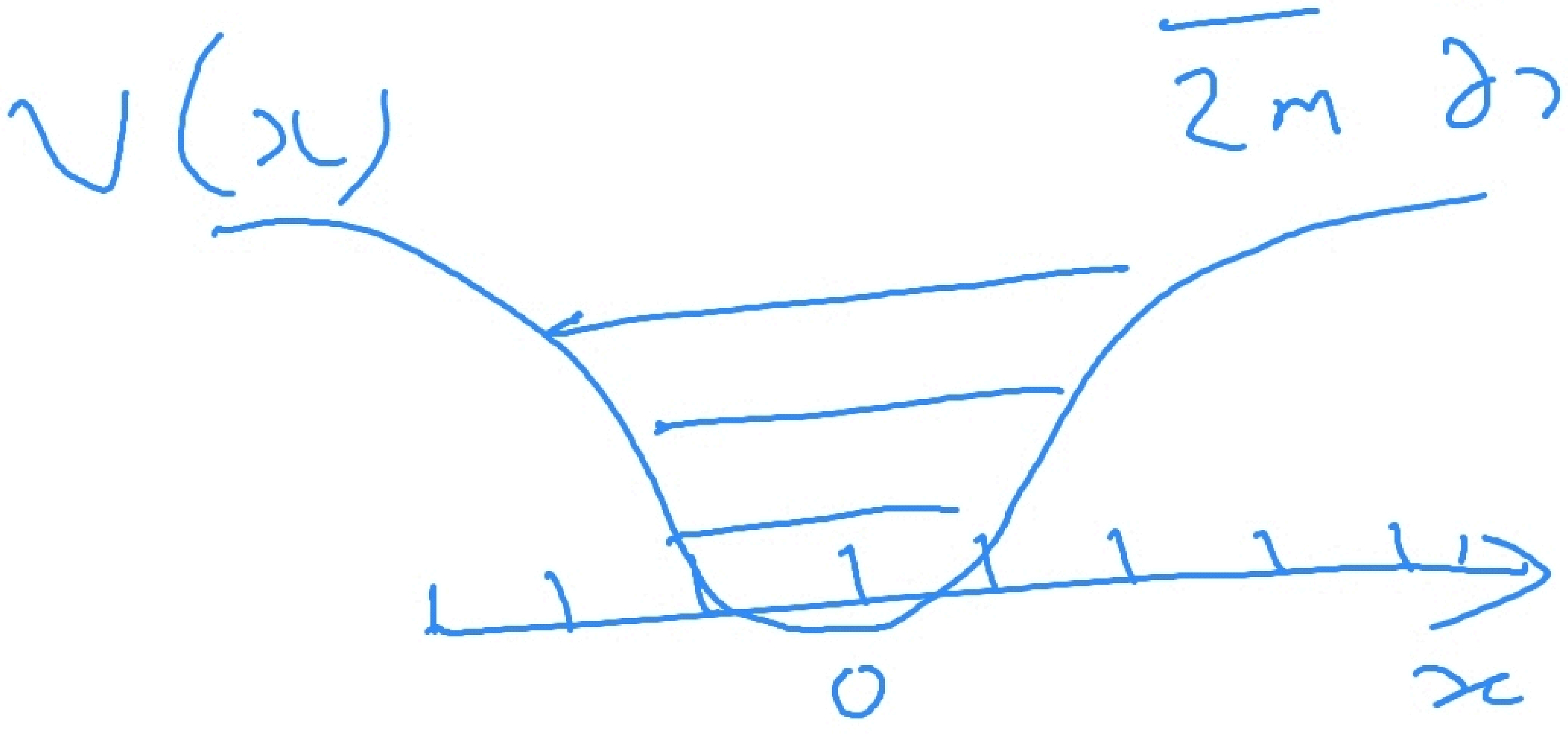
TISE $\quad -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi = E_k\psi$



$$\frac{d^2\psi}{dx^2} \approx \frac{\psi_{n-1} - 2\psi_n + \psi_{n+1}}{(\Delta x)^2}$$

