Chapter 8
Wave Field Synthesis

Methods of sound field synthesis aim at physically recreating a natural or any desired sound field in an extended listening area. As discussed in Sect. 5.1.2, sound field quantities to synthesize are mainly sound pressure and particle velocity or sound pressure gradients. If perfect control over these sound field quantities was achieved, virtual sources could be placed at any angle and distance and radiate a chosen source sound with any desired radiation pattern. This way, the shortcomings of conventional audio systems could be overcome: Instead of a sweet spot, an extended listening area would exist. Without the need for panning, lateral and elevated sources could be created and depth in terms of source distance could be implemented. If the sound radiation characteristics of an instrument was captured and synthesized, virtual sources could sound as broad and vivid as their physical counterpart. This means a natural, truly immersive, three-dimensional sound experience including the perception of source width, and motion, listener envelopment, reverberance and alike. Unfortunately, already the theoretic core of most sound field synthesis approaches imposes several restrictions. Current sound field synthesis implementations offer limited acoustic control under certain circumstances with several shortcomings. Still, due to elaborate adaptions of a sophisticated physical core sound field synthesis applications are able to create a realism which is unreachable with conventional audio systems.

In this chapter a short historic overview of sound field synthesis is given. The most prominent sound field synthesis approach includes the spatio-temporal synthesis of a wavefront that propagates through space as desired. In this book this specific approach is referred to as *wave field synthesis* (WFS) or *wave front synthesis* and is treated in detail in this chapter. The term *sound field synthesis* is used as umbrella terms covering several methods which aim at controlling a sound field. These methods include wave front synthesis, ambisonics and alike. The theoretic core of wave field synthesis is derived from several mathematical theorems and physical considerations.

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The derivation is explained step by step in the following section.[1] Several constraints make it applicable, as discussed in Sect. 8.3. These constraints lead to synthesis errors which are diminishable by adaptions of the mathematical core. Many sound field synthesis approaches model sound sources as monopole sources or plane waves. So a special treatment is given to the synthesis of the radiation characteristics of musical instruments in Sect. 8.4. Finally, some existing sound field synthesis installations for research and for entertainment are presented.

8.1 Sound Field Synthesis History

As already discussed in Sect. 7.2.2, methods of stereophonic recording and playback arose in the 1930s by Alan Dower Blumlein and others. At the same time Steinberg and Snow (1934a) conceptualized and implemented the *acoustic curtain* to capture and reproduce sound fields.[2] Authors like Ahrens (2012) and Friesecke (2007) consider this as the origin of wave field synthesis.[3] An acoustic curtain is depicted in Fig. 8.1. In principle, one plane wall of a recording room is covered by a mesh of microphones. These capture the auditory scene, e.g., a musical ensemble. In a performance room, one wall is covered by loudspeakers which are arranged in the exact same way as the microphones in the recording room. If now each recording is played back by the co-located loudspeaker, a copy of the auditory scene is created in the performance room. The constellation of the instruments is captured and reproduced this way. Listeners everywhere in the whole performance room have the impression that the instruments are actually there, arranged behind the loudspeaker curtain. Although they implemented the acoustic curtain with a pair or triplet of microphone-loudspeaker pairs only, they report to achieve a perceptually satisfying copy of the auditory scene. Later in this chapter we will see that an infinite acoustic curtain with infinitesimally spaced microphones and loudspeakers is necessary to capture and synthesize a sound field in a half space. This is exactly what the Rayleigh integral describes.

Another sound field recording and synthesis technique was developed in large part by Gerzon (1975) in the 1970s.[4] The sound pressure and the pressure gradients along the spatial dimensions are recorded at one receiver location by means of a microphone array. Two recording setups are illustrated in Fig. 8.2 for the two-dimensional case. These are equivalent if the microphones are proximate to one another compared to the considered wavelength. From these recordings, three channels W, X and Y can be derived as shown in the illustration. These contain the sound pressure W and the sound pressure gradients along two spatial dimensions X and Y. In a three-dimensional

[1] Mainly based on Pierce (2007), Williams (1999), Morse and Ingard (1986), Rabenstein et al. (2006), Ziemer (2018).

[2] See Steinberg and Snow (1934a, b).

[3] See Ahrens (2012), pp. 8f and Friesecke (2007), p. 147.

[4] See e.g. Gerzon (1973, 1975, 1981).

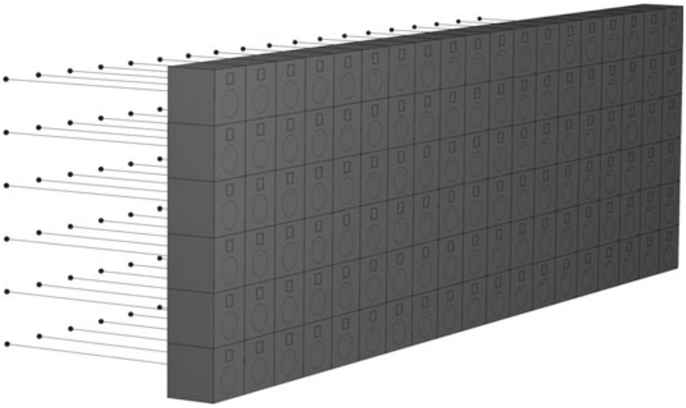

Fig. 8.1 Illustration of the acoustic curtain. After Ziemer (2016), p. 55

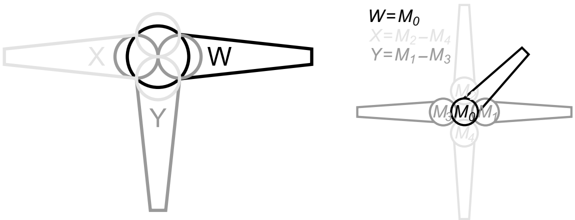

Fig. 8.2 Recording setups for first order ambisonics in two dimensions with different setups. After Ziemer (2017a), p. 315

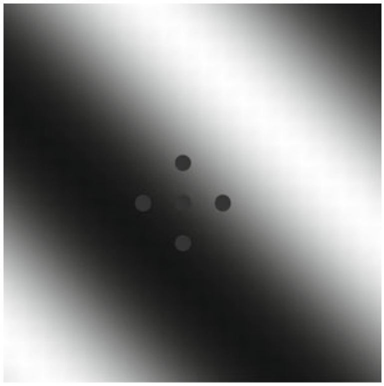

Fig. 8.3 Ambisonics microphone array in a sound field, after Ziemer (2017a), p. 316

setup an additional channel Z is encoded, containing the pressure gradient along the third dimension. Encoding these channels is referred to as *tetraphony* or *B-Format*. The sound pressure is a scalar and can be recorded by an omnidirectional pressure receiver. The pressure gradients can be recorded by figure-of-eight microphones or approximated by the difference of to opposing omnidirectional microphones which are out of phase. The recreation of the sound field described by these channels by means of a loudspeaker array is referred to as *periphony* or *ambisonics* (Fig. 8.3).

The omnidirectional component W is the spherical harmonic of the zeroth order $\Psi_0^0(\omega, \varphi, \vartheta)$ and X, Y and Z are the three spherical harmonics of the first order $\Psi_1^0(\omega, \varphi, \vartheta)$, $\Psi_0^1(\omega, \varphi, \vartheta)$ and $\Psi_1^1(\omega, \varphi, \vartheta)$ as discussed already in Sect. 5.3.1.1. So the microphone setup performs a spherical harmonic decomposition truncated at the first order. We can write these components in a vector Ψ and try to find loudspeaker signals $\mathbf{A}$ which recreate this encoded sound field. To achieve this, the sound propagation from each loudspeaker to the receiver position needs to be described by means of a matrix $\mathbf{K}$. Then, solving the linear equation system

$$\Psi = \mathbf{KA} \tag{8.1}$$

the loudspeaker signals $\mathbf{A}$ recreate the encoded sound field at the receiver position. The components in Ψ describe the desired sound field. The vector describes the sound pressure and at a central listening position and the pressure gradient along the spatial dimensions whose origin lies at this central position. The B-Format and higher order sound field encoding by means of circular or spherical harmonics are quasi-standardized. In contrast to conventional audio systems, as discussed throughout Chap. 7, the encoded channels are not routed directly to loudspeakers. Only the sound field information is stored. By solving Eq. 8.1 loudspeaker signals approximate the desired physical sound field or the desired sound impression. The solver is the ambisonics decoder. If the desired sound field contains as many values as loudspeakers present, the propagation matrix $\mathbf{K}$ in the linear equation system, Eq. 8.1, is a square matrix. In this case it can be solved directly. This can be achieved by means of an inverse matrix, or by numerical methods like Gaussian elimination. With more loudspeakers than target values the problem is under-determined: we have more known target values than unknown loudspeaker signals. In this case a pseudo inverse matrix can be used to approximate a solution. Unfortunately, this strategy comes along with several issues. First of all this approximate solution does not consider auditory perception. In the Moore Penrose inverse the Euclidean norm, i.e., the squared error, is minimized. This means that small errors in amplitude, phase, and time occur. These may be audible when they lie above the just noticeable difference of level or phase, or above just noticeable interaural level, phase or time difference.[5] The perceptual results of audible errors are a false source localization, especially for listeners that are not located at the central listening position. Other perceptual outcomes are audible coloration effects, spatial diffuseness, or an undesirably high loudness. A psychoacoustical solution to balance these errors would be desirable. Several ambisonics decoders have been proposed.[6] Psychoacoustic evaluation of existing decoders has been carried out.[7] However, psychoacoustic considerations should ideally be carried out already in the development process of the decoder. The radiation method suggested in this book is a physically motivated solution, and

[5]Details about auditory thresholds are discussed in Chaps. 4 and 6.

[6]An overview of ambisonic decoders can be found in Heller (2008). General solutions to inverse problems in acoustics are discussed in Bai et al. (2017).

