Modelling and optimising the housing of homeless populations: ten month PhD review

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Abstract

Modelling and optimisation are popular tools for supporting resourcing and capacity decisions in healthcare and homeless settings. We show how deterministic optimisation with a fluid flow model can support long-term capacity planning for a homeless care setting in the San Francisco Bay Area, California. Models of multi fidelity, including stochastic simulation, are available in this setting, and the solution space is integer-ordered. We therefore explore both multi fidelity and integer-ordered simulation optimisation methods and discuss potential research contributions at the intersection of these active fields of research.

1 Introduction

Homelessness is a growing problem faced by communities worldwide. An example is Alameda County in the San Francisco Bay area, California, where approximately 8000 people experienced homelessness in 2021 alone. Decision makers within these communities typically have some leverage over how relevant resources are allocated to help alleviate homelessness. In Alameda County, the local government must split their resources between emergency shelter and permanent social housing. Shelter is relatively cheap and quick to set up. It provides a safer alternative to street homelessness but does not provide a stable long-term living situation for its residents. Permanent social housing is more expensive than shelter and can take longer to set up, but it does offer the stable long-term living situationd that homelessness people need to improve their quality of life. Building capacity to alleviate homelessness takes time, especially since funds are typically available in varying amounts from year to year. Decision makers in communities like Alameda County must therefore make good time-varying capacity planning decisions to reduce homelessness now and in the future.

Operational research (OR) methods offer helpful tools to support such public sector decision-making. Optimisation helps decision-making by looking for a feasible solution which performs best across a (potentially infinite) set of alternative feasible solutions. To do this, a model of the performance of a solution is needed, and the quality of the model affects the quality of the subsequent optimisation results. We can model the homelessness care system as a queue. Homeless people must wait for permanent social housing to become available. Some of those in the queue for social housing can be accommodated in shelter, while the rest will be street homeless. Once a housing 'server' becomes availabe, a resident may stay their for some time (in some cases, the rest of their lives) before they ultimately leave the system.

The most accurate model of this queueing system is a high-fidelity stochastic simulation model. In this case, one can only estimate the performance of a solution and the subsequent optimisation falls in the realm of simulation optimisation (SO). There are different SO methods for different types of problem (which we later discuss) but the issue of limited computational resources pervades all SO methods. This issue stems from from the fact that a stochastic simulation is typically computationally expensive to run, and many simulation replications are required to be confident of a solution's performance, given the associated uncertainty.

As is common in queueing systems, lower fidelity models such as analytical queueing models offer a computationally cheaper alternative to stochastic simulation. They also offer helpful alternative perspectives on the dynamics of a queueing system. The drawback is that these models are typically less accurate, given the necessary assumptions which must be made. If one only uses a low-fidelity deterministic model to evaluate the performance of a solution, optimisation falls in the realm of deterministic optimisation. Performing this deterministic optimisation can be a helpful first step in the decision-making process. However, there is more we can do with our low fidelity models. Multi-fidelity simulation optimisation (MFSO) enables low-fidelity models to be used alongside high fideltiy stocahstic simulation in a SO algorithm which reduces the computational burden and therefore enables an optimal solution to be found more efficiently. Novel MFSO methods will be the main topic of this PhD research.

This document is organised as follows: in Section 2 we briefly review relevant literature on modelling and optimisation in homeless care settings and healthcare settings. There are many similarities between these two and the latter is more widely studied in the literature. We also review relevant SO methods including MFSO. In Sections 3 and 4 we discuss the main content of the PhD research to date. Section 3 introduces three models of multi-fidelity for homeless care systems. Section 4 introduces an optimisation formualation which addresses the time-varying capacity planning problem and we solve this problem in a determinsitc setting. In Section 5 we discuss how different types of uncertainty affect our decision-making process, and this discussion motivates the need for simulation optimisation. In Section 6 we discuss relevant interesting gaps in the current MFSO literature which the remainder of this PhD research seeks to address.

2 Literature Review

2.1 Modelling and optimisation in healthcare settings

Modelling hospital waiting lists using stochastic simulation e.g. Wood (2022) and using stocks and flows e.g. Worthington (1991). Optimisation such as Argyris et al. (2022) who balance efficiency and fairness in healthcare provision.

2.2 Modelling and optimisation in homeless care settings

Simulation modelling of homeless care system in Alameda County (Singham et al., 2023) and of shelters for runaway homeless youths (RHYs) (Kaya et al., 2022b). Optimisation such as Kaya et al. (2022a) who minimise the cost of matching demand with supply for RHYs who require beds and support services.

2.3 Simulation optimisation (SO)

2.3.1 Overview of SO methods

- Discrete SO: ranking & selection, adaptive random search, integer-ordered.
- Continuous SO: sample average approx, stochastic approx, meta models.

