Applied Statistics and Econometrics
Lecture 1

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Econometrics

- The use of mathematical and statistical methods:
  - to verify economic theories
  - to fit economic models to real data
  - to forecast future values of economic quantities
- Econometric techniques are also used by: sociologists, political scientists and other social scientists.

Why should you study econometrics?

Policy evaluation

Assist in evaluating effects of policies both before and after implementation

Financial markets

Forecasting, CAPM, APT, etc.

Strategic management

Inventory management and firm performances, analysis of divenstiture, etc.

Marketing

Demand functions for industries, study of consumer behavior, etc.

Macroeconomics

Models and business cycles, models of the monetary policy, growth forecast, etc.

Industrial organization

Price discrimination theories, estimation of market power, etc.

Why studying econometrics?

> *One hurdle is a talent and skills gap. The United States alone, McKinsey projects, will need* **140,000** *to* **190,000** *more people with "deep analytical" skills, typically experts in statistical methods and data-analysis technologies.*
>
> *McKinsey says the nation will also need 1.5 million more data-literate managers [...]. [...] the need for a sweeping change in business to adapt a new way of managing and making decisions that relies more on data analysis.*

Source: McKinsey (2011), *Big data: The next frontier for innovation, competition, and productivity*

Policy evaluation

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- but theory does not tell us its exact value
- yet, from a policy point of view it is fundamental to know the exact magnitude of the elasticity.

Financial market

Testing the validity of various versions of CAPMs attracts a lot attention in empirical finance. Various testing procedures and statistical methods have been proposed and studied.

CAPM

Fama and French (1993) form a three factor model to explain the expected excessive returns of assets. Broadly speaking, the three factors are:

- market index
- value equity of firms
- book-to-market value

Other examples

- What is the quantitative effect of reducing class size on student achievement?
- How does another year of education change earnings?
- Are "better" performing CEO payed more?
- What is the effect on output growth of a 1 percentage point increase in interest rates by the European Central Bank?
- What is the effect on housing prices on the environment?

In this course you will:

- Learn methods for estimating causal effects using observational data
- Learn some tools that can be used for other purposes, for example forecasting using time series data;
- Focus on applications - theory is used only as needed to understand the "why"s of the methods;
- Learn to evaluate the regression analysis of others - this means you will be able to read/understand empirical economics papers in other econ courses;
- Get some hands-on experience with regression analysis in your problem sets.

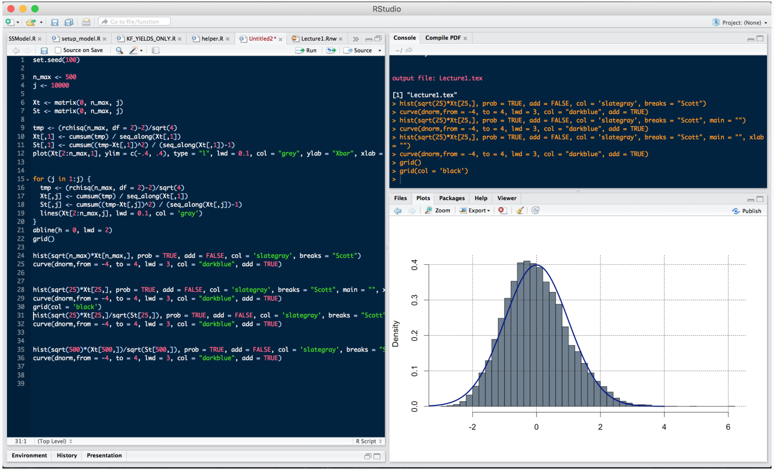
This course is “mostly” about using data to measure causal effects.

- Ideally, we would like an experiment
  - what would be an experiment to estimate the effect of class size on standardized test scores?
- But almost always we only have observational (nonexperimental) data.
  - returns to education
  - cigarette prices
  - monetary policy
- Most of the course deals with difficulties arising from using observational to estimate causal effects
  - confounding effects (omitted factors)
  - simultaneous causality
  - **correlation does not imply causation**

All you need is "data" ...