[7]See e.g. Zotter et al. (2014) and Spors et al. (2013).

not perceptually motivated. But it comes along with a number of psychoacoustic considerations, like the *precedence fade*.[8]

Encoding and reconstructing spherical harmonics of higher order is referred to as *higher order ambisonics* (HOA). With higher order, the sound field is not only synthesized correctly at the very receiver location but in a receiver area which increases with increasing order and increasing wavelength. Originally, the transfer function **K** was modeled as plane waves emanating from the loudspeakers. As discussed in Sects. 5.1.4 and 5.1.6, plane waves are a good approximation of very distant sources. Later, loudspeakers were modeled as monopoles. Monopoles are certainly a better approximation of the actual radiation characteristics of proximate loudspeakers. When their distance to the receiver location has a magnitude of some meters, they do have a relevant amplitude decay and a wave front curvature. These are the main differences between a plane wave and a monopole. When an approximate solution is found, small numeric errors can result in large amplitudes, especially for low frequencies. This is the case because the wave front curvature becomes large compared to the wavelength and the amplitude decay is large compared to the encoded pressure gradient of a plane wave. As a result, **K** has a bad condition number and Eq. 8.1 is ill-conditioned. Compensating these near field effects, e.g., by adoptions of **K** is referred to as nearfield compensated higher order ambisonics (NFC-HOA).[9]

In many applications ambisonics loudspeaker setups are circular or hemispherical. As long as the location of the receiver array and each loudspeaker is known, the necessary loudspeaker signals can be calculated for virtually any constellation. Ideally, the loudspeakers are arranged with regular spacing. Although possible, it is not necessary to encode a sound field by means of a microphone array. One can also freely define or simulate a desired sound field and save it in the B-format. The first approach is referred to as *data based rendering* since it contains measured data. The second approach is called *model based rendering* because source location and sound propagation are calculated. Consequently, the encoded sound field depends largely on the sound propagation model that was used in the calculation. Often, sources are modeled as monopole sources or plane waves. However, models like the *complex point source model* are also conceivable.[10]

Around the late 1980s a wave field synthesis approach was derived, developed and implemented at the Delft University of Technology. The approach was termed *acoustic control* and later *wave front synthesis*.[11] In these works, a mathematical core of sound field synthesis is formulated and interpreted in physical terms. From this core, wave front synthesis, ambisonics and other sound field synthesis methods can be derived. A lot of research and development took place in Delft especially

[8]The phychoacoustic sound field synthesis approach including the radiation method and the precedence fade are introduced in Chap. 9.

[9]More information on HOA and NFC-HOA can be found e.g. in Ahrens and Spors (2008a), Williams (1999), pp. 267ff, Spors and Ahrens (2008), Daniel et al. (2003), Menzies and Al-Akaidi (2007), Daniel (2003) and Elen (2001).

[10]The complex point source model is described in Sect. 5.3.1, applied in Ziemer (2014) and discussed extensively in Ziemer and Bader (2015a), Zimmer (2015a, 2017a).

[11]See e.g. Berkhout (1988) and later Berkhout et al. (1992).

throughout the 1990s.[12] From 2001 to 2003 they were supported by a number of universities, research institutions and industry partners in the CARROUSO research project funded by the European Community.[13] Achievements from this project were market-ready wave front synthesis systems.

Since then, mainly adoptions, extensions, refinements of methods or error compensation[14] and additional features like moving sources and complicated radiation patterns[15] have been implemented. A lot of research is still carried out in the field of wave field synthesis. For example interfaces and techniques for more accessible creation of content and control of wave field synthesis systems are being developed.[16] Another topic is to reduce the number of necessary loudspeakers either by a prioritized sweet area within the extended listening area or by considering psychoacoustic effects.[17] Although sound field synthesis is originally a physically reasoned approach, psychoacoustic considerations are not superfluous. It is the auditory perception that makes sound field synthesis systems sound as desired even though physical synthesis errors are present and easily measurable. An elaborated perceptual evaluation of synthesized sound fields receives more and more attention in the literature.[18]

8.2 Theoretical Fundamentals of Sound Field Synthesis

The general idea of sound field synthesis can be traced back to Huygens' principle. This principle can be described by means of the Kirchhoff–Helmholtz-Integral which is explained in this section. Although often considered as the mathematical core of wave field synthesis, this integral is barely implemented in wave field synthesis. Instead, the Kirchhoff–Helmholtz-Integral is reduced to the Rayleigh-Integral which can be applied rather directly by means of an array with conventional loudspeakers. The adoption process from the mathematical idea to the actual implementation is explained in the subsequent section for wave front synthesis applications.

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[12] See e.g. papers like Berkhout et al. (1993), de Vries et al. (1994), de Vries (1996), Berkhout et al. (1997) and Boone et al. (1999) and dissertations like Vogel (1993), Start (1997) and Verheijen (1997).

[13] Publications are e.g. Corteel and Nicol (2003), Daniel et al. (2003), Spors et al. (2003), Vaananen (2003) and many more. More information on CARROUSO can be found in Brix et al. (2001).

[14] See e.g. Gauthier and Berry (2007), Menzies (2013), Spors (2007), Kim et al. (2009), Bleda et al. (2005).

[15] See e.g. Ahrens and Spors (2008b), Albrecht et al. (2005) and Corteel (2007).

[16] See Melchior (2010), Fohl (2013) and Grani et al. (2016).

[17] See e.g. Hahn et al. (2016) and Spors et al. (2011), Ziemer (2018) for more information on *local wave field synthesis*, and Chap. 9 and Ziemer and Bader (2015b, 2015c), Ziemer (2016) for details on *psychoacoustic sound field synthesis*.

[18] See e.g. Start (1997), Wierstorf (2014), Ahrens (2016), Wierstorf et al. (2013) and Spors et al. (2013).

8.2.1 Huygens' Principle

Every arbitrary radiation from a sound source can be described as integral of point sources on its surface. In addition, each point on a wave front can be considered as origin of an elementary wave. The superposition of the elementary waves' wavefronts creates the advanced wave front. This finding is called *Huygens' principle* and is the fundament on which wave field synthesis is based on.

Figure 8.4 illustrates the Huygens' principle. Figure 8.5 clarifies this illustration by reducing it to two dimensions and splitting it into states at different points in time. The black disk in Fig. 8.5a represents the source at t_0 which creates a wavefront that spreads out concentrically. This wavefront is illustrated in dark gray in Fig. 8.5b with some points on it. Each point on this wave front can be considered the origin of an elementary source, which again create a wave front, represented by the gray disks in Fig. 8.5c. Together, these wave fronts form the propagated wave front of the original source at a later point in time illustrated in Fig. 8.5d. The distance between those elementary waves has to be infinitesimally small. A monopole-shaped radiation of these elementary waves would create a second wave front at time t_2. This second wave front would be inside the earlier wave front, closer to the original breathing sphere again. This can clearly be seen in both Figs. 8.4 and 8.5c: One half of the elementary waves are located inside the dark gray wave front. This is physically untrue; the elementary waves must have a radiation characteristic which is 0 geared towards the source. This radiation characteristic is described by the Kirchhoff–Helmholtz integral (K-H integral), discussed in the subsequent Sect. 8.2.2.

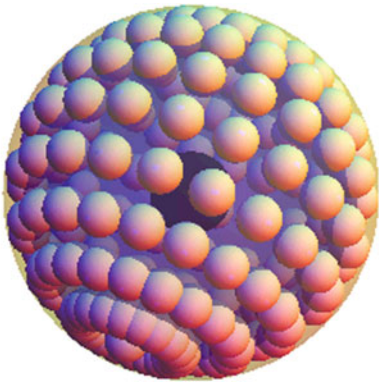
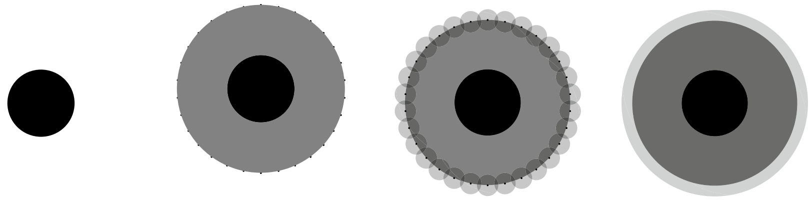


Fig. 8.4 Illustration of the Huygens' principle. Each point on a wavefront can be considered as the origin of an elementary wave. Together, the elementary waves create the propagated wavefront. From Ziemer (2016), p. 54



(a) t_0: breathing sphere (black). (b) t_1: elementary sources (black dots) on emanating wave front (gray). (c) t_2: wave fronts from elementary sources. (d) t_2: further emanated wave front from breathing sphere.

Fig. 8.5 Wave fronts of a breathing sphere at three points in time in 2D. The breathing sphere at t_0 (**a**) creates a wave front at t_1 (**b**). Points on this wave front can be considered as elementary sources which also create wave fronts at t_2 (**c**). By superposition these wave fronts equal the further emanated wave front of the breathing sphere (**d**). From Ziemer (2016), p. 55

8.2.2 *Kirchhoff–Helmholtz Integral*

The Gauss' theorem[19] states that spatial area integrals of a function over a volume V are equal to surface integrals of the normal components of a function over the volume's surface S

$$\int_V \nabla \mathbf{f}\, \mathrm{d}V = \int_S \mathbf{f}\mathbf{n}\, \mathrm{d}S \tag{8.2}$$

if it has a piecewise smooth boundary and the function $\mathbf{f}$ is a steady, differentiable vector function.[20] A special case of the Gauss' theorem is described by Green's second theorem[21]:

$$\int_V \mathbf{f}\nabla^2\mathbf{g} - \mathbf{g}\nabla^2\mathbf{f}\, \mathrm{d}V = \int_S \mathbf{f}\nabla\mathbf{g}\mathbf{n} - \mathbf{g}\nabla\mathbf{f}\mathbf{n}\, \mathrm{d}S \tag{8.3}$$

From Green's second theorem and the wave equations, Eqs. 5.4 and 5.16, the Kirchhoff–Helmholtz integral can be derived, which links the wave field of a source-free volume V with sources $\mathbf{Y}$ on its surface S:

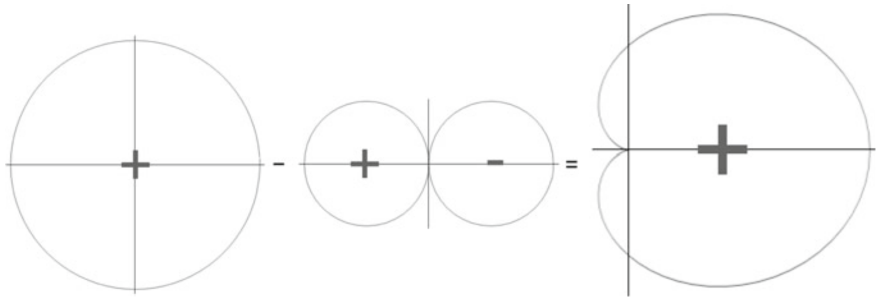
$$-\frac{1}{4\pi}\oiint_S \left(G(\omega, \Delta\mathbf{r}) \frac{\partial P(\omega, \mathbf{Y})}{\partial \mathbf{n}} - P(\omega, \mathbf{Y}) \frac{\partial G(\omega, \Delta\mathbf{r})}{\partial \mathbf{n}} \right) \mathrm{d}S = \begin{cases} P(\omega, \mathbf{X}), & \mathbf{r} \in V \\ \frac{1}{2} P(\omega, \mathbf{X}), & \mathbf{r} \in S \\ 0, & \mathbf{r} \notin V \end{cases} \tag{8.4}$$

Note that Eqs. 8.3 and 8.4 are nonlinear differential equations, which include the sought-after function and its derivative. The K-H integral states that the spectrum

[19] Also called "divergence theorem", see e.g. Pierce (2007), p. 58.

