Panel Data
An example

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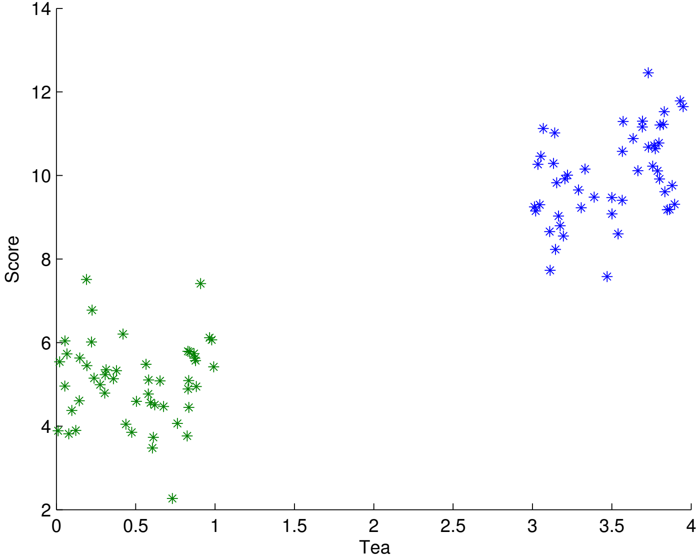
A trivial Example

Immagine to estimate the score on a English test using the tea consumptions:

$$S_i = \beta_0 + \beta_1 tea_i + u_i$$

Score on Tea



Assume a model:

$$y_{i,t} = \alpha_i + x_{i,t}\beta + u_{i,t}$$

You may rewrite the model as:

$$y_i = \begin{bmatrix} y_{i,1} \\ \dots \\ y_{i,T} \end{bmatrix} = \alpha_i \begin{bmatrix} 1 \\ \dots \\ 1 \end{bmatrix} + \begin{bmatrix} x_{i,1} \\ \dots \\ x_{i,T} \end{bmatrix} \beta + \begin{bmatrix} u_{i,1} \\ \dots \\ u_{i,T} \end{bmatrix}$$

$$y_i = \alpha_i \iota + X_i \beta + u_i$$

Again you can work a little bit more and write:

$$y = \begin{bmatrix} y_i \\ \dots \\ y_N \end{bmatrix} = \begin{bmatrix} \alpha_1 \iota \\ \dots \\ \alpha_N \iota \end{bmatrix} + \begin{bmatrix} X_1 \\ \dots \\ X_n \end{bmatrix} \beta + \begin{bmatrix} u_1 \\ \dots \\ u_N \end{bmatrix}$$

$$y = D\alpha + X\beta + u = Z\theta + u$$

where

$$D \equiv \begin{bmatrix} \iota & 0 & \dots & 0 \\ 0 & \iota & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \iota \end{bmatrix}$$

We are interested in estimating β NOT θ.

Partitioned Regression

Assume we have the following linear model:

$$w = X_1\beta_1 + X_2\beta_2 + \epsilon$$

BUT we are interested only in β_2. Then we can use the following condition:

$$\hat{\beta}_1 = (X_1'X_1)^{-1}X_1'X_1\hat{\beta}_1 = (X_1'X_1)^{-1}X_1'(Y - X_2\hat{\beta}_2)$$

Partitioned Regression

We can define the projection matrix:

$$M_{x_1} = I - X_1(X_1'X_1)^{-1}X_1'$$

and rewrite:

$$w^* = X_2^*\hat{\beta}_2$$

where $w^* = M_{x_1} \cdot w$ and $X_2^* = M_{x_1} \cdot X_2$ and find $\hat{\beta}_2$ as

$$\hat{\beta}_2 = (X_2^{*'}X_2^*)^{-1}X_2^{*'}w^*$$

We can compute the β under panel data as:

$$\hat{\beta} = (X' M_d X)^{-1} X' M_d Y$$

where

$$M_d = I - D(D'D)^{-1} D'$$

What is M_d?

$$D'D = T \cdot I_N$$

$$DD' = I_N \otimes \iota' \iota$$

where

$$A \otimes B = \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ \dots & \dots & \dots \\ a_{m,1}B & \dots & a_{m,n}B \end{bmatrix}$$