Figure 1: The cover of The Economist, November 2010

... and R



RStudio
set.seed(100)
n_max <- 500
j <- 10000
Xt <- matrix(0, n_max, j)
St <- matrix(0, n_max, j)
tmp <- (rchisq(n_max, df = 2)-2)/sqrt(4)
Xt[,1] <- cumsum(tmp) / seq_along(Xt[,1])
St[,1] <- cumsum((tmp-Xt[,1])^2) / (seq_along(Xt[,1])-1)
for (j in 1:j) {
lines(Xt[2:n_max,j], lwd = 0.1, col = 'gray')
abline(h = 0, lwd = 2)
grid()
hist(sqrt(n_max)*Xt[n_max,], prob = TRUE, add = FALSE, col = 'slategray', breaks = "Scott")
curve(dnorm,from = -4, to = 4, lwd = 3, col = "darkblue", add = TRUE)
output file: Lecture1.tex
[1] "Lecture1.tex"
Console
Files
Plots
Packages
Help
Viewer
Density

R as a statistical environment

We will use `R` as a statistical language.

- `http://r-project.org`, the web site of the `R` project
- `http://rstudio.org`, the web site of the Rstudio IDE
- `http://github.com/gragusa/ase`, our own package

Types of data: Time Series, Cross Section, Panel Data

Time series

data for a single entity (person, firm, country) collected at multiple time periods

Cross section

data on different entity entity (workers, consumers, firms, etc.) collected at a single time period

Panel data

data for multiple entities in which each entry is observed at two or more time periods.

Time series

```
The downloaded binary packages are in
        /var/folders/76/ms_fns3n1bn6r8hwrtj1zv0c0000gn/T//RtmpPkFCN3/downloaded_pa

             SP500
2015-11-30 2080.41
2015-12-31 2043.94
2016-01-29 1940.24
2016-02-29 1932.23
2016-03-31 2059.74
2016-04-29 2065.30
2016-05-31 2096.96
2016-06-30 2098.86
2016-07-29 2173.60
2016-08-31 2170.95
2016-09-30 2168.27
2016-10-31 2126.15
2016-11-30 2198.81
```

Cross section

```
salary    pcsalary  sales     roe       pcroe     ros       indus     finance
consprod  utility   lsalary   lsales

Obs:   208
  1. salary                   1990 salary, thousands $
  2. pcsalary                 % change salary, 89-90
  3. sales                    1990 firm sales, millions $
  4. roe                      return on equity, 88-90 avg
  5. pcroe                    % change roe, 88-90
  6. ros                      return on firm's stock, 88-90
  7. indus                    =1 if industrial firm
  8. finance                  =1 if financial firm
  9. consprod                 =1 if consumer product firm
 10. utility                  =1 if transport. or utilties
```

Dataset in R

```
> data(ceo); head(ceo, 4)

  salary pcsalary     sales       roe     pcroe ros indus finance consprod
1   1001       32  9958.000 10.900010 -30.60002  13     1       0        0
2   1122        9  6125.905 23.500000 -16.30001  14     1       0        0
3    578       -9 16246.000  5.900005 -25.70002 -21     1       0        0
4   1368        7 21783.220 13.800010  -3.00000  56     1       0        0
  utility
1       0
2       0
3       0
4       0
```

Cross section

```
dataset: CASchools
district school county      grades   students teachers calworks
lunch  computer expenditure income   english  read math testscore
Obs: 420
  1 school:      School name
  2 county:      County name
  3 grades:      Grade span of district
  4 students:    Student enrollment
  5 teachers:    Number of teachers
  6 calworks:    % of qualifying for CalWorks (income assistance)
  7 lunch:       % qualifying for reduced-price lunch
  8 computer:    Number of computers
  9 expenditure: Expenditure per student
 10 income:      District average income (in USD 1,000)
 11 english:     % of English learners
 12 read:        read test score
 13 math:        math test score
 14 testscore:   average of math and read
 15 str:         students/teachers
```

