

Radiometric dating equations (in one page)

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Part 1: The age equation

We begin with the observation that the rate of *decay* (i.e. the negative rate of change in the abundance) of a radioactive isotope with respect to time (dN/dt) is directly proportional to its abundance N at any given moment:

$$\frac{dN}{dt} = -\lambda N \quad (1)$$

...assuming a constant of proportionality λ , which we'll call N 's "decay constant."

Now, separate the derivatives in Eq. 1 and calculate their definite integrals over the time interval $[0, t]$.

$$\begin{aligned} \frac{1}{N} dN &= -\lambda dt \\ \int_0^t \frac{1}{N} dN &= \int_0^t -\lambda dt \\ \ln N - \ln N_o &= -\lambda t - -\lambda \cdot 0 \\ \ln \frac{N}{N_o} &= -\lambda t \\ \frac{N}{N_o} &= e^{-\lambda t} \\ N &= N_o e^{-\lambda t} \end{aligned} \quad (2)$$

...where N is the abundance of the radioactive isotope at some moment in time t , and N_o is its *initial* abundance at time $t = 0$.

Eq. 2 is the radiometric "age equation."

Part 2: Using the age equation

Start with Eq. 2, which describes the amount of radioactive isotope N (with initial abundance N_o) remaining after time t , given the isotope's decay constant λ (units of t^{-1}).

From this, we calculate the abundance of radiogenic isotope n^* produced by the decay of N

$$\begin{aligned} n^* &= N_o - N \\ n^* &= N_o - N_o e^{-\lambda t} \\ n^* &= N_o (1 - e^{-\lambda t}) \end{aligned} \quad (3)$$

This is lovely but requires knowledge of N_o , which is lost to time. All we can measure today is the value N . So, let's rearrange Eq. 2 to solve for N_o ...

$$N_o = \frac{N}{e^{-\lambda t}} = N e^{\lambda t} \quad (4)$$

...and substitute Eq. 4 into Eq. 3...

$$\begin{aligned} n^* &= (N e^{\lambda t}) (1 - e^{-\lambda t}) \\ n^* &= N (e^{\lambda t} - e^{\lambda t} e^{-\lambda t}) \\ n^* &= N (e^{\lambda t} - 1) \end{aligned} \quad (5)$$

A natural system may have originally incorporated some initial amount of radiogenic isotope n_o , so we calculate $n = n_o + n^*$ as

$$n = n_o + N (e^{\lambda t} - 1) \quad (6)$$

And *finally*, because it is, in fact, quite hard to measure things as absolute abundances but easier to measure isotopic ratios, we often rewrite Eq. 6 normalized to a stable isotope of the same element as n , which we'll call s_n .

$$\frac{n}{s_n} = \frac{n_o}{s_n} + \frac{N}{s_n} (e^{\lambda t} - 1) \quad (7)$$