Terminating sequences of Bunny Trainer Transforms

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In this note, we prove a necessary and sufficient condition for the termination of Bunny Trainer Transform sequences.

Definition. A *Bunny Trainer Transform* (BTT) is the following function on pairs of nonnegative integers (m, n):

$$BTT(m,n) \quad = \quad \begin{cases} BTT(n,m) & m > n \\ (2m, n-m) & 3m \leq n \\ (n-m, 2m) & m \leq n < 3m \end{cases}$$

The BTT represents the outcome of a match where the bunny trainer corresponding to the smaller integer (e.g. m in (m, n)) bets the entire position value m and wins. BTT is defined in such a way that if $(x, y) = BTT(m, n)$ then $x \leq y$.

Definition. A BTT sequence *terminates* or is *terminating* when $BTT^k(m, n) = (0, p)$ for some nonnegative integers k and p. If no such k and p exist, then the BTT sequence is *nonterminating*.

That is to say, the BTT sequence terminates when successive matches lead to one of the bunny trainers losing everything. We have $BTT(0, p) = (0, p)$, so the BTT sequence terminates if it eventually reaches a fixed point where the first integer in the pair is zero.

Definition. The *terminating condition* (TC) for a pair of nonnegative integers (m, n), at least one of which is positive, is the following:

$$\frac{m+n}{\gcd(m,n)} = 2^k$$

for some nonnegative integer k.

Theorem. *A pair of nonnegative integers (m, n) has a terminating BTT sequence if and only if $m = n = 0$ or the pair meets the terminating condition.*

Proof. □