[20] See Merziger and Wirth (2006), p. 551.

[21] See Merziger and Wirth (2006), p. 555.



(a) Monopole source. *- (b) Dipole source.* *= (c) Cardioid.*

Fig. 8.6 Two dimensional illustration of superposition. Monopole- and dipole-source form a cardioid-shaped radiation. After Ziemer (2018), p. 335. From Ziemer (2016), p. 57

$P(\omega, \mathbf{X})$ at each point $\mathbf{X}$ in a source-free volume V is the integral of the spectra $P(\omega, \mathbf{Y})$ at every point $\mathbf{Y}$ on the bounding surface S and their propagation function $G(\omega, \Delta\mathbf{r})$ in the direction of the normal vector $\mathbf{n}$ pointing inwards. $G(\omega, \Delta\mathbf{r})$ is a Green's function, a solution of the inhomogeneous Helmholtz equation, Eq. 5.22, and $P(\omega, \mathbf{Y})$ is a spectrum, a solution for the homogeneous Helmholtz equation, Eq. 5.9. $\Delta\mathbf{r}$ is the Euclidean distance $||\mathbf{Y} - \mathbf{X}||_2$. The sources $\mathbf{Y}$ on the boundary surface are secondary sources, excited by primary sources $\mathbf{Q}$ which lie in the source volume U. The first term of the closed double contour integral describes a wave which propagates as monopole since the propagation term $G(\omega, \Delta\mathbf{r}) = \frac{e^{-\imath k \Delta\mathbf{r}}}{\Delta\mathbf{r}}$ is a monopole. From the periodic motion equation, Eq. 5.14, it emerges that $\frac{\partial P}{\partial n}$ is proportional to sound particle velocity in normal direction $\mathbf{V}_n$. The second term of the integral is a wave which radiates as dipole, since $\frac{\partial G(\omega,\Delta\mathbf{r})}{\partial\mathbf{n}} = \frac{1+\imath k\Delta\mathbf{r}}{\Delta\mathbf{r}^2}\cos(\varphi)\, e^{-\imath k\Delta\mathbf{r}}$ is a dipole term. Sound field quantities P and V are convertible into each other after Euler's equation of motion, Eq. 5.1, so the K-H integral is over-determined and several approaches to a solution exist.

As already stated, the secondary sources on the surface of the source-free medium are monopole- and dipole-sources. In phase, they add up and inversely phased they are 0. So the radiation can double inwardly by constructive interference and become 0 outwardly by destructive interference. Combined, they create a *cardioid*, also referred to as *kidney* or *heart*. It is illustrated in Fig. 8.6.

The boundary surface could be the wave front around a source and the source free volume could be the room beyond this wave front. Then, the K-H integral is a quantified formulation of the Huygens' principle. It is illustrated in the two-dimensional Fig. 8.7. In contrast to the earlier illustration of Huygens' principle, Figs. 8.4 and 8.5, this modified version does not create a false wavefront that propagates inwards. This is because the elementary sources on the wavefront are cardioids that face away from the origin of the wave. However, the K-H integral could also describe a wave that propagates inwards. In this case the cardioids would face the focus point of the wavefront. These examples illustrate that both pressure and pressure gradient on a

Fig. 8.7 Kirchhoff–Helmholtz integral describing Huygens' principle for an outward propagating wave. From Ziemer (2018), p. 334

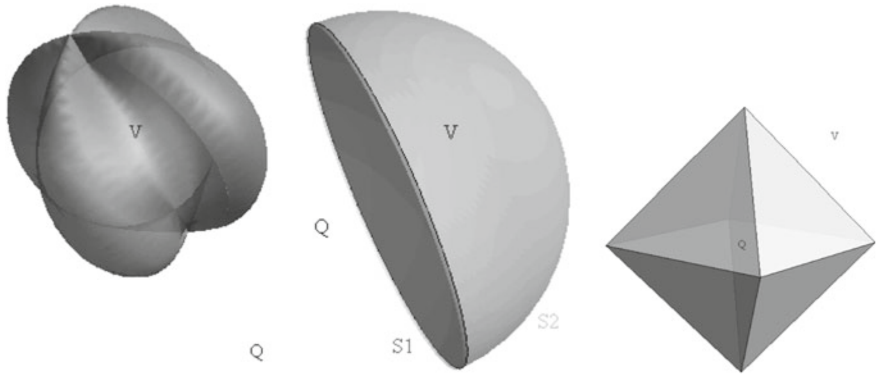
surface need to be known do describe the wave propagation direction. The Kichhoff–Helmholtz integral can describe wave fronts of monopole sources or plane waves as well as complex radiation patterns and diffuse sound fields with a random distribution of amplitudes, phases and sound propagation directions. In the illustrated example the elementary waves have different gray levels, indicating different complex amplitudes. So the amplitude and phase are different in any direction, as naturally observed in musical instruments, demonstrated, e.g., for the shakuhachi in Fig. 5.7, in Sect. 5.3.1.

The volume could also be any arbitrary other geometry. It could be the surface of a physically existing or non-existing boundary. This boundary is the separation surface between a source volume, which contains one or more sources, and a source-free volume, which contains the listening area. Any arbitrary closed boundary is conceivable as long as the premises of the Gauss' theorem are observed. Figure 8.8 illustrates three examples for a volume boundary, which will be regarded in later chapters. Two types of setups exist: Surrounding the listener with secondary sources—as in Fig. 8.8a and c—or surrounding the primary source(s), as illustrated in Fig. 8.8b.[22]

The Kirchhoff–Helmholtz integral describes analytically how spectrum and radiation on a volume surface are related to any arbitrary wave field inside a source-free volume. It is therefore the core of wave field synthesis.[23]

[22]Cf. Daniel et al. (2003), p. 3.

[23]See Berkhout et al. (1993), p. 2769.



(a) Arbitrary geometry. *(b) Hemisphere.* *(c) Octahedron.*

Fig. 8.8 Three volumes V with possible source positions Q. After Ziemer (2016), p. 58

8.3 Wave Field Synthesis

The Kirchhoff–Helmholtz integral is a theoretical construct which cannot simply be put into practice by technical means. It demands control of sound pressure and pressure gradients on a complete surface. That is a continuous distribution of an infinite number of secondary sources with infinitesimal distance, surrounding a volume entirely. Sound pressure and velocity need to be controllable everywhere on the volume surface, which is hardly possible by technical means. However, what we can control is the sound pressure of loudspeakers. But even with an infinite number of infinitesimally distanced loudspeakers completely separating a listening area from a source volume would be insufficient, as long as the pressure gradient cannot be controlled. So for a practical realization the reduction of secondary sources to a finite number of loudspeakers with discrete distances radiating approximately as monopoles or dipoles is feasible.[24] These have to be fed with the correct audio signal, often referred to as "driving function".[25] Surrounding an entire room with speakers is impracticable—as already mentioned in Sect. 4.4.1 and illustrated in Fig. 4.15—and requires enormous technical challenges, computational power, acquisition- and operating-costs. Therefore, concepts with plane arrays[26] and line arrays[27] of the speakers are proposed in the literature and commonly applied.

[24] See e.g. Spors et al. (2008).

[25] See e.g. Spors et al. (2008).

[26] See e.g. Oellers (2010).

[27] One line, see Gauthier and Berry (2007), Baalman (2008), Kolundzija et al. (2009a), Cho et al. (2010), Reisinger (2002, 2003) and Spors (2007), circular array, see Spors (2007), Rabenstein et al. (2006), Reisinger (2002, 2003) and Rabenstein and Spors (2008), and three to four lines surrounding the listening area, see Spors et al. (2003), Reisinger (2002, 2003), Rabenstein et al. (2006).

8.3.1 Constraints for Implementation

For implementing such Wave Field Synthesis (WFS) systems the K-H integral has to be adjusted to the restrictive circumstances, which leads to errors in the synthesis. A number of constraints simplify the K-H integral in a way which allows for a technical implementation of the theory by means of loudspeaker arrays[28]:

1. Reduction of the boundary surface to a separation plane between source-free volume and source volume
2. Restriction to one type of radiator (monopole or dipole)
3. Reduction of three-dimensional synthesis to two dimensions
4. Discretization of the surface
5. Introduction of a spatial border

The particular steps will be successively accomplished in the following subsections.

