Problem Set 1

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Solutions are due Wednesday, February 22 2017

1. Consider a standard normal distribution, i.e. $N(0,1)$. Evaluate:

   (a) $P(x \leqslant -1.96)$

   (b) $P(x \leqslant -1.64)$

   (c) $P(x \leqslant 0)$

   (d) $P(x \leqslant 1.64)$

   (e) $P(|x| \leqslant 1.96)$

2. Consider two random variables X and Z, with $E[X] = 2$, $E[Z] = 1$, $Var[X] = 1$, $Var[Z] = 1$, $a = 0.5$, $b = 3$, $c = 0.2$, $d = 2$.

   (a) Calculate $E[aX + b]$.

   (b) Calculate $Var[aX + b]$.

   (c) Assuming that X and Z are independent, calculate $Var[X + Z]$.

   (d) Assuming that $Cov(X, Z) = 1$, calculate $Cov(aX + b, cZ + d)$.

   (e) Generalize previous results for any finite $E[X]$, $E[Z]$, $Var[X]$, $Var[Z]$, a, b, c and d.

   (f) Assuming again $Cov(X, Z) = 1$, what can you say about $Corr(X, Z)$?

3. Let X be a standard normal random variable and $Y = X^2$.

   (a) Show that $E[Y|X] = X^2$.

(b) Show that E[Y]=1.

(c) Show that E[XY]=0 (recall that, for a variable $Z \sim N(0,1)$, $E[X^{2n+1} = 0]$ for all $n \in \mathbb{N}$).

(d) Show that $Cov(X,Y) = 0$, thus $Corr(X,Y) = 0$.

As you will find out in this example, Corr(X,Y)=0 does not imply E[Y|X]=0. However the opposite is true: $\mathrm{E}[Y|X] = 0 \Rightarrow Corr(X,Y) = 0$.

4. Let $x_1, x_2, \ldots, x_n$ be a sample of size n from an unknown distribution with expectation equal to μ. Considet the following estimators of μ:

$$\hat{\mu}_1 = \sum_{i=1}^{n} x_i/n, \quad \hat{\mu}_2 = x_1, \quad \hat{\mu}_3 = \frac{x_1}{2} + \frac{1}{2(n-1)} \sum_{i=2}^{n} x_i.$$

(a) Which of those are unbiased?

(b) Which of these are consistent?

(c) Are all unbiased estimators consistent?

5. A stockbroker who wants to compare mean returns of two stocks. He collects data on 90 days of trading. The aggregated data is reported in the table below:

First stock	Second stock
$n_1 = 90$	$n_2 = 90$
$\bar{x}_1 = 0.15$	$\bar{x}_2 = 0.08$
$s_1 = 0.10$	$s = 0.15$

Are there any significant differences in the mean returns?