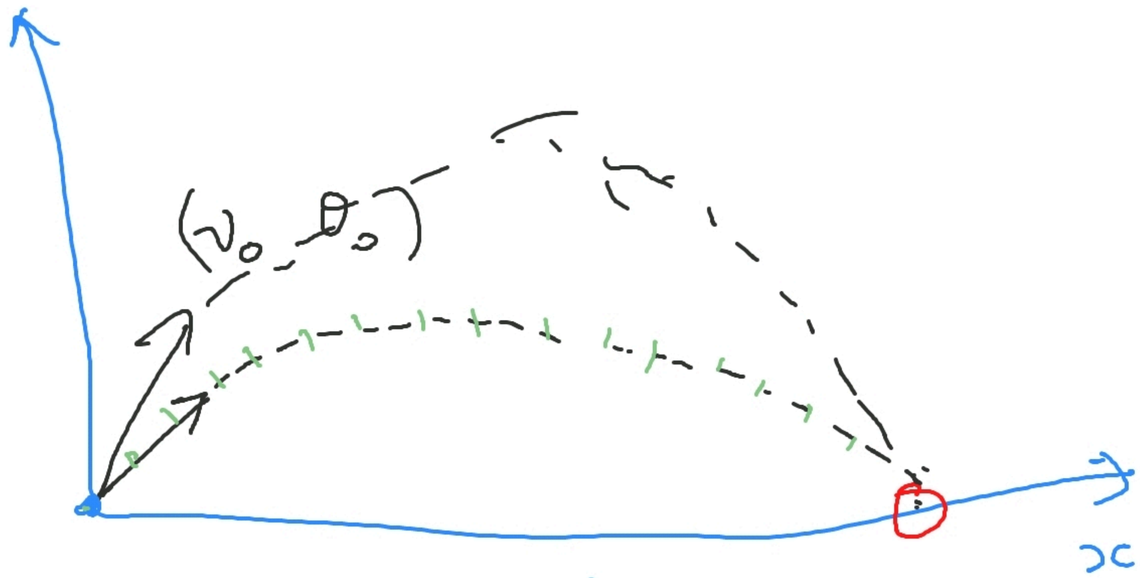
Shooting method



Two-point BVP

t

$t \in [t_1, t_2]$

$B_{1j}(t_1, \vec{y}) = 0 \quad j = 1, \ldots n_1$

$B_{2k}(t_2, \vec{y}) = 0 \quad k = 1, \ldots n_2$

IVP → specify, $\vec{y}(t = t_1)$. ⇒ Evolve in t. (steps in time)

[For IVP $\vec{y}_0 = (x_0, y_0, v_x^0, v_y^0)$

BVP $\vec{y} = (x, y, v_x, v_y)$

$x(t_1) = x_0, \; y(t_1) = y_0$

$= 0 \qquad = 0$

$x(t_2) = 100m, \; y(t_2) = 0$

$4 \text{ IVs} = N$

Guess (v_x^0, v_y^0)

$N = 4 \left[\frac{dx}{dt}, \frac{dy}{dt}, \frac{dv_x}{dt}, \frac{dv_y}{dt}\right]$

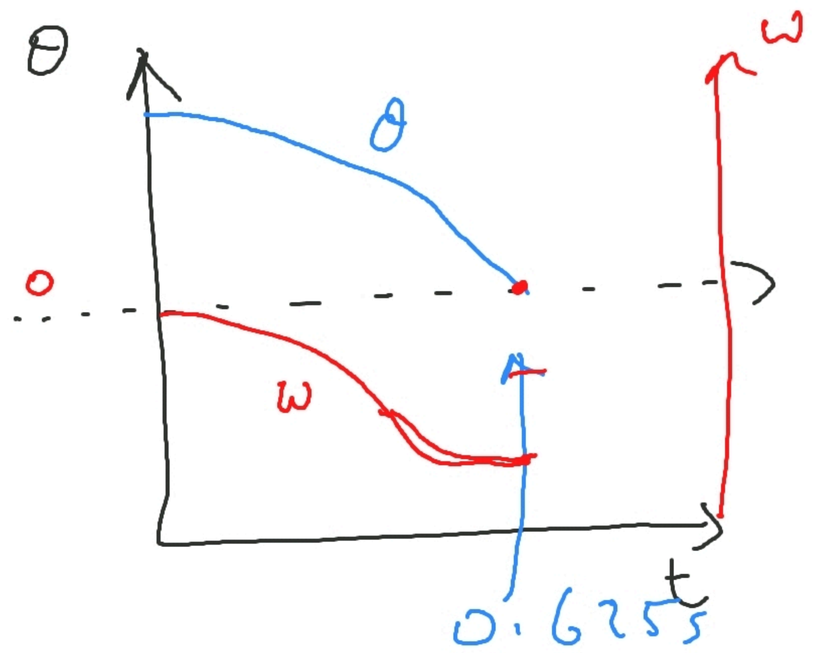
n_1 BCs at t_1

$n_2 = N - n_1$ BCs at t_2]

