Radiometric dating equations (in one page)

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Part 1: The age equation

We begin with the observation that the rate of *decay* (i.e. the negative rate of change in the abundance) of a radioactive isotope with respect to time (dN/dt) is directly proportional to its abundance N at any given moment:

$$\frac{dN}{dt} = -\lambda N \tag{1}$$

...assuming a constant of proportionality λ, which we'll call N's "decay constant."

Now, separate the derivatives in Eq. 1 and calculate their definite integrals over the time interval $[0, t]$.

$$\begin{aligned}
\frac{1}{N}\, dN &= -\lambda\, dt \\
\int_0^t \frac{1}{N}\, dN &= \int_0^t -\lambda\, dt \\
\ln N - \ln N_o &= -\lambda t - -\lambda \cdot 0 \\
\ln \frac{N}{N_o} &= -\lambda t \\
\frac{N}{N_o} &= e^{-\lambda t} \\
N &= N_o\, e^{-\lambda t} \qquad (2)
\end{aligned}$$

...where N is the abundance of the radioactive isotope at some moment in time t, and N_o is its *initial* abundance at time $t = 0$.

Eq. 2 is the radiometric "age equation."

Part 2: Using the age equation

Start with Eq. 2, which describes the amount of radioactive isotope N (with initial abundance N_o) remaining after time t, given the isotope's decay constant λ (units of t^{-1}).

From this, we calculate the abundance of radiogenic isotope $n*$ produced by the decay of N

$$\begin{aligned}
n* &= N_o - N \\
n* &= N_o - N_o\, e^{-\lambda t} \\
n* &= N_o \left(1 - e^{-\lambda t}\right) \qquad (3)
\end{aligned}$$

This is lovely but requires knowledge of N_o, which is lost to time. All we can measure today is the value N. So, let's rearrange Eq. 2 to solve for N_o...

$$N_o = \frac{N}{e^{-\lambda t}} = N\, e^{\lambda t} \tag{4}$$

...and substitute Eq. 4 into Eq. 3...

$$\begin{aligned}
n* &= \left(N\, e^{\lambda t}\right)\left(1 - e^{-\lambda t}\right) \\
n* &= N \left(e^{\lambda t} - e^{\lambda t} e^{-\lambda t}\right) \\
n* &= N \left(e^{\lambda t} - 1\right) \qquad (5)
\end{aligned}$$

A natural system may have originally incorporated some initial amount of radiogenic isotope n_o, so we calculate $n = n_o + n*$ as

$$n = n_o + N \left(e^{\lambda t} - 1\right) \tag{6}$$

And *finally*, because it is, in fact, quite hard to measure things as absolute abundances but easier to measure isotopic ratios, we often rewrite Eq. 6 normalized to a stable isotope of the same element as n, which we'll call ${}^{s}n$.

$$\frac{n}{{}^{s}n} = \frac{n_o}{{}^{s}n} + \frac{N}{{}^{s}n}\left(e^{\lambda t} - 1\right) \tag{7}$$