PHSX 615: Homework 5. Boundary value problem

Due date Thursday Dec. 5th

Explore numerical solution techniques of boundary value type problems like those arising from the equations of Laplace and Poisson.

Learning goals: Relaxation methods and graphical plotting of contour and vector maps.

Problem (30 pts) Boundary value problem. Solve the following problem in 2-d, similar to ones in Giordano 5.1, where we have a "double" capacitor (not sure this has any real application, but there are "inter-leaved" capacitors with a more complex geometry). It should be possible to see effects associated with the fringe-field.

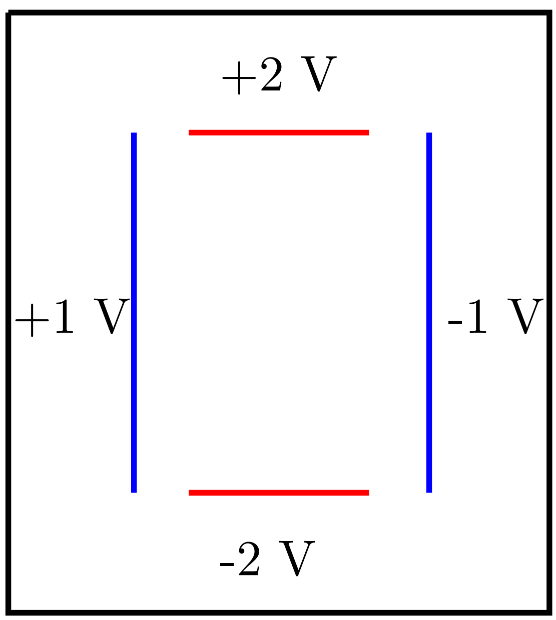

Figure 1: This consists of two sets of parallel conducting plates within a square conducting boundary of side-length, L, held at a potential of $V = 0$. The red set of plates are held at potentials of ± 2 V as indicated while the blue set are at $\pm$1V. The blue plates are $L/2$ in length. The red plates are $L/3$ in length. The centers of the red plates, and the centers of the blue plates are separated by $L/2$. The plate thicknesses may initially be taken as infinitesimal.

Solve Laplace's equation in 2-dimensions,

$$\nabla^2 V = 0$$

by discretizing on a numerical grid and using an appropriate relaxation method. Display your results graphically illustrating equi-potential lines and the corresponding electric field. A suitable grid spacing is likely to have grid points centered on the relevant boundaries. Taking L = 1.0 m, what E-field do you calculate at the center of the "capacitor"? What about at a coordinate with respect to the center of $(+L/8, +L/8)$ where fringe field effects may start to be important? Remember E-field is a vector quantity. How do the results vary as you increase the number of grid-points?

You may want to commision your relaxation method code, and check your implementation of for example, the Jacobi, Gauss-Seidel and Simultaneous Over-Relaxation methods on a simpler problem where you know the analytic solution (eg. a uniform field). Please be aware that the convergence rate of Jacobi and Gauss-Seidel can be really poor for large N. Remember GS is just SOR with $\alpha = 1$.

With this finite box, the boundary at which the potential goes to zero is not far from the plates. What happens if the side-length of the box is increased by a factor of 2? (Best to keep the grid spacing the same for the capacitor plates etc .., so you can compare simply the effect of distancing the boundary condition.).

Are your results sensitive to the thickness of the plates?

You should also report the number of iterations to reach a particular convergence criterion for the various methods. and report what value of the over-relaxation parameter, α, is most appropriate for your particular choice of the grid spacing.

Matrix-based methods are more appropriate for a really professional implementation of these kind of algorithms. In general though, the keep-it-simple-stupid philosophy probably means that you should not invest time trying to do this project that way unless you really have mastered the more straightforward techniques that I expect to see implemented in this project.