The Econometrics of DSGE models
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Problem Set #1

The file yss.csv contains data generated from the following state space models:

$$x_{t+1} = g_0 + g_1 x_t + m_1 \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$
$$y_t = h_0 + h_1 x_t + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2)$$

where

$$\begin{aligned} g_0 &= 0 \\ g_1 &= 0.9 \\ m_1 &= 1 \\ \sigma_\varepsilon &= 0.2 \\ h_0 &= 0 \\ h_1 &= 1 \\ \sigma_\eta &= 0.3 \end{aligned}$$

and

$$y_0 = 0, \quad x_0 = 0.$$

Let $\theta = \{g_0, g_1, m_1, \sigma_\varepsilon, h_0, h_1, \sigma_\eta\}$. The file kf_matlab.m contains code can be used to apply the Kalman's filter to $\{y_t\}_{t=1}^T$. The code reported in Listing 1. This example code call apply the Kalman's filter when $\theta = \{0, 0.9, 1, 0.2, 0, 1, 0.3\}$. The initial value are set to the unconditional mean and variance of x_t, which for this value of θ correspond to

$$\begin{aligned} a_0 &= E[x_t] = 0 \\ \sigma_0 &= Var[x_t] = \frac{\sigma_\epsilon^2}{1 - g_1^2} = \frac{0.2^2}{1 - 0.9^2} = 0.2105. \end{aligned}$$

```
% Load the yss.csv file
y = csvread('yss.csv', 1);

% Parameters
g0 = 0; g1 = .9; m1 = 1; sigmae = 0.2;
h0 = 0; h1 = 1; sigmaeta = 0.3;
% Initial values
x0 = 0;
s0 = 1/(1-g1^2)*sigmae^2;

% Call to kf_matlab
[loglik, filtered, Ptt, predicted, Pt] = ...
    kf_matlab(x0, s0, g0, g1, m1, sigmae, h0, h1,  sigmaeta, y)

>> loglik
loglik =
 -257.1391
```

Modify the code to estimate all the parameters (except m_1 that is to be kept fixed at $m_1 = 1$) using Maximum Likelihood

```
function [loglik, filt, Ptt, pred, Pt] = ...
             kf_matlab(x0, s0, g0, g1, m1, ...
                       sigmae, h0, h1, sigmaeta, y)
  T = size(y,1);
  % x_{t|t}
  filt  = zeros((T+1),1);
  Ptt   = zeros((T+1),0);
  Ptt(1,1)= s0;
  pred = zeros((T+1),1);
  Pt = zeros((T+1),0);

  for j=1:T
    % Predictions step
    pred(j,1) = g0+g1*filt(j,1);
    Pt(j,1) = g1*Ptt(j,1)*g1+m1^2*sigmae^2;
    % Updating step
    K = Pt(j,1)*h1*(h1^2*Pt(j,1)+sigmaeta^2)^(-1);
    filt(j+1,1) = pred(j,1) + K*(y(j,1)-h0-h1*pred(j,1));
    Ptt(j+1,1) = Pt(j,1) - K*(h1*Pt(j,1));
  end;

  mu = h0+h1*pred;
  sd = h1*Pt*h1+sigmaeta^2;
  loglik = -T*log(2*pi)/2 ...
           + sum(-log(sd(1:T))/2) ...
           -(y-mu(1:T)).^2./(2*sd(1:T)));
```

Code 1: Listings of `kf_matlab.m`.

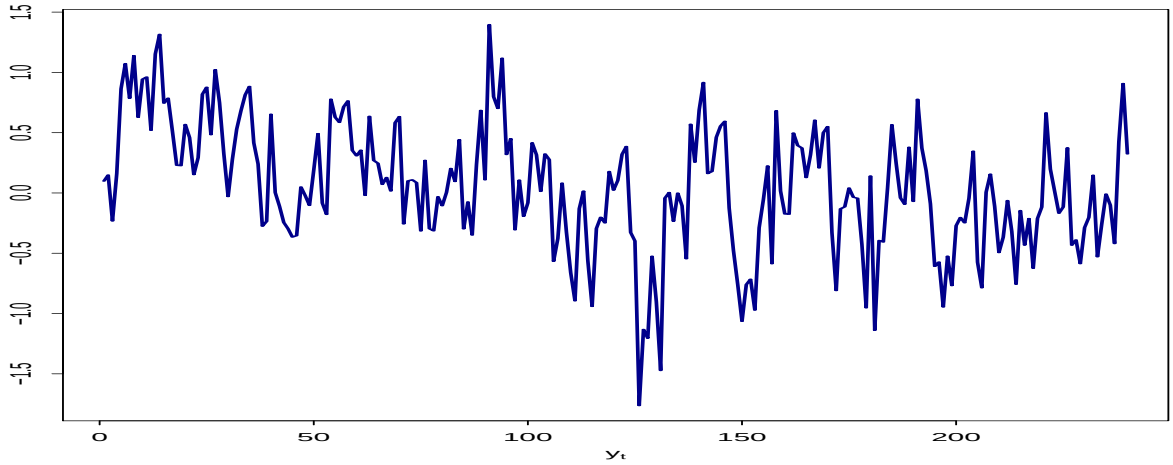

Figure 1: Plot of y variable.