8.3.2 Rayleigh-Integrals

Imagine a volume V consisting of a circular plane $S1$ closing a hemisphere $S2$, as illustrated in Fig. 8.8b, whose radius converges to ∞. The influence of the radiation from the source on $S2$ becomes 0 for the area in front of $S1$. This coherence satisfies the *Sommerfeld condition*. It remains a separating plane between source free volume and source volume. The K-H integral then consists of an integral over the plane $S1$ and thus fulfills the first simplification criterion from Sect. 8.3.1:

$$-\frac{1}{4\pi}\iint\limits_{S1}\left(G\left(\omega,\Delta\mathbf{r}\right)\frac{\partial P\left(\omega,\mathbf{Y}\right)}{\partial\mathbf{n}}-P\left(\omega,\mathbf{Y}\right)\frac{G\left(\omega,\Delta\mathbf{r}\right)}{\partial\mathbf{n}}\right)\mathrm{d}S=\begin{cases}P\left(\omega,\mathbf{X}\right), & \mathbf{X}\in V\\ 0, & \mathbf{X}\notin V\end{cases} \tag{8.5}$$

This step reduces the area of secondary sources from a three-dimensional surrounding of a source-free volume to a separation plane.

Since the Green's function, Eq. 5.22, is a linear combination of a special solution and a general solution, one term of the integral can be eliminated by adding a deftly chosen general solution to the free-field Green's function. So the radiation can be restricted to one type of radiator. If the Green's function is chosen to be

$$G_D\left(\omega,\Delta\mathbf{r}\right)=\frac{e^{-\imath k\Delta\mathbf{r}}}{\Delta\mathbf{r}}+\frac{e^{-\imath k\Delta\mathbf{r}'}}{\Delta\mathbf{r}'}, \tag{8.6}$$

$G_D\left(\omega,\Delta\mathbf{r}\right)$ is 0 on the surface S—which satisfies the so-called *homogeneous Dirichlet boundary condition*[29]—and the second term vanishes if $\Delta\mathbf{r}'$ is the mirrored position of $\mathbf{X}$, mirrored at the tangent of point $\mathbf{Y}$ on S. This implicitly models the boundary

[28] These or similar simplifications are also proposed by Rabenstein et al. (2006), p. 529.

[29] See e.g. Burns (1992).

as a rigid surface,[30] leading to the *Rayleigh I integral* for secondary monopole sources as already introduced in Eq. 5.29 in 5.3.3:

$$P(\omega, \mathbf{X}) = -\frac{1}{2\pi} \iint_{S1} \left(G_D(\omega, \Delta\mathbf{r}) \frac{\partial P(\omega, \mathbf{Y})}{\partial \mathbf{n}} \right) \mathrm{d}S. \tag{8.7}$$

Now, considering $\frac{\partial P(\omega,\mathbf{Y})}{\partial \mathbf{n}}$ the desired source signal, an explicit solution can be found e.g. by means of wave field expansion. This approach is called "simple source approach" and is the basis of some sound field reconstruction methods such as HOA.

Since the distance $|\Delta\mathbf{r}|$ between secondary source position $\mathbf{Y}$ and considered position in the source-free volume $\mathbf{X}$ equals the distance between the secondary source position and the mirror position $\left|\Delta\mathbf{r}'\right|$, $G_D(\omega, \Delta\mathbf{r})$ is nothing but a doubling of the free-field Green's function $G(\omega, \Delta\mathbf{r})$:

$$G_D(\omega, \Delta\mathbf{r}) = 2G(\omega, \Delta\mathbf{r}) \tag{8.8}$$

Assuming $\frac{G_N(\omega,\Delta\mathbf{r})}{\partial n}$ to be 0 satisfy the *homogeneous Neumann boundary condition*[31] and the first term of Eq. 8.5 vanishes. This is accomplished by choosing

$$G_N(\omega, \Delta\mathbf{r}) = \frac{e^{-\imath k \Delta\mathbf{r}}}{\Delta\mathbf{r}} - \frac{e^{-\imath k \Delta\mathbf{r}'}}{\Delta\mathbf{r}'}, \tag{8.9}$$

yielding the *Rayleigh II integral* for secondary dipole sources:

$$P(\omega, \mathbf{X}) = -\frac{1}{2\pi} \iint_{S1} \left(P(\omega, \mathbf{Y}) \frac{\partial G(\omega, \Delta\mathbf{r})}{\partial \mathbf{n}} \right) \mathrm{d}S. \tag{8.10}$$

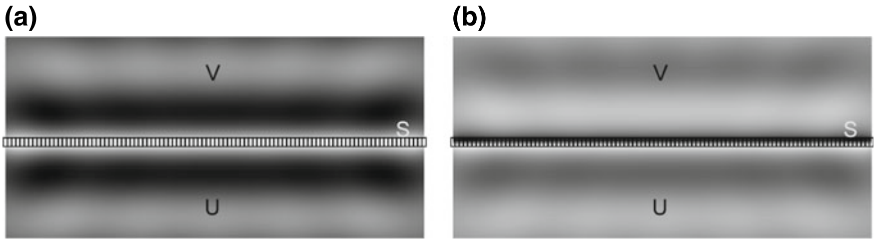
In both cases the second simplification criterion from Sect. 8.3.1 is satisfied. But since the destructive interference outside the source-free volume is missing, $P(\omega, \mathbf{X})$ for $\mathbf{X} \notin V$ is not 0. A mirrored sound field in the source volume is the consequence. In case of monopoles the sound field created by the secondary sources is identical with the one inside the source-free volume. This effect is similar to the earlier illustration of Huygens' principle, Figs. 8.4 and 8.5. In case of dipole sources the phase in the source volume is the inverse of the phase inside the source-free volume. Additionally, the sound pressure or, respectively the particle velocity, duplicate by adding the general solution of the Green's function. Both cases are illustrated in Fig. 8.9 for a one-dimensional loudspeaker array.

Both formulations do not apply for arbitrary volume surfaces but for separation planes only.[32] To ensure that any position around the listening area can be a source position, the listening area has to be surrounded by several separation planes. If Eqs.

[30] See Spors et al. (2008), p. 4 and Baalman (2008), p. 27.

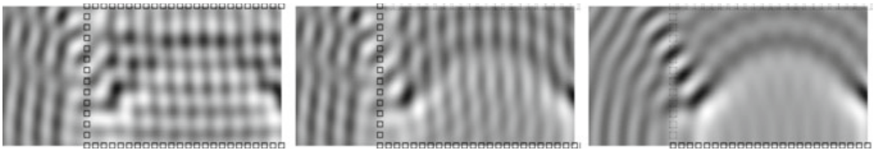
[31] See e.g. Burns (1992).

[32] See Spors et al. (2008), p. 5.



Monopole loudspeakers. *Dipole loudspeakers.*

Fig. 8.9 Desired sound field above and mirrored sound field below a separation plane according to the Rayleigh I integral for secondary monopole sources (**a**) and the Rayleigh II integral for secondary dipole sources (**b**). After Ziemer (2018), pp. 337 and 338



(a) Three active line arrays. *(b) Two active line arrays.* *(c) One active line array.*

Fig. 8.10 Illustration of the spatial windowing effect: A circular wave front superimposes with virtual reflections from two (**a**) or one (**b**) additional loudspeaker array(s). When muting those loudspeakers whose normal direction deviates from the local wave front propagation direction by more than 90° (**c**), the synthesized wave front is much clearer. Here, the remaining synthesis error is a truncation error, resulting from the finite length of the loudspeaker array. After Ziemer (2018), p. 338

8.7 and 8.10 are applied to other geometries, they still deliver approximate results.[33] In any case, the source-free volume has to be convex so that no mirrored sound field lies inside the source-free volume, i.e. volume (a) in Fig. 8.8 is inappropriate.[34] Since $S1$ is implicitly modelled as a rigid surface, several reflections occur when a listening area is surrounded by several separation planes. These unwanted reflections emerge from speakers whose positive contribution to the wave front synthesis lies outside the listening area. The portion of sound that propagates into the listening area does not coincide with the synthesized wave propagation direction. This artifact can be reduced by spatial "windowing"[35] technique applied to the Rayleigh I integral:

$$
\begin{aligned}
P(\omega, \mathbf{X}) &= d(\mathbf{Y}) \frac{P(\omega, \mathbf{Y})}{\partial \mathbf{n}} 2G(\omega, \mathbf{Y}) \\
d(\mathbf{Y}) &= \begin{cases} 1, & \text{if } \langle \mathbf{Y} - \mathbf{Q}, n(\mathbf{Y}) \rangle > 0 \\ 0, & \text{otherwise} \end{cases}
\end{aligned}
\tag{8.11}
$$

[33] See Spors et al. (2008), p. 5.

[34] See Spors and Ahrens (2008), pp. 4f.

[35] See de Vries et al. (1994), Spors et al. (2008), p. 5 and Gauthier and Berry (2007), p. 3.

Here, $d(\mathbf{Y})$ is the windowing function for spherical waves which is 1 if the local propagation direction of the sound of the virtual source at the position of the secondary source has a positive component in normal direction of the secondary source. If the deviation is $\frac{\pi}{2}$ or more, $d(\mathbf{Y})$ becomes 0 and the speaker is muted. That means only those loudspeakers whose normal component resembles the tangent of the wave front of the virtual source are active. The term $G(\omega, \Delta\mathbf{r})$ describes the directivity function of the secondary source, i.e. of each loudspeaker. The other terms are the sought-after driving functions D of the loudspeakers[36]:

$$D(\omega, \mathbf{Y}) = 2d(\mathbf{Y}) \frac{P(\omega, \mathbf{Y})}{\partial \mathbf{n}} \tag{8.12}$$

An example for the unwanted virtual reflections due to applying the Rayleigh integral although surrounding the listening area from three sides is given in Fig. 8.10. The same wave front is synthesized according to the Rayleigh integral in three ways. In (a), three linear loudspeaker arrays are active. Here, the desired wave front superimposes with virtual reflections from the two additional arrays. In (b), one loudspeaker line array is muted. The contribution of these loudspeakers to the wave front synthesis would lie above the figure, i.e. outside the listening area. Muting them does not decrease synthesis precision in the listening area. In (c), the second line array is muted. Now one can clearly see the desired wave front curvature. No virtual reflections are visible. The remaining sound field synthesis error is the so-called *truncation error*. It will be discussed in detail in Sect. 8.3.3.

