Practice 2 OLS

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1. (a) Write a code for

$$y = \sum_{i=0}^{1000} 0.5^i \qquad for \quad i \quad even$$

$$z = \sum_{i=1}^{1001} 0.5^i \qquad for \quad i \quad odd$$

> **Solution:** Solution will be provided in class.

(b) Write a code for

$$A = [1:100; 2:101; 3:102..........101:200];$$

> **Solution:** Solution will be provided in class.

2. Set $M = 10$ the numbers of Montecarlo simulations and $N = 10$ numbers of random draw. u_i is a random variable distributed as a Normal with mean zero and variance equal to one. y_i is defined as :

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

(a) Estimate M times $\hat{\beta}_0$ and $\hat{\beta}_1$ and their variance.

(b) Plot the distribution of $\hat{\beta}_0$ and $\hat{\beta}_1$ and their variance.

(c) Do the same increasing M and N. Does something change?

(d) Do (a - b-c) for u_i distributed as a χ^2 with 2 degrees of freedom. What does it change?

> **Solution:** Here you find the Matlab code.

```
clear;
```

```
clc;
%% Second Practice
s=1; % If loop s=1 chi-squared - s=2 Normal
N=10; % Number of sample draw
M=10; % Number of time we do the loop- Number of MonteCarlo
k=1; % Number of beta we want to estimate excluding constant

%% Initialize the Vector and Matrices
   beta=NaN(2,M);
   Sigma=NaN(2,2);
   Sigma_new=NaN(2,M);



%% IF:
% We do not want to loose time changing everywhere the name of
% distribution.
% Hence we say if k==1 then the distribution of u will be a chi-squared
% with 2 degree of freedom if instead k is something different then the
% distribution of u will be a normal with mean zero and variance one.

if s==1;
    distribution='chisq';
    Parameters = [2];

else
    distribution='norm';
    Parameters = [0,1];
end

%% Beta
% We decide which are going to be the exact value of beta. The first is the
% constant while the second is the one that multiply x-vector.
   Real_Beta= [1 ; 7];
   %% Loop
for m=1:M
    %% Matrix of x:
```

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   % We create a matrix of x where in the first column there will be all
   % ones while in the second column there will be data distributed as a
   % Normal with mean  5 and wariance equal to 1.

 x=[ones(N,1) normrnd(5,1,N,1)]; % X-Data-Vector
 %% u-vector:
 % U vector should be a vector N*1 with zero mean.
 % We want that E[u|x]=0 then E[u]=0 too. Indeed we can apply Law of
 % Iterated Mean to prove that E_x[E[u|x]]=E[u].
   % We know that if E[u|x]=0 -> E[u]=0 that is not equivalent to say if
   % E[u]=0 ->E[u|x]=0. But we know that if E[u]different from 0
   % ->E[u|x] different from zero too.
 u= randraw(distribution, Parameters, [N 1]);
 if s==1;
   u=(u-2)/(2);
 else
    u=u;
 end



%% Y
% We build y as y=x * beta+ u where u is orthogonal to x (independent).

y = x* Real_Beta+ u;



% We estimate beta as :
% \hat{Beta}= (x'x)^{-1}*x'y
beta(:,m)=(transpose(x)*x)\transpose(x)*y;

%% Estimation of u_hat
u_hat= y - x *beta(:,m);

%% Variance
%% Estimation of u_hat variance
variance=(u_hat'*u_hat)/(N-k-1);
```

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% Under Homoschedasticity we estimate the variance matrix of variance-covariance as:
% VAR(\hat_{\beta})=( \frac{1}{N-k-1}\sum_{i=0}^{N} \hat{u}_i^2) * (x'x)^{-1}

Sigma(:,:)=variance*inv(transpose(x)*x);

% We store the information in a vector :
% First element is the variance of the constant, Second and Third
% covariance, Fourth is the variance of \beta_1
Sigma_new(:,m)= diag(Sigma);

end

%% Plot the figure.

figure(1)
histfit(beta(1,:));
xlabel( '\beta_0' );
ylabel( 'frequency' );


figure(2)
histfit(beta(2,:));
xlabel( '\beta_1' );
ylabel( 'frequency' );



figure(3)
histfit(Sigma_new(1,:));
xlabel( '\sigma_0' );
ylabel( 'frequency' );


figure(4)
histfit(Sigma_new(2,:));
xlabel( '\sigma_1' );
```

```
ylabel( 'frequency' );
```

1 Problem Set 2

3. Set M, number of Montecarlo simulation equal to 100 and N number of sample draw equal to 10. u_i is distributed as a Normal with mean equal to zero and variance equal to 1. $x_{1,i}$ is distributed as a Normal with mean equal to 6 and variance equal to one. We set y_i as:

$$y_i = \beta_0 + \beta_1 x_{1,i} + u_i$$

where you decide the value of β_0 and β_1.

(a) Estimate β_0 and β_1 M times.(10 points)

(b) Estimate the variance of β_0 and β_1 M times. (10 points)

(c) Standardize β_1 using the true mean and the sample variance for the all M simulations. Count the number of times that the absolute value of the standardize value of β_1 is less than 1.96 and 1.64 and divide this value by M.(30 points)

(d) Do step (a)-(c) for $N = 100$ and $M = 1000$. (5 points)

(e) Do step (a)-(c) for u_i distributed as a χ^2 with 5 degree of freedom, $N = 10$ and $M = 100$. (Remember that you have to standardize the variable). (10 points)

(f) Do step (a)-(c) for u_i distributed as a χ^2 with 5 degree of freedom, $N = 100$ and $M = 1000$. (Remember that you have to standardize the variable).(5 points)

(g) What are you computing at point (c)?(15 points)

(h) Explain the different results you have. What happens when you increase either N or M? What is the difference between using a χ^2 and a Normal? (15 points)