Cross section

```
> data(CASchools)
> head(CASchools, 2)

  district                    school  county grades students teachers calworks   lunch
1    75119        Sunol Glen Unified Alameda  KK-08      195    10.90   0.5102  2.0408
2    61499    Manzanita Elementary     Butte  KK-08      240    11.15  15.4167 47.9167
  computer expenditure income  english  read  math testscore      str
1       67    6384.911 22.690 0.000000 691.6 690.0     690.8 17.88991
2      101    5099.381  9.824 4.583333 660.5 661.9     661.2 21.52466
```

Panel Data

```
dataset: Cigarettes
state      year       cpi        population packs      income
tax        price      taxs

  1. state:      State
  2. year:       Year
  3. cpi:        Consumer price index
  4. population: State population
  5. packs:      Number of packs per capita
  6. income:     State personal income (total, nominal)
  7. tax:        Average state, federal and average local
                 excise taxes for fiscal year
  8. price:      Average price during fiscal year, including
                 sales tax
  9. taxs        Average excise taxes for fiscal year, including
                 sales tax
```

Panel Data

```
  state year   cpi population  packs    income  tax  price  taxs
01   AL 1985 1.076    3973000 116.48   46014968 32.5 102.18 33.34
02   AR 1985 1.076    2327000 128.53   26210736 37.0 101.47 37.00
03   AZ 1985 1.076    3184000 104.52   43956936 31.0 108.57 36.17
[...]
49   AL 1995 1.524    4262731 101.085  83903280 40.5 158.37 41.90
50   AR 1995 1.524    2480121 111.042  45995496 55.5 175.54 63.85
51   AZ 1995 1.524    4306908  71.954  88870496 65.3 198.60 74.79
[...]
94   WI 1995 1.524    5137004  92.466 115959680 62.0 201.38 71.58
95   WV 1995 1.524    1820560 115.568  32611268 41.0 166.51 50.42
96   WY 1995 1.524     478447 112.238  10293195 36.0 158.54 36.00
```

Formal definition of data

The data are

- a sample of size n, denoted

$$\{Y_1, Y_2, \ldots, Y_n\}$$

  - Y_1 is the first observation, Y_2 is the second observation, etc.
  - for cross section the typical observation is the i^{th} observation, denoted Y_i
  - for time series the typical observation is customarily denoted by Y_t
- if we have data on more than one variable, we have a multivariate sample,

$$\{(Y_1, X_1), (Y_2, X_2), \ldots, (Y_n, X_n)\}$$

Data summary

Central tendency

(sample) mean, (sample) median

Dispersion

(sample) variance, (sample) standard deviation

Position

(sample) Percentiles, (sample) deciles, and (sample) quartiles

Central tendency

- The leading measure of central tendency is the sample mean, which is the arithmetic average of the data
- For a sample of size n, the sample mean

$$\bar{Y} = (Y_1 + Y_2 + \ldots + Y_n)/n$$

- Often, this formula is abbreviated using the summation convention

$$\bar{Y} = \sum_{i=1}^{n} Y_i/n$$

Central tendency

- The other leading indicator of central tendency is the (sample) median, which is the value of the sample that divides the data after ordering into two halves, the median bein the midpoint
- The median is relatively easy to calculate:

- Odd Number of Data Values (n is odd)
  1. arrange data in order from smallest to largest
  2. Find the data value in the **exact** middle

- Even Number of Data Values (n is even)
  1. arrange data in order from smallest to largest
  2. Find the mean of the **two** middle numbers

Dispersion

- the sample variance
  - is the average of the deviation of the data from the mean

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

- the sample standard deviation
  - is the square root of the sample variance

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$$

Measure of symmetry (skewness)

The skewness measures the symmetry of the distribution:

$$skew = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{Y_i - \bar{Y}}{s}\right)^3$$

- A **positive skewed** or **right-skewed** data have a much longer tail on the right (skew>0)
- A **negative skewed** or **left-skewed** data have a much longer tail on the right (skew<0)

Measure of peakedness (kurtosis)

The peakedness of the distribution and and fatness of the tails is measured by the kurtosis

$$kurt = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{Y_i - \bar{Y}}{s}\right)^4 - 3$$

- **positive kurtosis** indicates data that is relatively flat
- **negative kurtosis** indicates data that is relatively peaked

Position

The sample quartiles divides the data in 4 parts:

- the lower quartile (Q_1) is that point where one-quarter of the observed sample lies below and three-quarters of the order sample lies above
- the middle quartile (Q_2) is the sample median
- the upper quartile (Q_3) is that point where three-quarters of the ordered sample lies below and one-quarter of the order sample lies above.

