Papers on modelling capacity expansion to meet runaway homeless youth demand and some potential formulations of the PhD problem

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Runaway homeless youths

Here we discuss several papers which model short-term shelters which provide temporary housing and support services for runaway homeless youths (RHY) in the US. It has been noted that, amongst other things, RHY are particularly at risk of involvement with human trafficking if they are not given proper access to housing and other support such as counselling and medical care. Here we focus on aspects of these problems which bear similarity with the PhD problem of housing homeless populations.

Discrete Event Simulation

Kaya et al. (2022b) use a DES model of flow through a shelter which specifically serves LGBTQ+ RHY. Youths arrive in the shelter and seek either both a bed and specific support services, or only support support services. Service types include counselling, medical treatment and case management. Length of stay is modelled with a triangular distribution. Reneging is modelled using "bed patience" and "service patience" attributes for each arrival, also modelled with triangular distributions. A mode of 5 days for bed patience begs the question: where do youths wait when in the queue for a bed? If they do not wait in the shelter (i.e. if they remain unsheltered) there must be some administrative process of keeping them in the queue, presumably involving the shelter keeping their contact information and informing them when a bed is available. If this is indeed the case, then what does reneging mean in this context? Presumably it means that there is a point at which the youth is no longer willing to be admitted to a bed in the shelter, since even though there is no drawback for the youth in staying on the waiting list (the shelter simply keeps their contact details) they may have moved away from the area, found other short-term solutions or for some other reason not be in a position to accept a bed. The authors go on to simulate various capacity expansion options and analyse both the resulting number of queue abandonments and the resulting service capacity utilisation.

Optimisation methods and formulations

Kaya et al. (2022a) solve an integer linear program in order to optimally match demand (for both beds and support services) from heterogeneous RHY (i.e. arrivals have different needs) in Ney York City (NYC) with supply from shelters which have capacity to offer different types of service. Capacity for supply has three different levels:

- available capacity in existing shelters
- hypothetical new capacity in existing shelters (there is an upper bound on this for each shelter)
- overflow capacity in, say, nearby hotels (no upper bound on this capacity)

The use of each type of supply has different costs with available capacity being free or very cheap, hypothetical new capacity being more expensive to reflect the cost of building this new capacity and overflow capacity being even more expensive. Since there is no "unsheltered" population modelled, there is no "societal cost" included in the model and all costs are considered to be financial. The authors solve the problem with a range of randomly generated demand profiles, where length of stay is modelled with two normal distributions, one for the bulk of the demand and one for a small portion of the demand who are assumed to leave early for various positive and negative reasons. The authors' formulation is thus a **cost minimisation** formulation.

Maass et al. (2020) look at a similar problem but where services are not only for RHY but for anyone needing shelter who is involved in human trafficking. They also use an integer linear program, but here in order to optimise the location of new shelters across the US which can be built to accommodate the demand, given a limited budget. For each hypothetical new shelter, they calculate a societal benefit (using concepts from health and social welfare economics) which is comprised of:

- Disability-adjusted life years (DALYs) averted
- Criminal justice costs avoided
- Labour productivity gained

DALYs include years of life which could be lost or could be of poor quality if the shelters did not provide the service. DALYs averted are those lost years (and poor quality years) which are avoided with the existence and running of the shelter and its services. DALYs averted is converted to a financial benefit using a "willingness to pay" metric, which varies from state to state around a baseline of $\$20,000$ per DALY averted. The other two benefits are already in the units of USD. They then combine these societal benefits with financial building and operating costs into a single objective function and include a budget constraint. There are also bounds on the total extra capacity which can be built at each location. Their formulation is thus a **benefit-less-cost maximisation** formulation.

Finally, we discuss a fractional programming approach by Miller et al. (2022) which addresses the same underlying RHY shelter problem as Kaya et al. (2022a) of capacity expansion to meet the heterogeneous demand of RHY in NYC. One difference in their approach is they allow the building of new shelters as well as adding capacity to existing shelters. Similarly to Maass et al. (2020), the authors here estimate the social benefit of extra capacity in financial terms. In order to do this the author's borrow concepts from Bertotti et al. (2015) who conduct a detailed study on the "social return on investment" of a new homeless shelter in London, UK. In this study, a wide range of effects are evaluated in financial terms, from the increased employment rates to the improved mental and physical health of service users. Having estimated both the social benefit and financial costs of building and running new shelter capacity, Miller et al. (2022) optimise an objective function which is a benefit to cost ratio (BCR). They include constraints which ensure sufficient movement away from the status quo and thus optimal solutions represent the most efficient solutions which are considered a sufficient change from the status quo. Their formulation is a **benefit-cost-ratio maximisation** formulation. This type of formulation does not need the assumption (seen in benefit-less-cost maximisation) that social benefit and financial cost can have the same units. They solve the fractional program on a set of randomly generated problem instances using Dinkelbach's algorithm which solves a series of linearised fractional programs. The authors evaluate the solutions given by their formulation in comparison to benefit-less-cost maximal and cost-minimal formulations. They find that their solutions are slightly more costly than the cost-minimal solutions and not as beneficial as the benefit-less-cost maximal solutions, but their solutions are much less costly than the benefit-less-cost maximal solutions and give a much better benefit-to-cost ratio.

Potential formulations for the PhD problem

Taking inspiration from the aforementioned formulations relating to the problem of capacity expansion for services for RHY populations, we now list five possible formulations for the PhD problem of optimising the expected performance of a US county-wide homelessness response system where the decision variables are how much emergency shelter and long-term housing to build over a finite time horizon, where some elements of this "performance" can only be estimated using stochastic simulation. For each possible formulation, we list some advantages and disadvantages.

