Rolling horizon routine

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Algorithm 1 outlines a basic setup for creating an optimal building plan for the housing waiting list problem, over a rolling decision horizon of length T_D.

In plain English with worked example - with references to the variables in the algorithm: Suppose we want to make a decision covering the next 4 years (T_D), considering 10 years ahead (T_M) from the start of each year. First, we require a forecast ($\boldsymbol{\lambda}$) of annual arrival rates which allows us to model ahead equally far from each year in the decision horizon. In our example, we need a 13 year forecast, so that at the start of year 4, we have a 10 year forecast from year 4 to 13, inclusive.

With $T_D = 4$, a feasible building solution ($x \in \mathbb{X}$) is always 4 years long. Building rates further than 4 years ahead are fixed in the model (M) and are the same for any feasible solution. Other model parameters which apply to any solution are also fixed in M, for example the housing service rate.

We start in year 1, and optimise over our set of feasible building solutions ($\mathbb{X}$), with respect to objective function (Y). The optimisation routine will involve evaluating (or estimating) objective function values with a 10-year model run. This "intermediate" optimal solution (*plan*) will be a 4 year building plan, but we only add the 1st year of this *plan* to $\boldsymbol{x}^*$, our "final" optimal solution.

We then suppose that we have proceeded for one year using this optimal solution. Using the model outputs at the end of year 1 for this solution, we can estimate the new current demand (λ_0), along with the new current supply of housing and shelter (h_0, s_0), reflecting the position we would expect to find ourselves should we follow our plan for one year until the next decision point.

We then update our feasible solution space $\mathbb{X}$ to cover the next 4 years (i.e. years 2 to 5 inclusive). If we are imposing shape constraints, then this updated solution space will depend on our decision in year 1. We iterate the process of optimising over the following 4 year period (looking 10 years ahead). After each iteration we add the first year of the *plan* to $\boldsymbol{x}^*$, update our current demand and supply and update our feasible solution space for the next iteration. We stop when we have a 4-year long optimal solution $\boldsymbol{x}^*$.

Algorithm 1 Rolling horizon routine

T_D = decision horizon (years)
$\boldsymbol{x}^* = \{x^*_{yr}\}_{yr=1,\ldots,T_D}$ empty set to construct optimal solution
T_M = modelling horizon (years). $T_M \geq T_D$
λ_0 = current demand (number in system)
$\boldsymbol{\lambda} = \{\lambda_t\}_{t=1,\ldots,T_D+T_M-1}$ = future annual arrival rates
(h_0, s_0) = current supply of housing and shelter
$\mathbb{X}$ = feasible building solutions for next T_D years
Y = objective function to minimise
M = analytical queueing model
for yr = 1:T_D **do**
    $\boldsymbol{\lambda'} = \{\lambda_t\}_{t=\text{yr},\ldots,\text{yr}+T_M-1}$ (select arrival rates covering next T_M years)
    $plan = \text{Optimise}(Y, \mathbb{X}, \lambda_0, \boldsymbol{\lambda'}, (h_0, s_0), M)$ (Plan for next T_D years)
    $x^*_{\text{yr}} = plan[1]$ (Include 1st year of plan in optimal solution)
    Update (h_0, s_0) to be new current supply of housing/shelter
    Update λ_0 to be new current demand
    if yr $< T_D$ **then**
        Update $\mathbb{X}$ to cover period $(\text{yr}+1, \text{yr}+T_D)$
    end if
end for
Return $\boldsymbol{x}^*$