The Econometrics of DSGE Models

Giuseppe Ragusa
Luiss University

EIEF
Lecture 5: Estimate state space models

April 18, 2013

Estimation of DSGE Model

Likelihood approach

Recall the steps to obtaining a *state space* representation a DSGE model

1. Obtain first order conditions of the model
2. (log) linearize the system of equation, to obtain

$$\Gamma_0(\theta)E[x_{t+1}] = \Gamma_1 x_t(\theta) + C + \Psi(\theta)z_t$$

3. Solve the linear rational expectation system, to obtain

$$x_{t+1} = G_0(\theta) + G(\theta)x_t + M(\theta)\varepsilon_{t+1}$$

4. Measurement equation (linking data to model variables)

$$\underbrace{\underbrace{y_t}_{\text{observables}} = H(\theta)x_t \underbrace{(+m(\theta)\eta_t)}_{\text{meas. error}}}_{\text{observation equation}}$$

Estimation of DSGE Model

Likelihood approach

Recall the steps to obtaining a *state space* representation a DSGE model

1. Obtain first order conditions of the model
2. (log) linearize the system of equation, to obtain

$$\Gamma_0(\theta)E[x_{t+1}] = \Gamma_1 x_t(\theta) + C + \Psi(\theta)z_t$$

3. Solve the linear rational expectation system, to obtain

$$x_{t+1} = G_0(\theta) + G(\theta)x_t + M(\theta)\varepsilon_{t+1}$$

4. Measurement equation (linking data to model variables)

$$\underbrace{\underbrace{y_t}_{\text{observables}} = H(\theta)x_t \underbrace{(+m(\theta)\eta_t)}_{\text{meas. error}}}_{\text{observation equation}}$$

Estimation of DSGE Model

Likelihood approach

Recall the steps to obtaining a *state space* representation a DSGE model

1. Obtain first order conditions of the model
2. (log) linearize the system of equation, to obtain

$$\Gamma_0(\theta)E[x_{t+1}] = \Gamma_1 x_t(\theta) + C + \Psi(\theta)z_t$$

3. Solve the linear rational expectation system, to obtain

$$x_{t+1} = G_0(\theta) + G(\theta)x_t + M(\theta)\varepsilon_{t+1}$$

4. Measurement equation (linking data to model variables)

$$\underbrace{\underbrace{y_t}_{\text{observables}} = H(\theta)x_t \underbrace{(+m(\theta)\eta_t)}_{\text{meas. error}}}_{\text{observation equation}}$$

Estimation of DSGE Model

Likelihood approach

Recall the steps to obtaining a *state space* representation a DSGE model

1. Obtain first order conditions of the model
2. (log) linearize the system of equation, to obtain

$$\Gamma_0(\theta)E[x_{t+1}] = \Gamma_1 x_t(\theta) + C + \Psi(\theta)z_t$$

3. Solve the linear rational expectation system, to obtain

$$x_{t+1} = G_0(\theta) + G(\theta)x_t + M(\theta)\varepsilon_{t+1}$$

4. Measurement equation (linking data to model variables)

$$\underbrace{\underbrace{y_t}_{\text{observables}} \quad = H(\theta)x_t \underbrace{(+m(\theta)\eta_t)}_{\text{meas. error}}}_{\text{observation equation}}$$

State-space model

Consider a simplified version of the SS:

$$x_{t+1} = g_0 + g_1 x_t + m_1 \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$
$$y_t = h_0 + h_1 x_t + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2)$$

The objective is to estimate

$$\theta = \{g_0, g_1, m_1, \sigma_\varepsilon^2, h_0, h_1, \sigma_\eta^2\}$$

from a realization of the observable $\{y\}_{t=1}^{T}$. This can be done in 3 steps:

1. Apply the Kalman's filter
2. Calculate the likelihood functions
3. Apply a MCMC algorithm

Data

Data generation process

The data are generated with

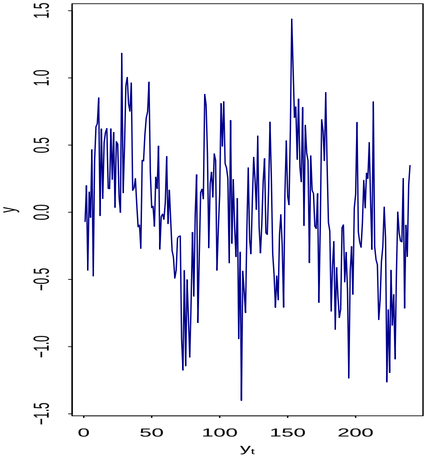
$$
\begin{aligned}
g_0 &= 0 \\
g_1 &= 1 \\
m_1 &= 1 \\
\sigma_\varepsilon &= 0.2 \\
h_0 &= 0 \\
h_1 &= 1 \\
\sigma_\eta &= 0.3
\end{aligned}
$$