HW4 pendulum

$L = 1.5\,m$, $g = 9.80\,m/s^2$

$\vec{y} = \begin{pmatrix} \theta \\ w \end{pmatrix}$

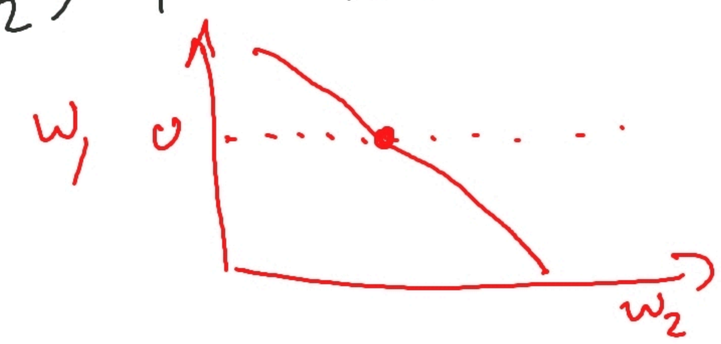


$\frac{d\theta}{dt} = w_1|_{t=t_1} = 0$ ← launch from rest

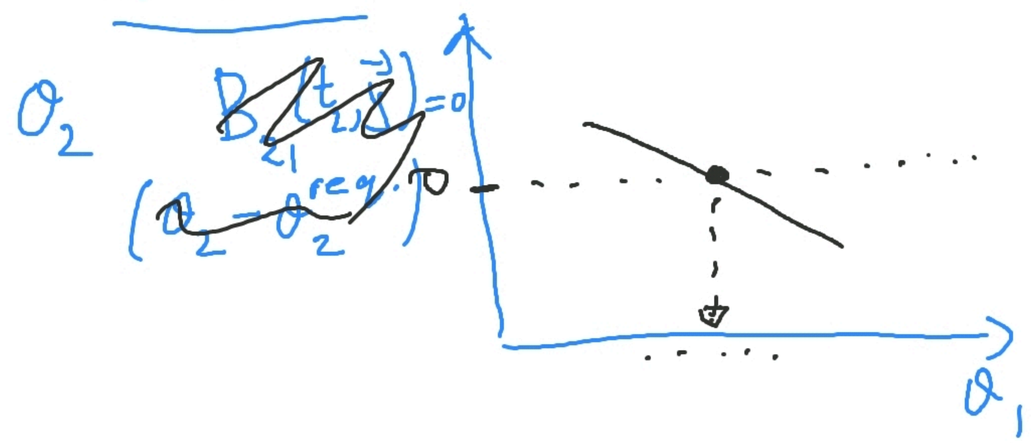
$\theta_2(t = t_2 = T/4) = 0$ with $T = 2.5\,s$

$\frac{d\theta}{dt}$
$\frac{dw}{dt}$ θ_1, w_1 = IVP

θ_2, w_1 = BVP

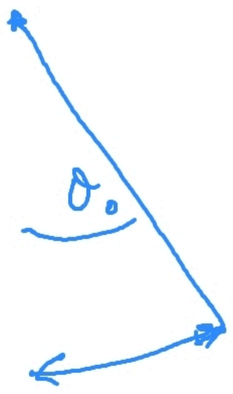


Guess θ_1



P

$T_{1/4}$?



2 ODEs

$\frac{d\theta}{dt}$, $\frac{dw}{dt}$

3 ODEs

$t_1 = 0$

$n_1 = 2$
$n_2 = 1$

$T_{1/4}$

$$\tau \equiv \frac{(t - t_1)}{T_{1/4}} = \frac{t}{T_{1/4}}$$

$\tau \in [0, 1]$

$t \in [0, T_{1/4}]$

$$\left.\begin{aligned} \frac{d\theta}{dt} &= w \\ \frac{dw}{dt} &= -\frac{g}{l}\sin\theta \\ \frac{dT_{1/4}}{dt} &= 0 \end{aligned}\right\} \rightarrow \left.\begin{aligned} \frac{d\theta}{d\tau} &= T_{1/4}\, w \\ \frac{dw}{d\tau} &= -\frac{g}{l} T_{1/4} \sin\theta \\ \frac{dT_{1/4}}{d\tau} &= 0 \end{aligned}\right\}$$

$\theta(\tau=0) = 60°$

$w(\tau=0) = 0$ rad/s

$\theta(\tau=1) = 0$

$\vec{y} = (\theta, w, T_{1/4})$