Position

Even more detailed divisions of the sample are possible.

- Deciles split the ordered sample into tenth and are used, for example, to summarize the distribution of individual income
- Percentiles split the order sample into hundredths. The p^{th} percentile is the value for which p percent of the observed values are equal to or less than the value

Summarizing data

Table 1: Descriptive Statistics, California Schools Dataset.

Statistic	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
students	2,629.0	3,913.0	379	950.5	3,008
teachers	129.1	187.9	19.7	48.6	146.4
calworks	13.2	11.5	4.4	10.5	19.0
lunch	44.7	27.1	23.3	41.8	66.9
computer	303.4	441.3	46	117.5	375.2
expenditure	5,312.0	633.9	4,906.0	5,214.0	5,601.0
income	15.3	7.2	10.6	13.7	17.6
english	15.8	18.3	1.9	8.8	23.0
read	655.0	20.1	640.4	655.8	668.7
math	653.3	18.8	639.4	652.4	665.8

Summarizing data in R

```
> data(CASchools); head(CASchools[,-c(1:4)], 4)

  students teachers calworks   lunch computer expenditure income   english  read
1      195    10.90   0.5102  2.0408       67    6384.911 22.690  0.000000 691.6
2      240    11.15  15.4167 47.9167      101    5099.381  9.824  4.583333 660.5
3     1550    82.90  55.0323 76.3226      169    5501.955  8.978 30.000002 636.3
4      243    14.00  36.4754 77.0492       85    7101.831  8.978  0.000000 651.9
   math testscore      str
1 690.0     690.8 17.88991
2 661.9     661.2 21.52466
3 650.9     643.6 18.69723
4 643.5     647.7 17.35714
```

Summarizing data in R

```
> summary(CASchools[c("students", "teachers", "math", "read")])

    students          teachers            math            read      
 Min.   :   81.0   Min.   :   4.85   Min.   :605.4   Min.   :604.5  
 1st Qu.:  379.0   1st Qu.:  19.66   1st Qu.:639.4   1st Qu.:640.4  
 Median :  950.5   Median :  48.56   Median :652.5   Median :655.8  
 Mean   : 2628.8   Mean   : 129.07   Mean   :653.3   Mean   :655.0  
 3rd Qu.: 3008.0   3rd Qu.: 146.35   3rd Qu.:665.9   3rd Qu.:668.7  
 Max.   :27176.0   Max.   :1429.00   Max.   :709.5   Max.   :704.0  
```

Summarizing data in R

```
> summary(ceo[c("salary", "sales", "roe", "ros")])

     salary           sales                 roe              ros        
 Min.   :  223   Min.   :  175.2   Min.   : 0.5   Min.   :-58.00  
 1st Qu.:  735   1st Qu.: 2200.3   1st Qu.:12.4   1st Qu.: 20.75  
 Median : 1032   Median : 3693.3   Median :15.5   Median : 52.00  
 Mean   : 1282   Mean   : 6824.4   Mean   :17.2   Mean   : 61.18  
 3rd Qu.: 1408   3rd Qu.: 7017.2   3rd Qu.:20.0   3rd Qu.: 81.00  
 Max.   :14822   Max.   :97649.9   Max.   :56.3   Max.   :418.00  
```

Graphical representation of data

Graphical methods used vary with the type of univariate data

Cross-section

histogram, boxplot, pie-chart

Time series

line chart

Cross section

Cross section

```
List of 57
 $ line                :List of 6
  ..$ colour       : chr "black"
  ..$ size         : num 0.5
  ..$ linetype     : num 1
  ..$ lineend      : chr "butt"
  ..$ arrow        : logi FALSE
  ..$ inherit.blank: logi TRUE
  ..- attr(*, "class")= chr [1:2] "element_line" "element"
 $ rect                :List of 5
  ..$ fill         : chr "#fafafa"
  ..$ colour       : chr "black"
  ..$ size         : num 0.5
  ..$ linetype     : num 1
  ..$ inherit.blank: logi FALSE
  ..- attr(*, "class")= chr [1:2] "element_rect" "element"
```

Box plots

A box plot or boxplot is a convenient way of graphically depicting groups of numerical data through their quartiles.

