Mesoscale model including reduced HH formulism

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Single neuron HH formulism

$$C\frac{dV}{dt} = g_L(E_L - V) + g_{Na}m_\infty(E_{Na} - V) + g_K n(E_K - V) + I_{app} + I_{syn}$$

$$\frac{dn}{dt} = \frac{n_\infty - n}{\tau_n} \qquad n_\infty = \theta_V(V_{n,1/2}, k_n)$$

$$\theta_V(V_{1/2}, K) = \frac{1}{1 + \exp\left[\left(V_{1/2} - V\right)/K\right]} \qquad m_\infty = \theta_V(V_{m,1/2}, k_m)$$

$I_{app}(t)$: applied external current

$I_{syn}(t) = I_{AMPA}(t) + I_{GABA}(t)$: synaptic current

Fast and slow membrane potential

Spike detection (for saving and activation of synapses) is accomplished using V_f (200 Hz high-pass filtered V). Additionally, to mimic how excitatory ($AMPA_R$) synapses are usually further away from the soma than inhibitory ($GABA_R$) synapses, the driving force for I_{AMPA} is from a 10 Hz low-pass filtered V.

$$\frac{dV_f}{dt} = \frac{dV}{dt} - \frac{V_f}{\tau_{vf}} \qquad \frac{dV_s}{dt} = \frac{V - V_s}{\tau_{vs}}$$

Applied input

The externally applied input I_app can be applied to a small, localized subset of neurons. The input can be a ramp, which could also be followed by an exponentially decayed or an abrupt fall to 0 current.

Synaptic currents

The synaptic current equations are:

$$I_{AMPA}(t) = g_{AMPA}(t)(E_{AMPA} - V_s(t))$$
$$I_{GABA}(t) = g_{GABA}(t)(E_{GABA} - V(t))$$

The equations for the dynamics of synaptic conductance is the same for both of these, just the constants and synaptic delays that differ ($cs = AMPA, GABA$). These are both alpha synapses (having the shape of $te^{-t/\tau_{cs}}$)

$$\frac{dg_{cs}}{dt} = z_{cs} \qquad \alpha = \frac{1}{\tau_{cs}}$$

$$\frac{dz_{cs}}{dt} = \alpha x + \beta z_{cs} + \gamma g_{cs} \qquad \beta = -2\alpha$$

$$x = g_{max}\sum_k \delta(t - t_k - t^{syn}_{delay}) \qquad \gamma = -\alpha^2$$

Additional notes on synaptic connections

The maximal synaptic conductance can be scaled by either the number of neurons in the network or the square root of such number.

The maximal synaptic conductance between 2 neurons can also depend on the spatial distnce between tbem, proportional to either $\exp(-d_{ij}/k)$ or $\exp(-d_{ij}^2/k)$.