Model Predictive Control (5LMB0)

Graded Project

Assignment 3

March 28, 2022

Dr. M. Lazar

MSc. V. D. Reyes Dreke

Table of Contents

Organization

- The **assignment 1 of the Graded project** work must be performed in groups of at most two students, who will submit a common report. If you are alone, try to find a partner during lectures. If you cannot find a partner, you can work together in a group of 3, as an exception, but you must submit 2 reports, one for 2 students and 1 for the additional student. The reports should be written independently, i.e., a copy of the same report for different groups of 1 or 2 students is not accepted. All students should be able to answer questions about all the parts of the assignments and reports.

- A report of **maximum 2 pages** showing the results of the project must be submitted in **PDF format** (see the report template). Appendices can be added to include additional plots and software functions.

- **The final report should be submitted accompanied by the source code in a .7z archive by e-mail to v.d.reyes.dreke@tue.nl and m.lazar@tue.nl before April 22, 23:59. The e-mail subject must begin with "Assignment 3 MPC 2022". Both source code files and e-mail subject should contain the surname (without initials) of the students that worked on the report, like MPC_Project_2022_Smith_Ploeg.7z.**

- The assessment will be made based on the quality (accuracy and critical nature of observations) of the report, correctness of the MPC design methods and of the MATLAB code, and creativity in terms of solving the assignments of the project.

- Questions regarding the assignment must be posted on the CANVAS forum or asked during the Seminars. Additional contact hours after the last lecture can be arranged upon request.

Inverted Pendulum System

Figure 1 depicts a diagram of the electrically driven inverted pendulum. This project aims to control the states of the inverted pendulum to an equilibrium point of interest. To control the states, a dc voltage (u) is used as the input of the motor that moves the mass (m). For modelling this system, the angle (q), the angular velocity ($\dot{q}$), and the current motor (i) are considered first, second and third state variables, respectively.

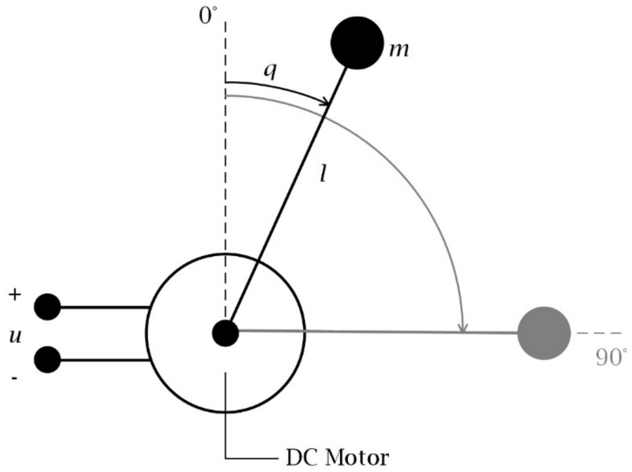

Figure 1 The electrically driven inverted pendulum.

Equations (1) and (2) describe the behavior of the system. In Eq (1), using Newton's second law, the behavior of the forces acting upon the mass is described, as follows

$$J\frac{d\dot{q}(t)}{dt} = mgl \cdot \sin\big(q(t)\big) - b \cdot \dot{q}(t) + k \cdot i(t). \tag{1}$$

In Eq (2), using Kirchoff's law, the voltage of the circuit elements is calculated, as follows

$$L\frac{di(t)}{dt} = -k \cdot \dot{q}(t) - R \cdot i(t) + u(t). \tag{2}$$

The inverted pendulum is subject to the following constraints:

$$\mathbb{X} = \{x \in \mathbb{R}^3 | |x_1| \leq 2\pi, |x_2| \leq 12, |x_3| \leq 8\}, \tag{3}$$

$$\mathbb{U} = \{u \in \mathbb{R} | \, |u| \leq 10\}. \tag{4}$$

Table 1 presents the system parameters for this project.

Table 1 System parameters.