2.3.2 Integer-ordered SO methods

- Retrospective search with piecewise-linear interpolation and neighborhood enumeration (R-SPLINE) (Wang et al., 2013).
- Discrete Stochastic Approximation (Lim, 2012)
- Gaussian Markov Random Fields (L. Salemi et al., 2019)

2.3.3 Multi fidelity SO methods

- Using deterministic optimisation results to begin simulation optimisation e.g. Jian and Henderson (2015).
- Ordinal transformation with optimal sampling (Xu et al., 2016).
- Modelling the error of a low-fidelity model
  - Polynomial error terms e.g. Chong and Osorio (2018).
  - Gaussian Process error terms e.g. Huang et al. (2006).
- Multi-fidelity Expensive Black Box (Mf-EBB) Optimisation

3 Models of multi-fidelity

Here we introduce three models for homeless care systems from low- to high-fidelity. Our low-fidelity model is a fluid flow model, which assumes housing

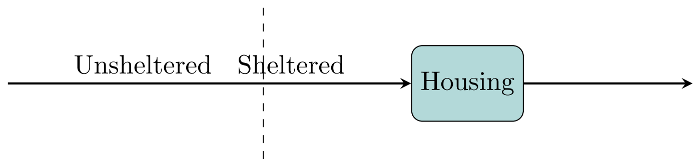

Figure 1: Simple queueing system

servers are always busy and treats flow in and out of the system like a liquid with a continuous-valued volume. Our medium-fidelity model is an $M_t/M/h_t$ queueing model which relaxes the server-always-busy assumption. We incorporate stochasticity and using a Markov chain analysis we can compute the expected number of people housed, sheltered and unsheltered at some point in time, given initial conditions. Our high-fidelity model is a discrete-event simulation model. Here, as well as caputuring stochasticity we are able to model additional real-world processes such as the conversion of shelter to housing, delays between houses being vacated and re-occupied by others and non-Markovian service time distributions. We now discuss each model in turn.

Figure 1 illustrates a simple queueing system which we model with a fluid flow model.

This section introduces a fluid flow queueing model which tracks the number of housed, sheltered, and unsheltered clients over time. The two main inputs to the model are a changing arrival rate over time, and a housing service rate which changes as housing is built. Additionally, the amount of shelter space available to support the queue for housing may change over time. This models the dynamics in Figure 1. Due to the current large queue for housing and continued inability for housing rates to keep up with arrivals to the system, the assumption that the servers will always be busy is not only reasonable, but significant enough to negate the usual assumptions of steady-state queueing behavior where the servers are idle with some positive probability.

In our fluid flow model we ignore the randomness in the arrival process and the service process for homeless people entering and leaving the homeless response system. Instead we assume that "fluid" flows into the system continuously at a rate $\lambda(t)$ and flows out at rate $\mu(t) = \mu_0 h(t)$ where μ_0 is the service rate of a single housing unit and $h(t)$ is the continuous-valued number of houses at time t. Given the initial number of people in the system X_0, at time t we can calculate the subsequent number of people in the system, $X(t)$, as

$$X(t) = X_0 + \int_0^t \lambda(t)dt - \int_0^t \mu_0 h(t)dt.$$

We split the queue for housing into an unsheltered and a sheltered part. We denote by $s(t)$ the continuous-valued number of shelters at time t. The size of the unsheltered queue $u(t)$ is then

$$u(t) = X(t) - h(t) - s(t) \tag{1}$$

$$= X_0 + \int_0^t \lambda(t)dt - \int_0^t \mu_0 h(t)dt - h(t) - s(t), \tag{2}$$

where we assume that capacities $h(t)$ and $s(t)$ are sufficiently small compared to the given arrival rate $\lambda(t)$ so that these resources are always full, and the use of a fluid flow model remains appropriate. In other words, the number of people housed and the number in shelter are the same as the housing and shelter capacities $h(t)$ and $s(t)$, respectively. In reality, there may be some friction in the system in that housing may be idle while units are experiencing turnover and the next person in the queue is being located, but this time can be incorporated into the service time distribution.

When analyzing the dynamics of the fluid flow model over a modeling horizon, we discretize time into days. We now let λ_d, h_d^D, s_d^D and u_d for all $d \in \{1, ..., D\}$ be the discretized equivalents of $\lambda(t), h(t), s(t)$ and $u(t)$, respectively, where D is the modeling horizon in days and is used as a superscript where we must later distinguish between daily and annual capacities. In order to evaluate our various objective functions (which we describe in Section 4) we typically approximate (2) with the sum

$$u_d = X_0 + \sum_{d'=1}^{d} \lambda_{d'} \delta t - \sum_{d'=1}^{d} \mu_0 h_{d'}^D \delta t - h_d^D - s_d^D, \tag{3}$$

where μ_0 is the daily service rate of a single housing unit and the stepsize $\delta t = 1$ day. In Figure 2 we give an illustrative example of the dynamics of u_d given by our fluid flow model, calibrated using realistic inputs for X_0, μ_0 and λ_d, h_d^D, s_d^D for all $d \in \{1, ..., D\}$ which we take directly from Singham et al. (2023).