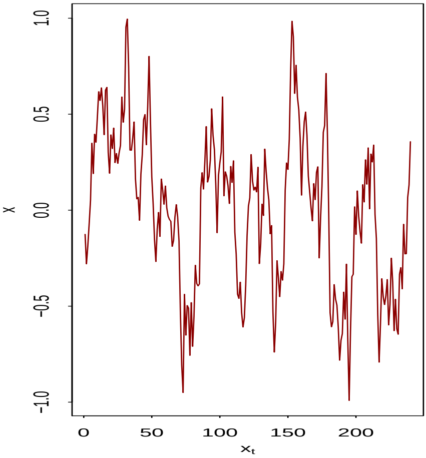
and

$$y_0 = 0, \quad x_0 = 0$$

Data

Plot generated data





Kalman's filter

- We start by applying the Kalman filter for a given value of the parameter vector
- We start the algorithm at the lung run values

$$x_0 = 0, \quad \sigma_0^2 = \frac{1}{1-\rho^2}\sigma_\varepsilon^2$$

- The output of the Kalman's filter is

$$\{x_{t|t}\}_{t=1}^T, \quad \{x_{t+1|t}\}_{t=1}^T, \quad \log L(\theta)$$

```
function [loglik, filt, Ptt, pred, Pt] = kf_matlab(x0, s0, g0, g1↩
    , m1, sigmae, h0, h1,  sigmaeta, y)
    T = size(y,1);
    % x_{t|t}
    filt  = zeros((T+1),1);
    Ptt   = zeros((T+1),0);
    Ptt(1,1)= s0;
    pred = zeros((T+1),1);
    Pt = zeros((T+1),0);

    for j=1:T
        % Predictions step
        pred(j,1) = g0+g1*filt(j,1);
        Pt(j,1) = g1*Ptt(j,1)*g1+m1^2*sigmae^2;
        % Updating step
        K = Pt(j,1)*h1*(h1^2*Pt(j,1)+sigmaeta^2)^(-1);
        filt(j+1,1) = pred(j,1) + K*(y(j,1)-h0-h1*pred(j,1));
        Ptt(j+1,1) = Pt(j,1) - K*(h1*Pt(j,1));
    end;

    mu = h0+h1*pred;
    sd = h1*Pt*h1+sigmaeta^2;
    lik = -T*log(2*pi)/2+sum(-log(sd(1:T))/2-(y-mu(1:T)).^2./(2*↩
        sd(1:T)));
```

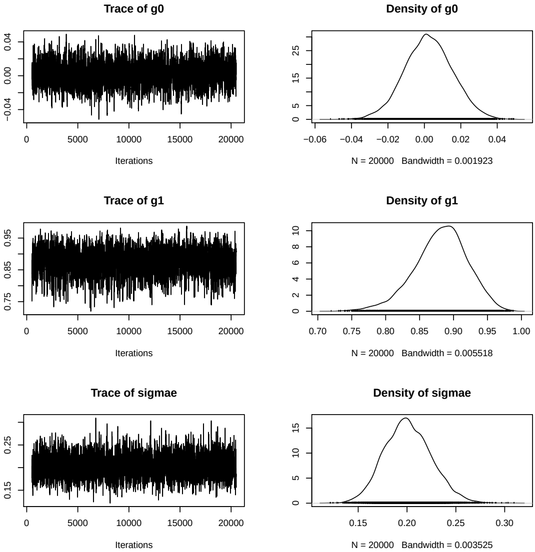
```matlab
function [chain] = kf_mcmc(sim, y)

% fix m1, h0, h1
% to be estimated g0, g1, sigmae,

par0 = [0 log(.8/(1-.8)) log(0.2)];

[theta_s,FVAL,EXITFLAG,OUTPUT,GRAD,HESSIAN] = fminunc(@(x)-post(x, y), par0);
Sigma         = inv(HESSIAN);
gamma_s       = post(theta_s, y);
for s=1:sim
    % Draw candidate
    theta_star = mvnrnd(theta_s', Sigma);
    % Construct gamma(theta*)
    gamma_star = post(theta_star, y);
    % Construct r
    r = min(exp(gamma_star-gamma_s),1);
    % Draw U
    U = rand(1);
    if(U<=r)
        theta_s = theta_star;
    end
    chain(s,:) = theta_s;
end
```

```
function [post] = post(par, y)
    x0 = 0;
    rho = exp(par(2))/(1+exp(par(2)));
    sigmae = exp(par(3));
    s0 = sigmae^2/(1-rho^2);

% likelihood
    lLik = kf_matlab(x0, s0, par(1), rho, 1, sigmae^2, 0, 1, ↩
        0.3, y);

    % prior
    post = lLik + log(normpdf(par(1), 0, 10)) + log(betapdf(rho, ↩
        5, 1.4)) + gampdf(sigmae^2, 1, 3);
```

Maximum Likelihood

```
## $par
## [1]  0.002222  1.910343 -1.597765
##
## $value
## [1] 119.9
##
## $counts
## function gradient
##       24       24
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"
##       g0       g1   sigmae
## 0.002222 0.871058 0.202348
```

MCMC


Trace of g0
0.04
0.00
−0.04
0
5000
10000
15000
20000
Iterations
Density of g0
25
15
5
0
−0.06
−0.04
−0.02
0.00
0.02
0.04
N = 20000 Bandwidth = 0.001923
Trace of g1
0.95
0.85
0.75
0
5000
10000
15000
20000
Iterations
Density of g1
10
8
6
4
2
0
0.70
0.75
0.80
0.85
0.90
0.95
1.00
N = 20000 Bandwidth = 0.005518
Trace of sigmae
0.25
0.15
0
5000
10000
15000
20000
Iterations
Density of sigmae
15
10
5
0
0.15
0.20
0.25
0.30
N = 20000 Bandwidth = 0.003525

Fit