Box plots have lines extending vertically from the boxes (**whiskers**) indicating variability outside the upper and lower quartiles, hence the terms box-and-whisker plot.

Cross section

Time series

Bivariate data analysis

Bivariate data analysis considers the relationship between two variables, such as education and income, or price and house size, or testscore and str.

Data summery tools:

- scatterplot (graphical)
- covariance and correlation (numerical)

Scatter plot

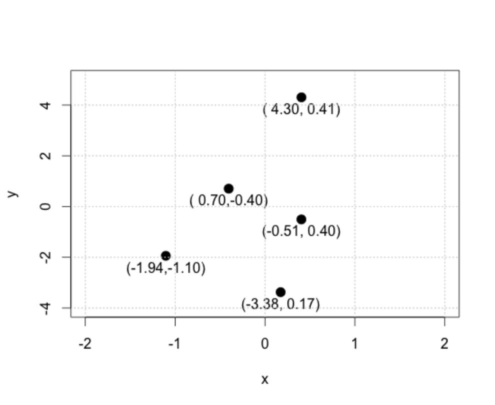
The data is displayed as a collection of points.

Example (Data)

Suppose to have the following bivariate sample:

	Y	X
1	-0.51	0.40
2	0.70	-0.40
3	-3.38	0.17
4	-1.94	-1.10
5	4.30	0.41

Scatterplot



Scatter plot: `testscore` and `str`

Scatter plot: `salary` and `roe`

Scatter plot: `salary` and `roe`

Covariance

A measure of association between two variables, say x and y is the covariance:

$$s_{XY} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})$$

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The covariance measures only linear association between y and x

Covariance between `testscore` and `str`

The covariance between Test Score and str is negative:

```
> with(CASchools, cov(str, testscore))
```

```
[1] -8.159323
```

- the unit of measure of the covariance are **difficult** to interpret

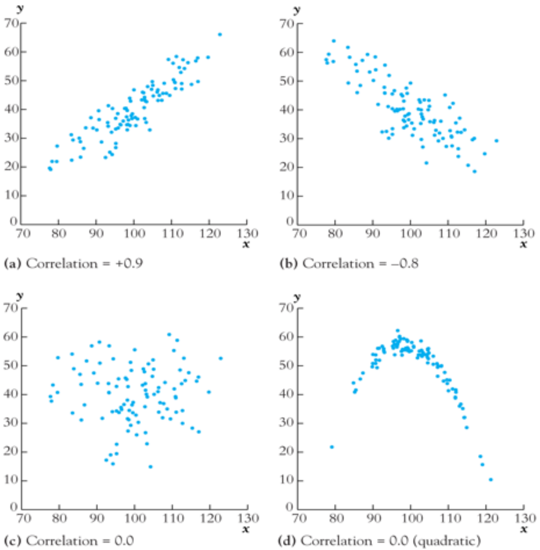
Correlation coefficient

The correlation coefficient is defined in terms of the covariance:

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

- $-1 \leq r_{XY} \leq 1$
- $r_{XY} = 1$ mean perfect **positive** linear association
- $r_{XY} = -1$ means perfect **negative** linear association
- $r_{XY} = 0$ means **no** linear association

The correlation coefficient measures linear association



Correlation `testscore` vs. `str`

How big do you think it is?

```
> with(CASchools, cor(str, testscore))
[1] -0.2263627
```

Correlation coefficient

The correlation coefficient is defined as

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

where s_X and s_Y are the sample standard deviations of X and Y, respectively.

Remarks

- treat X and Y symmetrically: $r_{XY} = r_{YX}$
- while r_{XY} detects (linear) association, it is neutral on whether it is X that is causing Y or Y that is causing X
- it can be shown that r_{XY} measure the number of standard deviations that Y changes by when X changes by one standard deviation