Computer Assignment 2

Deliverable: The solutions for the two exercises and a Python computer program that uses the simulation approach outlined below (and in computer assignment 1) as the solution to the given problem. To earn credit for the assignment the program must satisfy the rubric below.

Title: Three Solutions, Repeated Independent Trial Problem

Textbooks often have the answers to the problems in it in the back of the text. The student uses the answers provided in the text to verify the correctness of their answers to problems. If you attempt to solve a problem and the answer is not provided, you may not be able to verify whether you got the correct answer. Assuming there does not exist an answer for the problem you have solved, how do you know your answer is correct? To know that you have solved a problem correctly (or at least have confidence in your answer) you need to solve it several different ways and then see if the answers are consonant. Each approach to solving the problem must have a fundamentally different paradigm to support the argument that the answer is valid.

You will both from the standpoint of theory and from the standpoint of simulation using a Python program address the following probability problem.

Problem: You have a coin with the probability of heads being p. Toss the coin until a head comes up for the first time. What are the chances of that happening on an odd-numbered toss?

Exercise 1: Solve the problem by the direct approach of repeated independent trials. To do this it is expected that you will make use of an infinite geometric series.

Hints:

With $P(\{H\}) = p$ we can look at the possible favorable outcomes and the associated probabilities. The subscript indicates the number of the toss. (Refer to the note on geometric series.)

Outcome	Probability
H_1	p
TTH_3	$(1-p)^2 p$
$TTTTH_5$	$(1-p)^4 p$

and so forth indefinitely. Each trial is composed of independent events but all of the trials are mutually exclusive. Write your solution with the relevant steps.

Exercise 2: Solve the problem using total probability. The event E is 'first head on an odd toss'. Cut the sample space into two cases: first toss is heads H_1or first toss is tails T_1. N.B.: The subscript indicates first toss.

$\{E\} = \{H_1\} \cup \{E \cap T_1\}$

Hints: Apply conditional probability $P(A \cap B) = P(A)P(B|A)$ and model the sample space.

H *Cut the sample space into two mutually exclusive events.*

TH

TTH

$TTTH$

$TTTTH$

$TTTTTH$

$TTTTTTH$

and so forth ad infinitum.

The first outcome in the sample space, H, is one of the desired outcomes. All the rest of the outcomes share the same feature of tails on the first toss. Notice in this subset that the desired outcomes are alternate outcomes. When you condition the event E on tails on the first flip T_1what is the new sample space? What is the complement of the event E? Finally, how are $P(E|T_1)$ and $1 - P(E)$ related?

Write your solution with the relevant steps.

Exercise 3: Frequency Simulation

Write a Python program that solves the problem using simulation. (Similar as to what was done in computer assignment 1.) Simulation in this context means a computer program that models the probability experiment and then repeat this experiment a large number of times. Correctly evaluating the results of the simulation will make it possible to determine an approximate answer to the probability problem. Some suggestions and comments are the following:

1.) The Python program simulating a Bernoulli random variable found in the note 'Markov, Introduction' can be used to model the coin flip.
2.) Consider the Bernoulli trials repeating until the first 'head'.
3.) In the Python language the code for the modulus or congruence operation is '%'. This can be used to determine if the 'head' was on an odd flip.
4.) Keep a record of the length of each run.
5.) Use an accumulator to calculate the total number of wins.
6.) Possibly there will be a need to reset (assign initial value) variables.
7.) The number of iterations of the outer loop is significant.

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Additional Instructions: Use the solution from the exercises to obtain the exact (theoretical) results and use the simulation to obtain the approximate results for the probability of heads: $p = \frac{1}{5}, \frac{1}{2}$, and $\frac{2}{3}$.

Rubric

Name and I.D. # Name of Assignment Submission Date	Not Satisfactory: Data requested absent.	Satisfactory: Has all data requested.
Exercise 1. Solves problem using geometric series approach.	Not Satisfactory: Missing steps. No general solution.	Satisfactory: Shows steps using geometric series to obtain general solution.
Exercise 2. Solves problem using total probability approach.	Not Satisfactory: Missing steps. No general solution.	Satisfactory: Shows steps using total probability to obtain general solution.
A block comment at beginning of program summarizing it. Line comments throughout the program explaining its operation.	Not Satisfactory: Conveying incomplete thoughts.	Satisfactory: A brief two or three sentence explanation at the beginning. Sufficient line comments explaining the code.
Identify lines in program that model the experiment intrinsic to the probability problem.	Not Satisfactory: Experiment not identified or blocked off.	Satisfactory: Experiment clearly identified.
List references.	Not Satisfactory: No references listed.	Satisfactory: References – assignment handout, internet, students, and etc. listed
Run the program with the three values of p listed and show the corresponding output.	Not Satisfactory: Not run for the values listed or no output.	Satisfactory: Run for three values of p listed with three output.
Submit solutions to exercises 1 & 2, the .py file of the program to dropbox and a **PDF of the program** with the output to dropbox.	Not Satisfactory: Absent solutions to exercises 1 & 2. Absent files. Absent output.	Satisfactory: Solutions to exercises 1 & 2, a copy of the Python file, the output all in the PDF and the .py file These two complete files are submitted to dropbox.