We first propose that for each solution $\boldsymbol{x}$ there is a deterministic financial cost $C(\boldsymbol{x})$ and for the jth simulation run with solution $\boldsymbol{x}$ one can calculate a corresponding social benefit $b(\boldsymbol{x};\boldsymbol{u}_j)$ which is a realisation of random variable $B(\boldsymbol{x};\boldsymbol{U})$. The form of the function $b(\boldsymbol{x};\boldsymbol{u}_j)$ is not defined here, and may be different (and have different units) in the following formulations - this is for future discussion. The function $b(\boldsymbol{x};\boldsymbol{u}_j)$ could, for example, quantify the proportion of unsheltered people in the system who were given shelter during the time horizon of the simulation run.

1. Benefit maximisation

This formulation maximises the expected social benefit, given a financial budget c:

$$\begin{aligned} \max \quad & \mathbb{E}[B(\boldsymbol{x},\boldsymbol{U})] \\ \text{s.t.} \quad & C(\boldsymbol{x}) < c \\ & \boldsymbol{x} \in \mathbb{X} \end{aligned}$$

Advantages:

- There is no need to express B in financial terms, but any component parts of B must be in the same units as each other.
- There are several SO methods available for this type of problem such as stochastic approximation and meta-modelling (if $\mathbb{X}$ is continuous) or ranking & selection, Gaussian Markov random fields,

sample average approximation or adaptive random search methods (if $\mathbb{X}$ is discrete).

Disadvantages:

- An optimal solution will likely spend the full budget, and potentially dismiss a solution which spends less than the full budget but with greater efficiency.

2. Cost minimisation

This formulation minimises the financial cost, given a stochastic constraint on the social benefit.

$$\begin{aligned} \min \quad & C(\boldsymbol{x}) \\ \text{s.t.} \quad & \Pr\{B(\boldsymbol{x}, \boldsymbol{U}) > b\} > 1 - \alpha \\ & \boldsymbol{x} \in \mathbb{X} \end{aligned}$$

Advantages:

- There is no need to express B in financial terms, but any component parts of B must be in the same units as each other.

Disadvantages:

- An optimal solution will likely only just meet the constraint on social benefit, and thus potentially dismiss a solution which gives a much better social benefit for a cost which is only slightly higher than the minimum possible feasible cost.
- Stochastic constraints are difficult to deal with in simulation optimisation.

3. Benefit-less-cost maximisation

This formulation maximises the expected difference between social benefit and financial cost. The constraint ensures that the financial cost is viable. In this formulation, the social benefit must be evaluated in financial terms. This could include a combination of disability-adjusted life years (DALYs) averted (multiplied by a "willingness to pay" factor), criminal justice savings and healthcare savings, as discussed by Maass et al. (2020).

$$\begin{aligned} \max \quad & \mathbb{E}[B(\boldsymbol{x}, \boldsymbol{U}) - C(\boldsymbol{x})] \\ \text{s.t.} \quad & C(\boldsymbol{x}) < c \\ & \boldsymbol{x} \in \mathbb{X} \end{aligned}$$

Advantages:

- The objective function includes both parts of interest - the social benefit and the financial cost.
- There are several SO methods available for this type of problem, as for formulation 1 (benefit maximisation).

Disadvantages:

- Social benefit B must be expressed in financial terms.
- There remains the potential to dismiss a solution which efficiently spends a less-than-full budget, since the objective function is only interested in the difference between the benefit and cost.

4. Benefit-cost-ratio maximisation

This formulation maximises the social benefit to financial cost ratio. The constraint ensures that the financial cost is viable. Action constraints $\hat{X}$ are introduced to ensure sufficient movement away from the status quo, as discussed in Miller et al. (2022).

$$\begin{aligned} \max \quad & \mathbb{E}\left[\frac{B(\boldsymbol{x},\boldsymbol{U})}{C(\boldsymbol{x})}\right] \\ \text{s.t.} \quad & C(\boldsymbol{x}) < c \\ & \boldsymbol{x} \in \hat{\mathbb{X}} \cap \mathbb{X} \end{aligned}$$

Advantages:

- The objective function includes both parts of interest - the social benefit and the financial cost.
- There is no need to express B in financial terms, but any component parts of B must be in the same units as each other.
- An optimal solution will lead to the most efficient use of resourcese - i.e. the best possible benefit per unit cost.
- There are several SO methods available for this type of problem, as for formulation 1 (benefit maximisation).

Disadvantages:

- One must define suitable action constraints to ensure sufficient movement from the status quo

5. Bi-objective simulation optimisation

This formulation simultaneously optimises a social benefit objective function and a financial cost objective function, thus balancing conflicting objectives. The constraint ensures that the financial cost is viable.

$$\begin{aligned} \min \quad & \{-\mathbb{E}[B(\boldsymbol{x},\boldsymbol{U})], C(\boldsymbol{x})\} \\ \text{s.t.} \quad & C(\boldsymbol{x}) < c \\ & \boldsymbol{x} \in \mathbb{X} \end{aligned}$$

Advantages:

- The objective function includes both parts of interest - the social benefit and the financial cost
- There is no need to express B in financial terms, but any component parts of B must be in the same units as each other.

- An optimal solution will lead to the most efficient use of resourcese - i.e. the best possible benefit per unit cost.

Disadvantages:

- Multi-objective simulation optimisation methods are not very well established in the literature.

References

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