Although considered as source- and obstacle-free field, it is to a certain extent possible to recreate the wave field of a virtual source within the source-free volume. This is achieved by assuming an inverse propagation and calculating a concave wave front at the surface which focuses at the position of the virtual source and creates a convex wave front from then on. These sources are called "focused sources".[37] Figure 8.10 already exemplifies a focused source. More examples will be given throughout the chapter. Of course, focused sources will not work for listeners between the active loudspeakers and the focus. For them, the wave front seems to arrive somewhere from loudspeaker array and not from the focus. In contrast, listeners behind the focus do not experience the concave wavefront. They simply hear the convex wave front which seems to originate in the focus point. So focused sources reduce the extent of the listening area.

8.3.2.1 Two Dimensions

For applications in which the audience is organized more or less in plane, it is sufficient to recreate the wave field correctly for that listening plane only, rather than in

[36] See Spors et al. (2008), p. 5.

[37] The derivation of the secondary source signals and further information on these sources can be found e.g. in Kim et al. (2009), Geier et al. (2010), Ahrens and Spors (2009).

the whole listening volume. Furthermore, the best source localization resolution of the human auditory system is in the horizontal plane as discussed in Sect. 4.4. This is the main reason why conventional audio systems mostly focused on horizontal audio setups, as presented in Chap. 7. Luckily, when listening to music, listeners are often organized roughly in plane, like in many concert halls, opera houses, cinemas, theaters, in the car, on the couch in the living room etc. Furthermore, one or several one-dimensional distributions of loudspeakers are easier implementable than covering a complete room surface with loudspeakers. Reducing the three-dimensional wave field synthesis to two dimensions reduces the separation plane $S1$ to a separation line $L1$. In theory, one could simply reduce the surface integral to a simple integral and the Rayleigh integrals would take the forms

$$P\left(\omega, \mathbf{X}\right) = \frac{1}{2\pi} \int_{L1} \left(G\left(\omega, \Delta\mathbf{r}\right) \frac{\partial P\left(\omega, \mathbf{Y}\right)}{\partial \mathbf{n}} \right) \mathrm{d}S1 \tag{8.13}$$

and

$$P\left(\omega, \mathbf{X}\right) = \frac{1}{2\pi} \int_{L1} \left(P\left(\omega, \mathbf{Y}\right) \frac{\partial G\left(\omega, \Delta\mathbf{r}\right)}{\partial \mathbf{n}} \right) \mathrm{d}S1. \tag{8.14}$$

In these cases $\mathbf{X}$ is two-dimensional

$$\mathbf{X} = \begin{bmatrix} x \\ y \end{bmatrix}. \tag{8.15}$$

This solution was satisfying if no third dimension existed, e.g. if wave fronts of the secondary sources had no spherical but a circular or cylindrical propagation.[38] Then, the propagation function $G\left(\omega, \Delta\mathbf{r}\right)$ was different, having an amplitude decay of $\frac{1}{\sqrt{r}}$ instead of $\frac{1}{r}$. This is owed to the fact that the surface S of a circle or cylinder doubles with a doubled circle radius r_{circle}

$$S = 2\pi r_{\text{circle}} \tag{8.16}$$

in contrast to the spherical case in which it squares with the doubled radius as already indicated in Eq. 5.24 in Sect. 5.1.6. In this case

$$I \propto \frac{1}{r} \tag{8.17}$$

and thus

$$p \propto \frac{1}{\sqrt{r}}\,. \tag{8.18}$$

[38]See e.g. Spors et al. (2008) pp. 8f, Rabenstein et al. (2006), pp. 521ff.

So the practical benefit of 8.13 and 8.14 is minor since transducers with a cylindrical radiation in the far field are hardly available.[39] An approximately cylindrical radiation could be achieved with line arrays of loudspeakers.[40] But replacing each individual loudspeaker by a line array of speakers contradicts our goal to reduce the number of loudspeakers. Simply replacing cylindrically radiating speakers by conventional loudspeakers which have a spherical radiation function leads to errors in this wave field synthesis formulation due to the deviant amplitude decay.

The Huygens' principle states that a wave front can be considered as consisting of infinitesimally distanced elementary sources. An infinite planar arrangement of elementary point sources with a spherical radiation could (re-)construct a plane wave, since the amplitude decay which is owed to the 1/r-distance law is compensated by the contribution of the other sources. Imagining secondary line sources with a cylindrical radiation, linear arrangement of sources would be sufficient to create a planar wave front. In a linear arrangement of elementary point sources, the contribution of the sources from the second dimension is missing, resulting in an amplitude decay. Therefore, a "2.5D-operator" including a "far field approximation" which modifies the free-field Green's function to approximate a cylindrical propagation is used.[41] This changes the driving function to

$$D_{2.5\mathrm{D}}\left(\omega, \mathbf{Y}\right) = \sqrt{\frac{2\pi\ \left|\mathbf{Y} - \mathbf{X}_{\mathrm{ref}}\right|}{\imath k}} D\left(\omega, \mathbf{Y}\right) \tag{8.19}$$

with $\mathbf{X}_{\mathrm{ref}}$ being a reference point in the source-free volume. This yields the "2.5-Dimensional" Rayleigh integral[42]:

$$P\left(\omega, \mathbf{X}\right) = -\int_{-\infty}^{\infty} D_{2.5D}\left(\omega, \mathbf{Y}\right) G\left(\omega, \Delta\mathbf{r}\right) \tag{8.20}$$

Taking reference points $\mathbf{X}_{\mathrm{ref}}$ parallel to the loudspeaker array, the wave field can be synthesized correctly along a reference line. Between the speakers and the reference line, the sound pressures are too high, behind it they are too low.

Until now, free-field conditions are assumed. However, if not installed in the free field, reflections may occur and superimpose with the intended wave field created by the loudspeaker system. Under the term "listening room compensation" a variety of methods are proposed to reduce the influence of reflections. The simplest form is passive listening room compensation which means that the room is heavily damped. This is an approved method, applied e.g. in cinemas. However, for some listening rooms, for example living rooms, damping is impractical. Therefore, active solutions are proposed, like adding a filtering function which eliminates the first reflections of

[39] Cf. Spors and Ahrens (2008), p. 6 and Goertz (2008), p. 444.

[40] As often applied in PA systems for concerts, see e.g. Friedrich (2008), pp. 316ff.

[41] See e.g. Spors et al. (2008), pp. 9f or Wittek (2007), p. 58.

[42] See Spors et al. (2008), p. 11, Baalman (2008), pp. 28–46 and Verheijen (1997), pp. 37–49 and pp. 153–156. The derivation of the 2.5D-operator is given in Ahrens (2012), pp. 288f.

the room to the calculated loudspeaker signals.[43] "Adaptive wave field synthesis"[44] uses error sensors which measure errors occurring during WFS of a test stimulus emerging e.g. from reflections. Then any WFS solution is modified by a regularization factor which minimizes the squared error. This is of course a vicious circle since compensation signals corrupt the synthesized wave field and are reflected, too, adding further errors. This problem is related to the error compensation of head-related audio systems. Due to an exponentially increasing reflection density it is hardly possible to account for all higher order reflections. Thus, the approach is limited to first order reflections.

8.3.2.2 Discretization

A discretization of the Rayleigh integrals adopts the continuous formulation to discrete secondary source positions:

$$P(\omega, \mathbf{X}) = \frac{1}{2\pi} \sum_{\mathbf{r}_Y=-\infty}^{\infty} \left(G(\omega, \Delta\mathbf{r}) \frac{\partial P(\omega, \mathbf{Y})}{\partial \mathbf{n}} \right) \Delta\mathbf{r}_Y \tag{8.21}$$

and

$$P(\omega, \mathbf{X}) = \frac{1}{2\pi} \sum_{\mathbf{r}_Y=-\infty}^{\infty} \left(P(\omega, \mathbf{Y}) \frac{\partial G(\omega, \Delta\mathbf{r})}{\partial \mathbf{n}} \right) \Delta\mathbf{r}_Y \tag{8.22}$$

Thereby the Nyquist–Shannon sampling theorem has to be regarded: The sampling frequency has to be at least twice the highest frequency of the signal to be presented for no aliasing to occur. The highest frequency to be represented error-free is the *critical frequency* or *aliasing frequency*. In this case the sampling frequency is spatial; the speaker distance $\Delta\mathbf{Y}$ has to be maximally half the distance of the largest presentable wavelength

$$f_{\max} = \frac{c}{2\Delta\mathbf{Y}} \tag{8.23}$$

between the speakers. The spatial sampling of the secondary source distribution is a process of sampling and interpolation; the interpolator is given by the radiation characteristics of the loudspeakers.[45] For the trace wavelength between the speakers

$$\lambda_{\Delta\mathbf{Y}} = \lambda\,|\sin\alpha| \tag{8.24}$$

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[43] See Horbach et al. (1999), Corteel and Nicol (2003), Spors et al. (2003, 2004, pp. 333–337, 2007b).

[44] See Gauthier and Berry (2007).

[45] See Spors (2008), p. 1. An adaption of WFS to the radiation characteristic of the loudspeakers is derived in de Vries (1996).

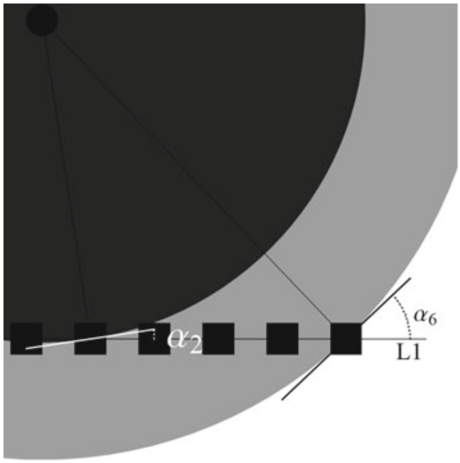


Fig. 8.11 Several incidence angles for one source position. From Ziemer (2016), p. 68

is valid, where α is the angle between the normal direction of a loudspeaker and the wave when striking this loudspeaker. Respectively, it can be considered as angle between separation line $L1$ and the tangent of the wave front when striking the speaker position. This leads to an adjustment of Eq. 8.23 to

$$f_{\max} = \frac{c}{2\Delta\mathbf{Y}\sin\alpha}. \tag{8.25}$$

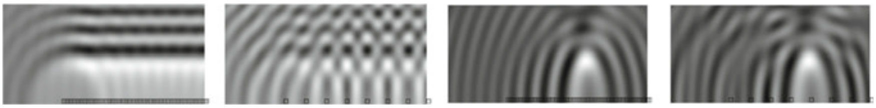
The angle α may vary depending on position and radiation of the source in a range between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. Two examples for α are illustrated in Fig. 8.11. to clarify the coherency. The black disk represents the source, the dark and light gray disks the wave front at two different points in time, just as in Figs. 8.4 and 8.5 in Sect. 8.2.1.

