Computer Assignment #2

1) Solve using Geometric Series: $P(T_{oss}=\text{Odd}) = P(T=1)+P(T=3)+P(T=5)$

$S_\infty = \frac{a}{(1-r)}$ $P(T=1)=p$, $P(T=3)=(1-p)^2p$, $P(T=5)=(1-p)^4p$

(Solve for General Form)

$$S_\infty = p\cdot(1-p)^0 + p\cdot(1-p)^2 + p\cdot(1-p)^4 \ldots$$

$$= p\cdot\underset{(1)}{(1-p)^0}\left[1 + (1-p)^2 + (1-p)^4 \ldots\right]$$

$a=1$, $r=(1-p)^2$

$$= p\left(\frac{1}{1-(1-p)^2}\right) = p\,\frac{1}{1-(1-2p+p^2)} =$$

$$= \frac{\cancel{p}}{\cancel{1}-\cancel{1}+2\cancel{p}-p^{\cancel{2}}}$$

$$\boxed{S_\infty = \frac{1}{2-p}}$$

Eg: $p=\frac{1}{2}=.5$

$$S_\infty = \frac{1}{2-(.5)} = \frac{1}{1.5} = \frac{1}{\frac{3}{2}} = \boxed{\frac{2}{3}}$$

Solve using Total probability

2) $\bar{E}$

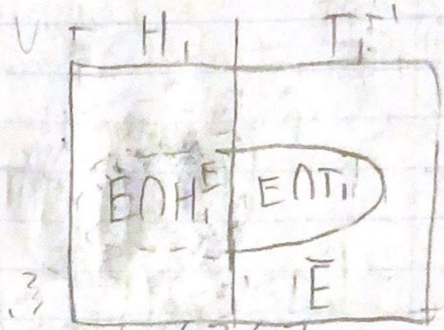
E = first head on an odd toss

H_1 = first toss is Heads

T_1 = first toss is Tails

$\{E\} = \{H\} \{T_1 \cap E\}$

$\{E\} = \{H, TTH, TTTTH, \ldots\}$



U $U = \{H_1\} \{T_1\}$

$\{\bar{E}\} = \{TH, TTTH, TTTTTH \ldots\}$

Assume $P(E|T_1) = 1 - P(E)$

$P(\text{Toss} = \text{odd}) =$ (B/c $P(E|H_1) = 1$)

$\boxed{\{E\} = \{H\} \cup \{T_1 \cap E\}}$

$\boxed{\{E\} \cap \{T_1\} = \{E\} - \{H_1\}}$

$\{\bar{E}\} = \overline{\{H\} \cap \{T \cap E\}}$ $\{T_1\} = \{T_1 \cap E\} \cap \{T_1 \cap E\}$

$\{\bar{E}\} = U - \{E\}$

$\{T_1\} \cap \overline{\{T_1 \cap E\}}$

$\{\bar{E}\} = \{H_1\} + \{T_1\} - \{H\} -$

$\{T_1\} \cap \{H_1 \cup E\}$

$(\cancel{\{T_1\} \cap \{H_1\}}) \cup (\{T_1\} \cap \bar{E}\})$

$\boxed{\{\bar{E}\} = \quad (\{T_1 \cap \bar{E}\})}$

$\{\bar{E}\} = \varnothing \quad \cup \{T_1 \cap \bar{E}\}$

$P(U) = 1 = P(E) + P(\bar{E})$

$\boxed{\{\bar{E}\} = \{T_1 \cap \bar{E}\}}$

$P(\bar{E}) = 1 - P(E)$

$U = \{H\} \cup \{T \cap E\} \cup \{T \cap \bar{E}\}$

$P(\bar{E}) = P(E|T_1)$

$P(\bar{E}) = P(T_1 \cap \bar{E}) =$

$P(E) = P(H_1) + P(T_1 \cap E)$

$P(\bar{E}) P(T_1 | \bar{E})$

P

$P(E) = P(H) + P(T_1) P(E|T_1)$

$P(\bar{E}) = P(T_1) P(\bar{E}|T_1) = P(E|T_1) - P(E|T_1) = \dfrac{P(E) - P(H)}{P(T_1)}$

$P(\bar{E}) = \dfrac{P(E) - P(H)}{P(T_1)}$

$$P(E) = \frac{P(E) - P(H)}{P(T_1)} \qquad P(H_1) = p$$

$$P(T) = q = (1-p)$$

$$1 - P(E) = \ldots$$

$$P(T_1) - P(T_1)P(E) = P(E) - P(H)$$

$$P(T_1) + P(H_1) = P(E) + P(T_1)P(E)$$

$$P(T_1) + P(H_1) = P(E)(1 + P(T_1))$$

$$P(E) = \frac{P(T_1) + P(H)}{1 + P(T_1)} = \frac{1-p+p}{1+1-p} = \boxed{\frac{1}{2-p}}$$