Description	Symbol	Value	Unit
Electrical resistance	R	1.0	Ω
Electrical inductance	L	1.0 10^{-3}	H
Motor constant	k	6.0 10^{-2}	NA^{-1}
Friction coefficient	b	1.0 10^{-3}	Nsm^{-1}
Pendulum mass	m	7.0 10^{-2}	kg
Pendulum length	l	1.0 10^{-1}	m
Pendulum inertia	$J = ml^2$	7.0 10^{-4}	kgm^2
Standard gravity	g	9.81	ms^{-2}

Simulink Model Description

The following Simulink model presented in Figure. 2, called "Pendulum_Nonlinear_System.slx", will be used in some of the assignments and it will be provided to the students. In addition to this Simulink file, we provide a MATLAB's script, called "MPC_Project_Pendulum_2022.m", which contains all the parameters needed to run the assignments. The first, second and third initial state are represented with the variables in the MATLAB's script "**x1_0**", "**x2_0**" and "**x3_0**", respectively. Furthermore, these variables are used as the initial conditions in the integrator blocks called "**Angle**", "**Velocity**" and "**Current**", respectively. The simulation time of this model is defined by the formula $T\_sim * T\_sample$, where "**T_sim**" is the number of time-steps of the simulation and "**T_sample**" is the sample time.

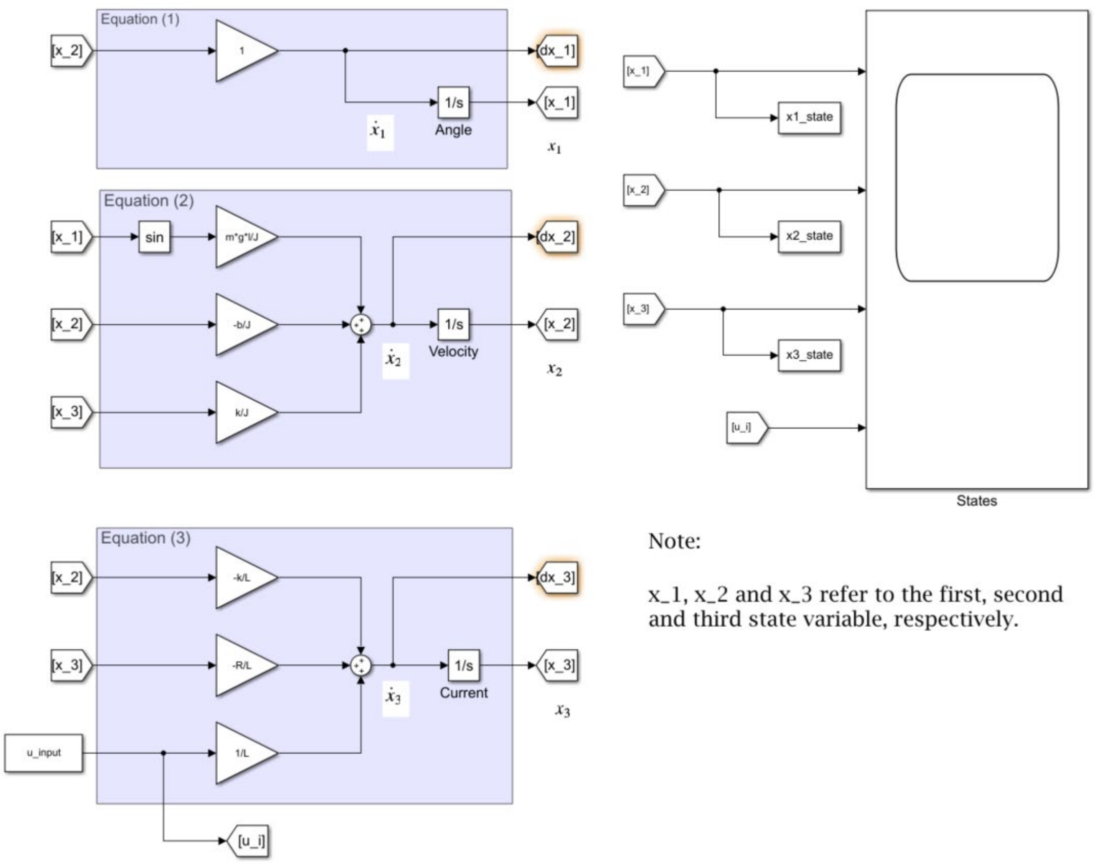

Figure 2 Simulink Model of the Pendulum System

Graded Assignment 3

For grading, there are **10 points for the assignment** in total. Each task of the different parts of the assignment is graded depending on its complexity. The value of each task is highlighted with a blue font. The final grade of the homework project is obtained as the average of the individual assignments. Every task is graded based on the correctness of the software and the critical discussion of the results in the report.