Undersampling creates erroneous wavefronts above $f_{\max}$. These erroneous wavefronts contain the frequencies above the critical frequency, cause perceivable changes in sound color and disturb the localization of the virtual source.[46] Two examples of spatial aliasing are illustrated in Fig. 8.12. These illustrations contain an additional error due to the finite number of loudspeakers. It is called truncation error and will be discussed in detail in the subsequent subsection. Aliasing wave fronts create a spatial comb filter effects which colors stationary signals and smear transients. They can be heard as high-frequency echoes following the desired wave front. In the case of focused sources, they create high-frequency pre-echoes preceding the desired wave front. As long as the condition

$$|\sin\alpha\,(\omega)| < \frac{c}{2\Delta\mathbf{Y}\,f_{\max}} = \frac{\pi c}{\Delta\mathbf{Y}\;\omega_{\max}} \tag{8.26}$$

is satisfied no aliasing wavefronts will occur.

[46] See Spors et al. (2008), p. 14 and Daniel et al. (2003), p. 15.



(a) Plane wave without aliasing. *(b) Plane wave with aliasing.* *(c) Focused source without aliasing.* *(d) Focused source with aliasing.*

Fig. 8.12 Virtual sources with (**b** and **d**) and without (**a** and **c**) aliasing. Erroneous wave fronts superimpose with the desired wave fronts. All synthesized wave fronts exhibit a truncation error which has to be compensated. After Ziemer (2016), p. 69

One suggestion to reduce artifacts is to process frequencies above the critical frequency not by means of WFS but by conventional stereophonic sound between two to three loudspeakers. This method is called "Optimized Phantom Source Imaging" (OPSI)[47] and combines WFS with conventional amplitude based panning. Thus, OPSI reintroduces psychoacoustic considerations to WFS. In that manner no aliasing echoes as such occur but the common disadvantages of stereophonic sound become effective: A comb filter effect arises, the display of depth becomes worse and high frequencies are only located correctly on the sweet spot. At other positions two to three wave fronts arrive slightly shifted in time. Also the radiation characteristic of the instrument cannot be displayed. López et al. (2005) suggest a related approach, called "sub-band approach", playing frequencies above the aliasing frequency through the one loudspeaker with the most similar direction to the virtual source only.[48] This approach does not bring along the disadvantages of stereo but still a more or less correct localization is only possible in a small part of the listening area. Furthermore, the presentation of the complicated radiation characteristic of high frequencies is not possible. By randomizing the phase of the high frequencies of the loudspeakers artifacts are smeared.[49] An example is illustrated in Fig. 8.13. Due to aliasing, undesired spatially regular amplitude fluctuations occur. By phase randomization the distribution of errors becomes irregular. This reduces the sound coloration but the synthesized wave front still deviates strongly from the desired spherical or plane wave front and the localization accuracy is reduced strongly.[50] A reconstruction of the radiation characteristic of musical instruments is impossible with these methods.

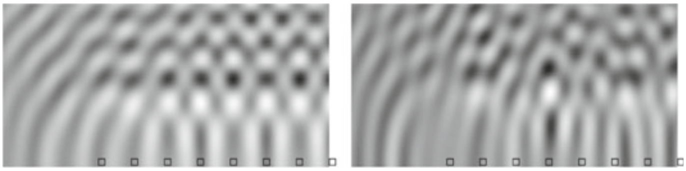
In all three cases the signal is divided by the critical frequency into two frequency regions. For the lower frequency region the theory of WFS is applied. Frequencies in the higher region are not processed to recreate an original wave field but to obtain the natural temporal and spectral properties as well as an approximately correct source position. The methods are based on the same psychoacoustic considerations: Partials of a sound tend to fuse. Higher frequencies tend to be masked by lower frequencies. Altogether the audible portion of sound will be integrated into one auditory stream

[47] See Spors et al. (2008), p. 15, Wittek (2007), pp. 96–105, Reisinger (2002), pp. 42ff, Huber (2002), pp. 20–54.

[48] See López et al. (2005).

[49] See Spors et al. (2008), p. 17.

[50] See Wittek (2007), p. 88.



(a) Regular amplitude fluctuations due to spatial aliasing. *(b) Irregular aliasing error due to phase randomization.*

Fig. 8.13 Above the critical frequency, regular amplitude errors occur (**a**). By phase randomization (**b**) the amplitude and phase distribution becomes irregular. After Ziemer (2018), pp. 340 and 341

with one group source position. Then, the lower frequency region—which offers very precise localization cues due to the correct reconstruction of the wavefield—is crucial for a distinct and correct localization and the wrong localization cues of higher frequencies are neglected by the auditory system. All these psychoacoustic phenomenons have been illuminated already in Chap. 4, especially Sects. 4.3 and 4.5.

Of course, these methods work best if the chosen distance between adjacent speakers is so small that the aliasing frequency is as high as possible. Then it can even be speculated that the influence of the frequencies above the critical frequency is weak concerning sound coloration and localization. Spors et al. (2008) confirm this assumption:

> However, the human auditory system seems to be not too sensible to spatial aliasing if the loudspeaker spacing is chosen in the range $\Delta x = 10 \ldots 30$ cm.[51]

Quite a different method is to recreate the wave field not for the discrete loudspeaker positions but for discrete listening positions sampling the listening area. The approach is called "sound field reconstruction" or "sound field reproduction" applying least-squares solution.[52] Sampling positions are chosen under the assumption that if a wave field is reproduced correctly on a grid satisfying the Nyquist–Shannon sampling theorem, the wave field is correct everywhere inside the grid. This approach can be combined with crosstalk cancellation—as discussed in Chap. 7—to create a realistic binaural signal at discrete listening positions.[53]

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[51] Spors et al. (2008), p. 17. Note that Spors et al. (2008) name the speaker positions "x", in this book they are called $\mathbf{Y}$.

[52] Cf. Kolundzija et al. (2009b) and Kirkeby and Nelson (1993).

[53] Proposed and implemented by Menzel et al. (2006).

8.3.3 Spatial Border

A constraint of the discrete Rayleigh integrals, Eqs. 8.21 and 8.22, to a finite number of speaker positions is the 5th simplification of the list in Sect. 8.3.1. This creates two borders from which the created wave front curvatures fade to the wave front of the speaker itself. This effect is called "truncation".[54] It appears like diffraction through a gap and has the effect that the wave field cannot be synthesized in the area beyond the border. Furthermore, a more or less spherical wave front propagates from the border originated in the last speaker,[55] since the compensatory effect of adjacent speakers is missing. The truncation effect can be compensated by reducing the amplitudes of the outermost speakers. This does, however, slightly reduce the listening area extent. Figure 8.14 shows this artifact and its correction by applying a half cosine filter at the left end of the loudspeaker array. This gradual amplitude attenuation is referred to as *tapering*. It can be seen that, due to tapering, the amplitude of the virtual wavefront decays towards the outer positions in the listening area. The truncation error in generally weaker in corners, where two line array meet. An example is illustrated in Fig. 8.15. Compensation sources can compensate truncation by using speakers with antiphased signals at the array ends.

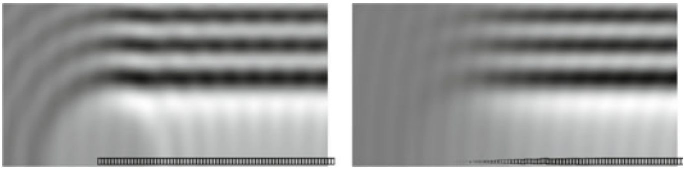
8.3.4 Listening Room

Until now, we assumed a free field and added loudspeaker arrays as secondary sources. So even in a highly damped room or outdoors, the assumption of a free field barely holds. This is especially true if we have actual listeners. Luckily, loudspeakers and listeners cause similar absorption, reflections and diffraction, no matter whether the impinging wave front is natural or synthesized. So the presence of loudspeakers and listeners does not seem to corrupt the wave front synthesis system. But this might change if we consider focused sources. Listeners between the focus point and the loudspeakers not only have trouble localizing the virtual focused source. They also corrupt the concave wave front and, as a consequence, the synthesized wave front is erroneous. This effect seems to be weak, as to the author's knowledge the effect is not addressed in the literature. Probably because the human body barely affects low frequencies. These deflect perfectly round the listener. However, higher frequencies may not deflect perfectly around the listener and create an undesired wave shadow. Even high frequencies are largely absorbed, but the wave fronts of high frequencies are erroneous, anyway, due to spatial aliasing. Furthermore, many wave front synthesis systems are installed slightly above the audience. Consequently, listeners are barely in the direct path between the loudspeakers and other listeners.

Much more critical is the presence of physical borders, i.e., room walls, floor and ceiling. Reflections from these surfaces superimpose with the desired wavefront. If

[54] See Start (1997), pp. 47ff, Verheijen (1997), pp. 50ff and Baalman (2008), pp. 37ff.

[55] See Spors et al. (2008), p. 14.



(a) Truncation effect in the case of a planewave.

(b) Reduction of the truncation effect by applying a cosine filter.

