurate, succinct and critical about what you write and plot. **The first page is reserved for the title: mention your student ID and name. 2 additional pages are allowed for the results of the assignments**. The necessary details expected for each assignment in the report are described next. Please indicate the course material you have used for the crucial steps throughout the assignments or external references from literature, if applicable. Define the axis label, title, caption and legend in each plotted figure. Also, the plotted signals should be visible. **Please, for all the tasks presented, the results obtained in Graded Assignment 1 can be used.**

Part 1a).

- Explain the modification done in the cost function to design an MPC with stability and recursive feasibility guarantees. Please, present the value of **all the weighting matrices** used in the cost function and the MATLAB commands implemented.
- Plot the projection on x_1 and x_2 of the admissible invariant set.
- Plot the projection on x_1 and x_2 of the feasible set of states for the given N prediction horizon.
- Are the initial conditions a feasible set of states?
- Present a figure which contains a plot of the closed-loop trajectories of the states and control inputs using the linear model. Please, simulate the system for more than 800 steps.
- Present a figure which contains a plot of the closed-loop trajectories of the states and control inputs using the nonlinear model.
- Explain the performance difference when the linear MPC is solved with both models.

Part 1b).

- Plot the projection on x_1 and x_2 of the admissible invariant set.
- Plot the projection on x_1 and x_2 of the feasible set of states for the given N prediction horizon.
- Are the initial conditions a feasible set of states?
- Describe the difference between the projection of the feasible set of states using the new P and the projection of 1a).
- Present a figure which contains a plot of the closed-loop trajectories of the states and control inputs using the linear model. Please, simulate the system for more than 800 steps.
- Present a figure which contains a plot of the closed-loop trajectories of the states and control inputs using the nonlinear model.