Figure 2 shows an example of how one might come close to reaching functional zero in five years. The level of housing investment steadily increases over time. There is some initial increase in shelter, though in general there is less investment in shelter over the long term than in housing. The unsheltered population is stabilized and then eventually decreases approaching zero. The arrival rates λ_d are projected based on a current estimate of 10/day, along with the assumption that arrivals will increase in the coming years due to repercussions of COVID-19. Eventually, prevention methods will take effect and the arrival rate will hopefully decline (Alameda County, 2022). The daily service rate μ_0 is equivalent to the mean of a triangular distribution with lower limit 0 weeks, upper limit 400 weeks and mode 300 weeks.

In Section 4 we will evaluate (3) using annual housing and shelter capacity vectors $\boldsymbol{h} = \{h_t \; \forall t \in 0, ..., T\}$ and $\boldsymbol{s} = \{s_t \; \forall t \in 0, ..., T\}$ where T is a time horizon in years. In this case we assume that any annual increase or decrease

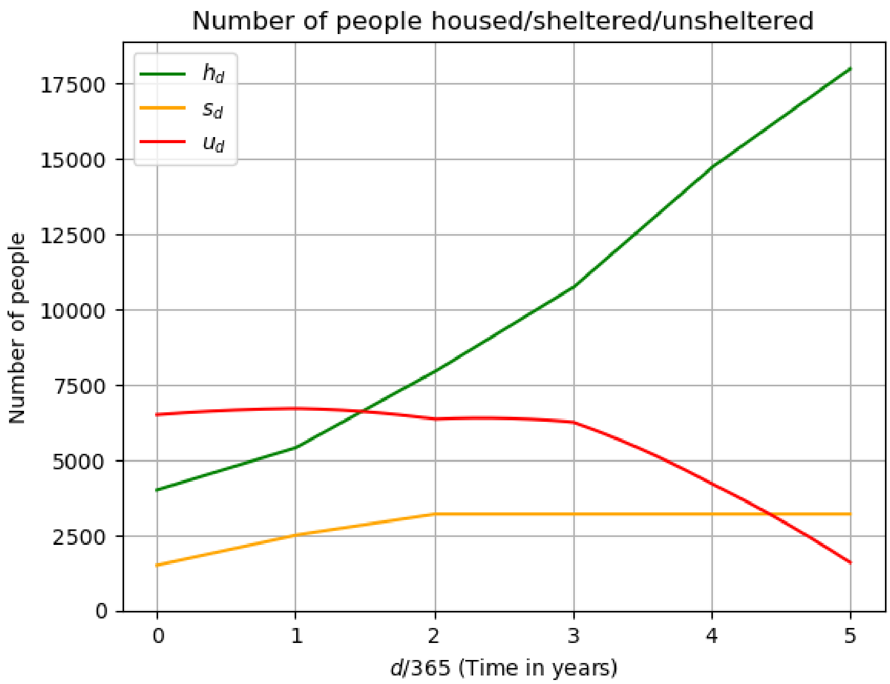

Figure 2: Dynamics of u_d, s_d^D and h_d^D. $X_0 = 12000$, $\mu_0 = 6.106 \times 10^{-4}$, Daily arrival rates λ_d in each year: $10.0, 11.9, 13.1, 13.1, 11.8$.

in capacity is spread evenly throughout the year, and (3) becomes

$$u_d(\boldsymbol{h}, \boldsymbol{s}) = X_0 + \sum_{d'=1}^{d} \lambda_{d'} \delta t - \sum_{d'=1}^{d} \mu_0 h_{d'}^D(\boldsymbol{h}) \delta t - h_d^D(\boldsymbol{h}) - s_d^D(\boldsymbol{s}), \tag{4}$$

where

$$h_d^D(\boldsymbol{h}) = h_{\lfloor \frac{d}{365} \rfloor} + \frac{d - \lfloor \frac{d}{365} \rfloor}{365} (h_{\lceil \frac{d}{365} \rceil} - h_{\lfloor \frac{d}{365} \rfloor}) \tag{5}$$

and

$$s_d^D(\boldsymbol{s}) = s_{\lfloor \frac{d}{365} \rfloor} + \frac{d - \lfloor \frac{d}{365} \rfloor}{365} (s_{\lceil \frac{d}{365} \rceil} - s_{\lfloor \frac{d}{365} \rfloor}). \tag{6}$$

4 Deterministic optimisation with low-fidelity model

- Optimisation formulations
- Numerical results

5 Discussion of uncertainty

- Stochastic uncertainty in homeless care problem (arrival/service processes).
- Input model uncertainty: good input models for today cannot reliably predict future events.

6 Potential contributions at intersection of integer-ordered and multi fidelity SO

- Using low-fidelity models to quickly compute gradients in RSPLINE/DSA.
- Adding prior information to GMRF using low-fidelity model.
- Modelling errors of low fidelity models using GMRF.

7 Conclusion

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