The necessary details expected for each assignment in the report are described in Appendix A.

Part 1. Nonlinear MPC design using an LPV approach

Consider the following discrete-time approximation:

$$x(k+1) = x(k) + T_s \cdot f\big(x(t), u(t)\big) \tag{5}$$

which is the discrete-time approximation state-space representation of equations (1) and (2).

a) Linear Parameter varying Formulation (3 points):

Formulate model (5) in **linear parameter varying** form as described in Lecture 6.

Hint: Please, review the recordings from Seminars 6 and 7 of MPC course in 2021.

b) Designing an MPC using an LPV approach (6 points):

Design an iterative QP nonlinear MPC controller based on LPV model in closed-loop with the nonlinear Simulink model and **compare the simulations results with the linear MPC controller (from Graded Assignment 1) for the same $\boldsymbol{N}, \boldsymbol{P}, \boldsymbol{Q}$ and $\boldsymbol{R}$**. Use $x(0) = [\mathbf{1.1\pi/2} \quad \mathbf{10} \quad \mathbf{-1.549}]^T$ as initial state.

Recall

$$N = 10, \qquad Q = \begin{bmatrix} 120 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \qquad P = Q, \qquad R = 10$$

where N is the prediction horizon, Q is the cost matrix for states, P is the terminal cost R is the cost for the input.

c) Computational Time Comparison (1 points):

Compare the average computation time of the linear MPC and LPV based MPC controllers in closed-loop with the Simulink nonlinear system. In this case, the initial states are $\boldsymbol{x(0)} = [\mathbf{1.1\pi/2} \quad \mathbf{10} \quad \mathbf{-1.549}]^{\mathrm{T}}$

Notice: For avoiding numerical errors during the simulations of the MPC, please formulate the equilibrium point as follows

- $\boldsymbol{x_{ss}} = \left[\boldsymbol{x_1^{ss}} \quad \boldsymbol{0} \quad -\frac{\left(\boldsymbol{m*g*l*(sin(x_{1_{ss}}))}\right)}{\boldsymbol{k}} \right]^{\boldsymbol{T}}$
- $\boldsymbol{u_{ss}} = [\boldsymbol{x_3^{ss}}]$

APPENDIX A: Report requirements

The report is limited in size, so please be accurate, succinct and critical about what you write and plot. **The first page is reserved for the title: mention your student ID and name. 2 additional pages are allowed for the results of the assignments**. The necessary details expected for each assignment in the report are described next. Please indicate the course material you have used for the crucial steps throughout the assignments or external references from literature, if applicable. Define the axis label, title, caption and legend in each plotted figure. Also, the plotted signals should be visible. **Please, for all the tasks presented, the results obtained in Graded Assignment 1 and 2 can be used.**

Part 1a).

- Explain and show the LPV embedding formulation of Equation (5).
- Highlight how you defined the scheduling variable ($\rho(k)$).

Part 1b).

- Explain the design of the Nonlinear MPC controller using a LPV approach
- Present a figure which contains a plot of the closed-loop trajectories of the states and control inputs using the linear and the Simulink models. Please, simulate the system for more than 1000 steps.
- **Bonus questions**: Analyze and compare the behavior of both MPC implementations when **the prediction horizon is equal to 4**. Plot closed-loop trajectories of states and control inputs from the given initial states. Explain the results

Part 1c).

- Explain the time difference between both MPC implementations. Do not present any plot of the MPC trajectories.
- Show the average time from both simulations.