$x_i = i\,\Delta x \qquad i \in (-\infty, \infty)$

$i \in [-100, 100]$

$\underline{\underline{\psi(x)}}$ for eigenvalue, E_k

$$\int_{-\infty}^{+\infty} \psi^*\psi \, dx = 1$$

ψ_i

$\psi(x) \to 0$
as $x \to \pm\infty$

Two-point BVP

N coupled 1st order ODEs $\qquad \vec{y}_{N\times 1}$

$[t_1, t_2]$ $\quad \left\{ \begin{array}{l} n_1 \text{ BCs at } t_1 \\ n_2 \text{ BCs at } t_2 \end{array} \right.$ $\qquad$ IVP $\quad N \quad \vec{y}_{0\,N\times 1}$

$n_1 + n_2 = N$

$$\frac{d\vec{y}}{dt} = f(t, \vec{y})$$

$$B_{1j}(t, \vec{y}) = 0 \qquad j = 1, \dots n_1$$

$$B_{2k}(t, \vec{y}) = 0 \qquad k = 1, \dots n_2$$

HW4 pendulum

$L = 1.5\,m$

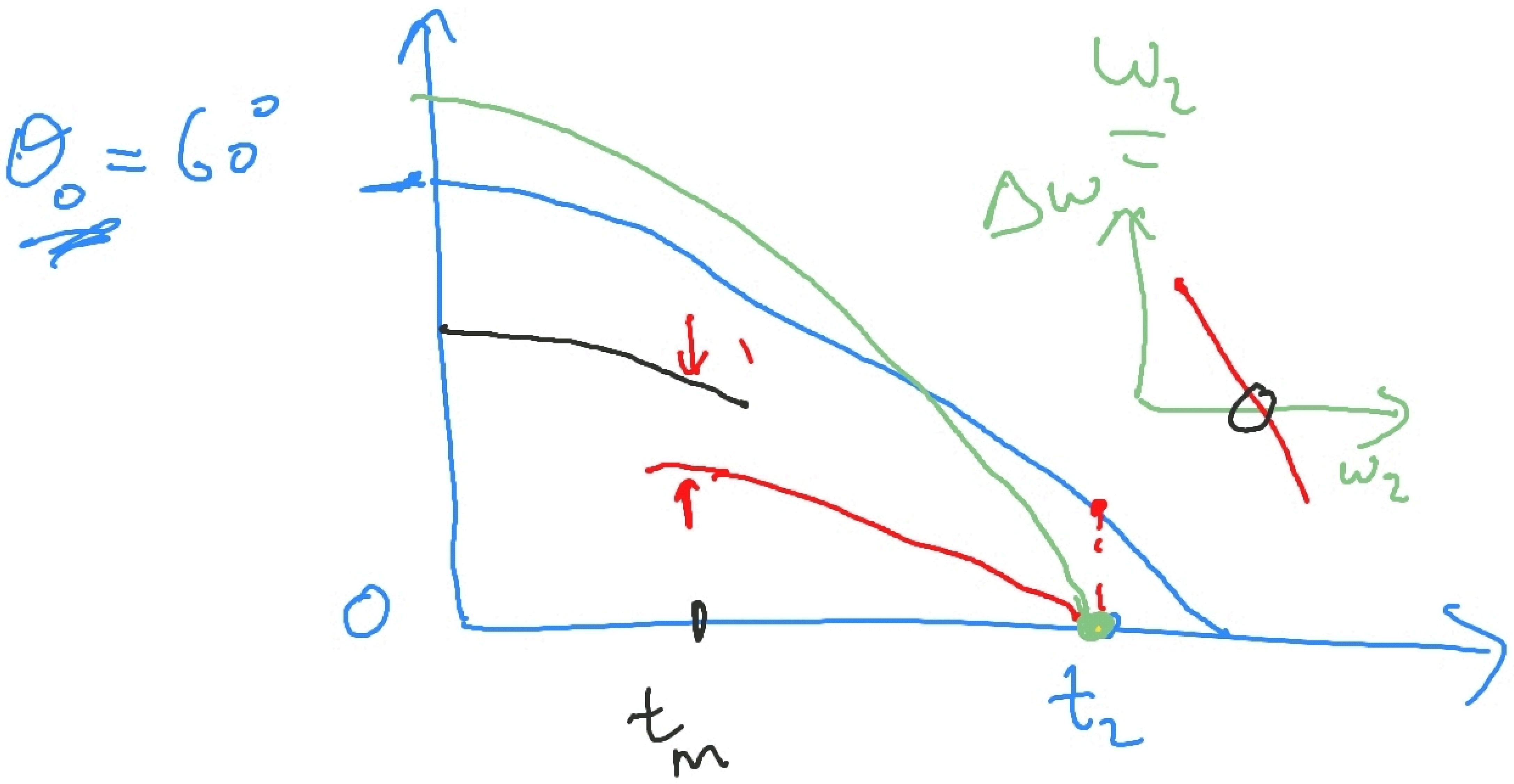
$g = 9.80\,m/s^2$

$$\frac{d\theta}{dt} = \omega(t_1 = 0) = 0\ \text{rad/s} \checkmark$$

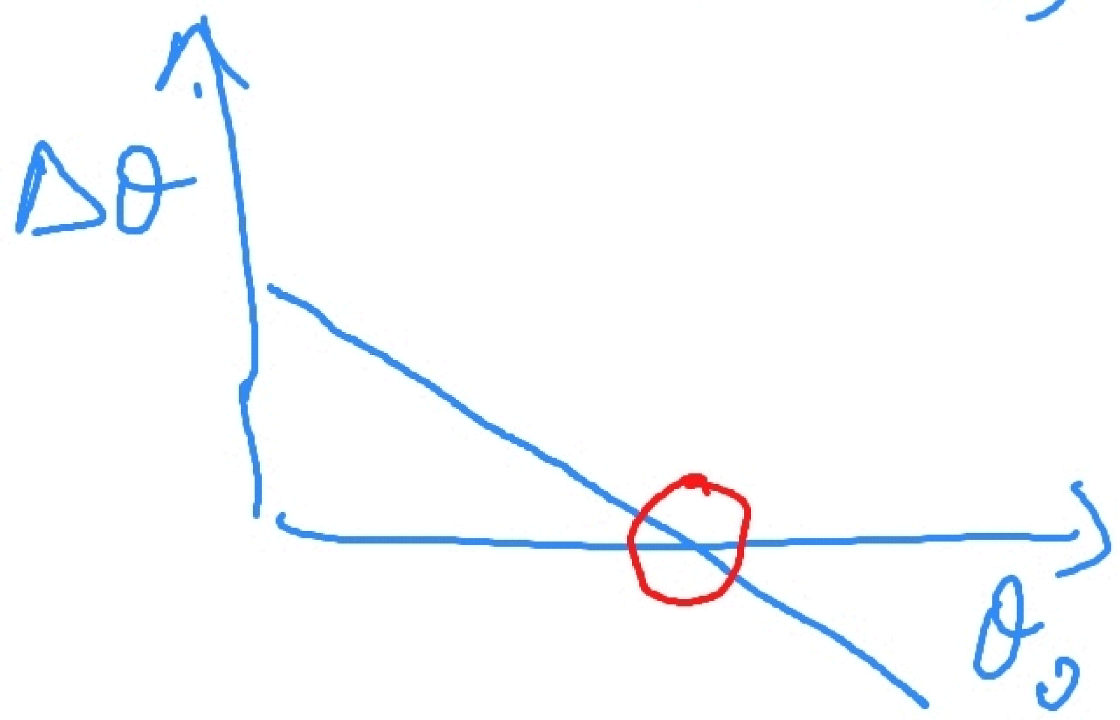
$$\theta\left(t_2 = \frac{T}{4} = 0.625\,s\right) = 0\ \text{rad}$$