Fig. 8.14 Truncation effect of a virtual plane wave (**a**) and its compensation by applying a cosine filter (**b**). The spherical truncation wave emanating from the left end of the loudspeaker array is eliminated. The remaining error occurs from the untapered right end of the array. After Ziemer (2016), p. 71

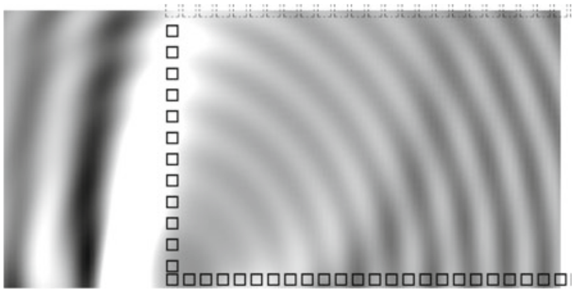
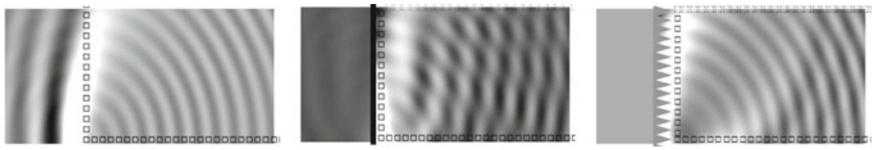


Fig. 8.15 A virtual point source in the corner. When two linear loudspeaker arrays meet, the truncation error is weak. After Ziemer (2018), p. 343

the wavefronts of direct sound was synthesized correctly, the room acoustics would sound perfectly natural. But as discussed throughout this chapter, this is not the case. First of all, the sound field is typically synthesized in one plane only. And even in this plane, the amplitude decay is too strong, aliasing errors occur and at the ends of the loudspeaker array synthesis errors are produced, may it be due to truncation or due to tapering. Outside this plane, wave fronts are not controlled at all and deviate from natural wave fronts. It follows, that especially reflections from floor and ceiling are unnatural. Synthesizing not only direct sound but additional room acoustics is difficult. These would always superimpose with the reverberation of the listening room. An example of the same synthesized wave front in a free field, in presence of a highly reflective wall, and a highly absorbing wall is illustrated in Fig. 8.16.

Damping the listening room is probably the easiest way to avoid undesired reflections. The downside is that it makes wave field synthesis systems even less flexible. The high number of loudspeakers already affects the interior of the room and so the installation of additional absorbers may be difficult and undesired. Therefore, technical solutions have been developed. Reducing the undesired reverberation of the room by technical means is referred to as *active listening room compensation*.



(a) Synthe sized wave front in a free field. *(b) Synthe sized wave front super imposed with wall reflection.* *(c) Synthe sized wave front with highly absorbing wall.*

Fig. 8.16 Wave field in a free field (**a**), in presence of a reflective wall (**b**) and highly absorbing wall (**c**). After Ziemer (2018), p. 343

Unfortunately, this can become a vicious circle, as in cross talk cancellation, discussed in Sect. 7.2.7. Details of active listening room compensation are out of scope of this book but can be found in the literature.[56]

8.4 Sound Field Synthesis and Radiation Characteristics

Radiation synthesis of musical sound is a research topic which receives growing interest.[57] Synthesizing the wave field as propagating from a source with a complex radiation pattern is demanding in many ways. One approach to recreate the natural sound radiation of musical instruments is to use an array of densely spaced loudspeakers. This approach is called "sound radiation synthesis".[58] The idea has been derived and applied by several researchers.[59] They use platonic solids, other regular polyhedrons or a spherical loudspeaker arrangement, as illustrated in Fig. 8.17, to create a high number of spherical harmonics. This approach could be considered as *spherical harmonic composition*, the inverse operation of spherical harmonic decomposition. In the literature it is sometimes referred to as "spatial additive synthesis".[60] The more loudspeakers are used the more complex radiation patterns can be synthesized. Assuming each loudspeaker to radiate as a monopole, the application is straightforward. However, implementing the actual radiation pattern of each loudspeaker and considering diffraction is challenging. An overview of methods is given in Zotter (2009).[61] From auralizations in virtual physical models and from impulse response measurements in actual rooms it has been found that the directivity of musi-

[56]The reader can refer e.g. to Spors et al. (2003, 2007a, b), Corteel and Nicol (2003).

[57]See Ahrens (2012), p. 13.

[58]See e.g. Ahrens (2012), p. 13.

[59]See e.g. Avizienis et al. (2006), Pollow and Behler (2009) and Kassakian and Wessel (2004), Ziemer (2009).

[60]See e.g. Warusfel and Misdariis (2004), p. 3.

[61]See Zotter (2009), pp. 111–152.

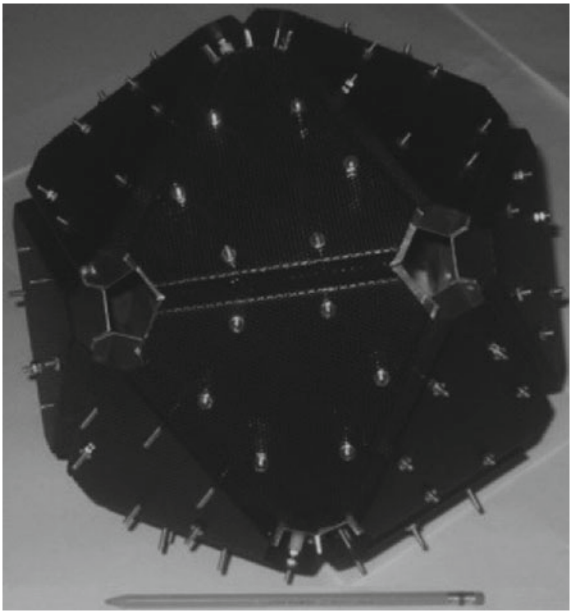


Fig. 8.17 120 loudspeakers mounted on the surface of a dodecahedron for matters of sound radiation synthesis. From Avizienis et al. (2006), with the permission of the Audio Engineering Society

cal instruments significantly affects the room response and leads to changes in the perceived naturalness and loudness.[62]

The high quantity and quality of research in the field of wave field synthesis led to market-ready loudspeaker systems which are able to create impressively realistic sounds with a distinct location of the source. But typically, virtual monopole sources or plane waves are created, which have small perceived dimensions.[63] There have been many attempts already to recreate the sound radiation characteristics of musical instruments via sound field synthesis. Menzel et al. (2006) proposed a WFS method to create binaural signals for a single listening position.[64] Baalman (2008) uses several monopole sources on the body of the virtual sound source to recreate its radiation patterns.[65] This approach is promising but the application is a compromise: A small number of monopole sources does not meet the complexity of many sound sources. A high number of monopole sources on the other hand may lead to an optimal recreation of the radiation characteristic but the computational costs are enormous, as already mentioned in Sect. 5.3.3 about equivalent sources methods in microphone array measurements. However, in more than 70% of the cases subjects of listening tests reported a higher "naturalness" for sources with complex radiation

[62] As already mentioned in Sect. 6.1. See also Martín et al. (2007), p. 395, Otondo and Rindel (2004), p. 1183.

[63] See e.g. Ahrens (2012), p. 198ff.

[64] See Menzel et al. (2006).

[65] See Baalman (2008), p. 97ff.

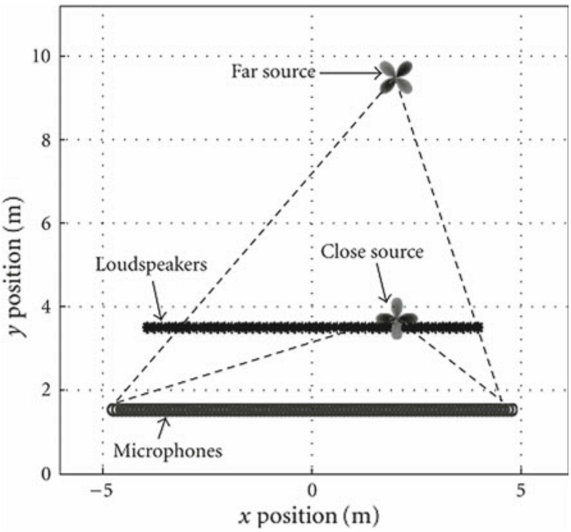


Fig. 8.18 Setup for simulation and actual implementation of synthesizing a complex radiation pattern using wave field synthesis. From Corteel (2007), p. 4, provided under Creative Commons License

patterns compared to virtual monopoles.[66] Corteel (2007) uses a combination of spherical harmonics to calculate driving functions for WFS.[67] He created single and combined spherical harmonics with functions of degree −2 to 2 using closed cabinet loudspeakers and multi-actuator panels (MAPs). A setup for his simulation and actual implementation is illustrated in Fig. 8.18. Although artifacts appear, increasing with decreasing source distance and increasing radiation complexity, he found that such complex source radiations created natural variations while wandering through the listening area as well as an increased perceived "presence" compared to monopole sources.[68] However, no elaborate listening tests have been performed. Böhlke (2016) decomposed the radiation characteristics of a violin to circular harmonics of order 64 and synthesized the radiated sound by means of 128 densely spaced virtual monopole sources in a wave front synthesis system. Listeners observed that the sound field varies when walking through the room, and the source sounds wider and less localizable.[69]

In Ahrens (2012) two formulations to create virtual sound sources with complex radiation patterns via wave front synthesis are given: A finite line sources which is divided into sections vibrating with alternating algebraic sign and spheres vibrating in higher modes. However, he states that these approaches are intermediate steps to a solution to the problem of unnatural spatial radiation characteristics of virtual sources. Firstly, because the computational costs are enormous. And secondly, to him, proper knowledge about the parameters that cause the perception of a certain

[66]See Baalman (2008), p. 19.

[67]See Corteel (2007).

[68]See Corteel (2007), p. 15.

[69]The complete work can be found in Böhlke (2016), a compact version in Böhlke and Ziemer (2017b), and an abstract in Böhlke and Ziemer (2017a).

source extent is needed to enable us to create a sound field that creates the desired spatial impression psychoacoustically, even if the physical wave field is different from a natural wave field emitted by a musical instrument.[70]

As discussed thoroughly in Chap. 5, actual musical instruments may radiate their sound from several vibrating surfaces and through multiple openings. This way wave fronts interfere and create the complicated patterns that make the sound broad and vivid. The radiation characteristics result from the extent of the body. Therefore, it may seem paradox to simplify musical instruments as point sources, especially if the sound radiation characteristics are to be measured, analyzed and synthesized. It is not physically correct but mathematically simple to simplify a musical instrument as a point. Such a point source has a singularity at its origin. From there on the sound wave propagates as a monopole. However, it is possible to define a direction-dependent function that describes a modification of amplitude and phase for each direction. Then, this wave front travels spherically, like a monopole. But this wave front is not necessarily an isobar. Amplitude and phase may vary over the spherical wavefront. Simplifying sound sources and propagation this way is referred to as *complex point source model*.[71] It could be shown that propagating a source sound of musical instruments by means of Eq. 9.1 yields a plausible sound field. The interaural phase- and level differences of a virtual listeners decrease as his distance to the source increases. When applying the complex point source model, the actual source extent of musical instruments could be fairly predicted from propagated sound field quantities. Based on the complex point source model sound radiation characteristics could be measured and synthesized for discrete listening points in space. This can give listeners the impression that the sound radiation characteristics are kept in the loudspeaker playback. The approach has been implemented in an octahedron-shaped loudspeaker array,[72] shown in Fig. 8.19.