$T = 2.5\,s.$

Guess θ_0 $(\theta(t_1 = 0))$



$$\Delta\theta = \theta(t_2) - \overset{BC}{\theta}(t_2)$$
$$= \theta(t_2)$$



B1 $\quad \omega(t_1 = 0) = 0$

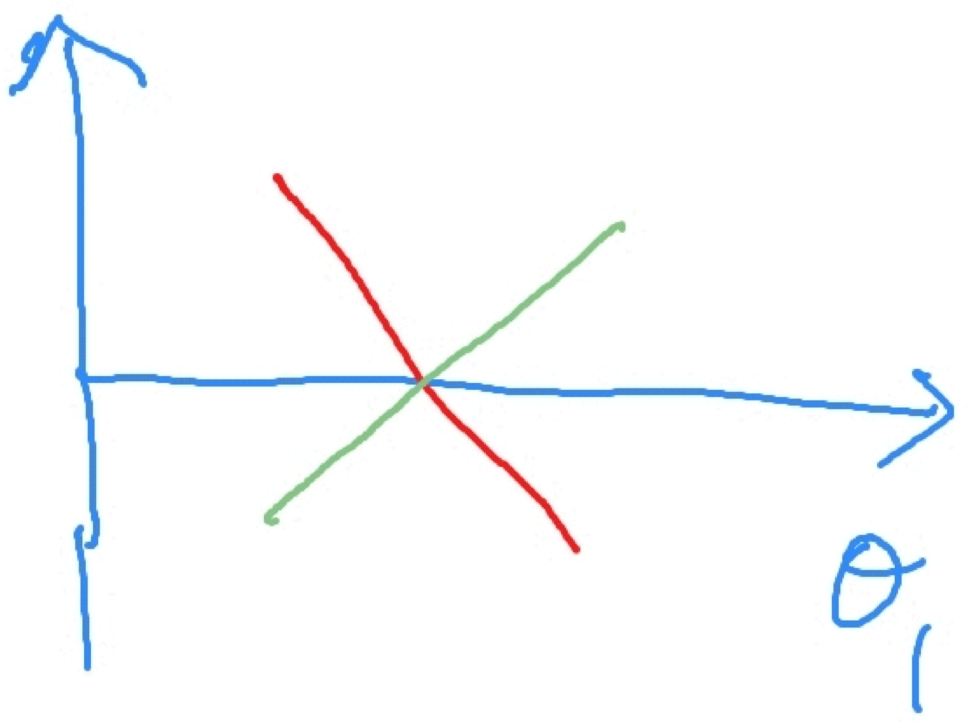
B2 $\quad \theta(t_2 = T/4) = 0 \qquad T = 2.5s$

G1 $\quad \theta(t_1 = 0) = \theta_1$

G2 $\quad \omega(t_2 = T/4) = \omega_2 = -\sqrt{\frac{2g}{l}(1-\cos\theta_1)}$

$\Delta\theta$

$\Delta\omega$



N ODEs

n_1 specified values

$n_2 = N - n_1$ free parameters.

$y_i(t_1; V_1, V_2, \ldots, V_{n_2})$

$\vec{V}_{n_2 \times 1}$

$$\vec{F}_{n_2 \times 1} = B_{2k}(t_2, \vec{y}) \qquad k = 1 \ldots, n_2$$

$$v_1' = v_1 - \frac{f(v_1)}{f'(v_1)}$$

$$\vec{V}^{new} = \vec{V}^{old} + \delta\vec{V}$$

$$J_{n_2 \times n_2} \cdot \delta\vec{V}_{n_2 \times 1} = -\vec{F}_{n_2 \times 1} \qquad J_{ij} = \frac{\partial F_i}{\partial V_j}$$

$A\vec{x} = \vec{b}$

$\vec{x} = A^{-1}\vec{b}$

$$\frac{\partial F_i}{\partial V_j} \approx \frac{F_i(V_1, V_2, \ldots V_j + \Delta V_j, \ldots) - F_i(V_1, V_2, \ldots V_j \ldots)}{\Delta V_j}$$