8.5 Existing Sound Field Synthesis Installations

Sound field synthesis systems are still at a stage of research and development. Systems are installed in universities and in research and development departments of companies. But in addition to that, several systems are already in practical use. They serve for immersive audio in the entertainment sector, like cinemas, theaters, clubs,

[70] See Ahrens (2012), pp. 198ff.

[71] More details on the complex point source model and investigations on the relationship between musical instruments and the calculated sound field can be found in Ziemer (2014, 2015a, 2017a), Ziemer and Bader (2015a).

[72] The approach is described in Ziemer (2009, 2011a). Demonstrations have been given on the Second International Conference of Students of Systematic Musicology 2009 (SysMus09) conference in Ghent, Belgium.

Fig. 8.19 Octahedron-shaped loudspeaker array to synthesize the sound radiation characteristics of musical instruments at 8 discrete locations. From Ziemer (2016), p. 155

and themed environments, or for communication.[73] Some exemplary sound field synthesis installations are presented in the following.

In the field of research, many wave field synthesis systems contain 24 or 32 loudspeakers in front of or around a listening area, which is large enough for about 5 listeners.[74] One example is the installation at the University of Sherbrooke, shown in Fig. 8.20.

A polygonic loudspeaker setup for wave front synthesis and for an alternative approach with a drastically reduced number of loudspeakers had been installed at the Fraunhofer IDMT institute in Ilmenau. Figure 8.21 shows the wave front synthesis layer with densely spaced loudspeakers and the alternative approach with the fourfold spacing of speakers.

[73]Most of all the wave field synthesis system of the Technical University Berlin in cooperation with Deutsche Telekom Laboratories, or IOSONO systems, the wave field synthesis system of Fraunhofer IDMT. Further information on installed wave field synthesis systems can be found e.g. in Baalman (2008), pp. 47ff, Montag (2011), Chaps. 5 and 6, Slavik and Weinzierl (2008), pp. 656f and 664ff and IOSONO GmbH (2008).

[74]See e.g. Gauthier and Berry (2008), p. 1994, Spors et al. (2003), Ahrens et al. (2010), p. 3, Reisinger (2002), pp. 37–39, Reisinger (2003), pp. 40–44, Vogel (1993), pp. 139f, Baalman (2008), p. 48 and Verheijen (1997), p. 103.

Fig. 8.20 Circular wave field synthesis setup for research. Reproduced from Gauthier and Berry (2008, p. 1994) with the permission of the Acoustical Society of America

Fig. 8.21 Wave field synthesis setup for research and development at Fraunhofer IDMT

Fig. 8.22 Psychoacoustic Sound Field Synthesis System at the University of Hamburg. From Ziemer (2016), p. 157

A psychoacoustic sound field synthesis system for music has been developed and tested at the Institute of Systematic Musicology of the University of Hamburg[75] and will be introduced in detail in the subsequent chapter, Chap. 9. It consists of 15 loudspeakers synthesizing a desired sound field in a listening area of around 1 m^2. Just as in many ambisonics systems, the spacing of 0.65 m between the loudspeakers is rather large. In contrast to wave front synthesis and some ambisonics approaches, every loudspeaker is active for every virtual source position. Perceptual mechanisms of the auditory system are considered in the sound field synthesis approach so that a precise localization and a natural and spatial sound impression are created despite physical synthesis errors. Even beyond the listening area, the localization is rather precise. Figure 8.22 is a photo of the installed system.

A full-duplex wave field synthesis system for communication is being developed at the Nippon Telegraph and Telephone (NTT) lab in Tokyo.[76] An individual combination of a loudspeaker- and a microphone array is installed in two separate rooms. In a conference phone call, several subjects can talk on both sides and even move, while all listeners on the other side can localize the speakers well. Of course, the proximity of microphones to loudspeakers on both sides of the line can cause serious problems. So this system focuses on echo-cancellation to suppress feedback loops. Another important topic is real-time implementation on a singe PC, including the signal processing for the microphone array, the wave field synthesis rendering, and

[75]Its developmental progress can be followed by referring to Ziemer (2009, 2011a, b, c, d, 2014, 2015a, b, 2016, 2017a, b, c, 2018), Ziemer and Bader (2015a, b, c, d, 2017).

[76]Details can be found e.g. in Emura and Kurihara (2015).

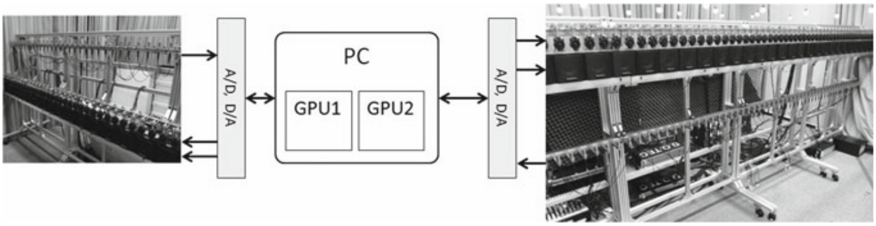


Fig. 8.23 Full duplex wave field synthesis system for communication. From Emura and Kurihara (2015), with the permission of the Audio Engineering Society

the echo-cancellation. This is achieved by fast rendering on two GPUs. The two systems are shown in Fig. 8.23.

The wave front synthesis system at the University of Applied Sciences Hamburg is coupled to a motion capture system.[77] This way, focused sources can be created in such way that they are always between one tracked individual and the loudspeaker array. On the one hand, this brings back the sweet-spot limitation of conventional spatial audio. But on the other hand, a tracked individual can now walk around a virtual source or be surrounded completely by a moving focused source. This offers a new degree of user interaction, which is beneficial, e.g. for virtual reality applications. Furthermore, listeners can control virtual source locations by trackers in their hands. Figure 8.24 is a photo of this system. The wave field synthesis system is linked to a head-mounted display for graphical, three-dimensional virtual reality. With this powerful combination, the WFS system of the University of Applied Sciences Hamburg is used to investigate the potential and limits of *redirected walking*.[78] Here, the translation and/or rotation of a subject can be under- or overemphasized in the virtual auditory and visual scene. This creates the illusion that the subjects walk paths that exceed the actual physical room.

A wave field synthesis system for research and public events can be found in the auditorium of the Berlin University of Technology,[79] illustrated in Fig. 8.25.

The wave field synthesis system developed in Berlin is also installed at the University of Music and Drama Hamburg. It is in use for both concerts and research, especially in the field of network music performance. The installed system can be seen in Fig. 8.26. The mobile system is transported to event venues like Kampnagel center for performing arts for demonstrations and concerts.

One WFS system containing 832 loudspeakers delivers an immersive sound experience at the Seebühne Bregenz,[80] illustrated in Fig. 8.27. In contrast to conventional PA systems with delay lines, a wave front synthesis system does not create echoes from the rear. These can be annoying to the audience, reduce speech intelligibility and may create conflicting source localization cues. Furthermore, the amplitude

[77] See e.g. Fohl and Nogalski (2013), Fohl (2013), Fohl and Wilk (2015) for details.

[78] See e.g. Nogalski and Fohl (2015, 2016, 2017), Meyer et al. (2016) for details on the approaches.

[79] Some details can be found in Baalman (2008), Chap. 3 and Slavik and Weinzierl (2008), p. 670.

[80] See Slavik and Weinzierl (2008), p. 656.

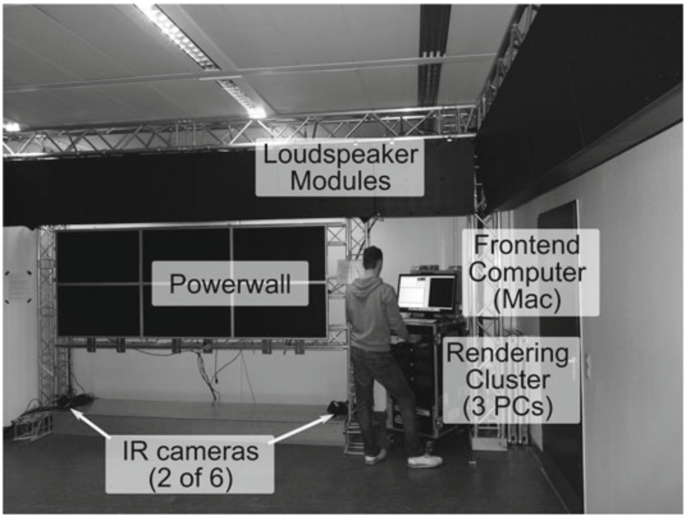


Fig. 8.24 Wave Field Synthesis System at the University of Applied Sciences Hamburg coupled to motion capture technology. Original photo by Wolfgang Fohl, provided under Creative Commons License. The photo is converted to grayscale

Fig. 8.25 Panoramic picture of the WFS loudspeaker system in an auditorium of Berlin University of Technology containing 832 channels and more than 2700 loudspeakers. Pressestelle TU Berlin, with friendly permission by Stefan Weinzierl

decay of line arrays lowers with increasing length. PA systems tend to be too loud in proximity to the loudspeakers. This is necessary to ensure that the sound pressure level is still high enough at the rear seats despite the large amplitude decay over distance. Wave front synthesis systems can create a lower amplitude decay and therefore have a high potential as a PA-alternative for stages with a large audience. Wave field synthesis systems have another advantage for theater. Most large theaters

Fig. 8.26 Wave field synthesis system for music installations and networked music performance at the University of Music and Theater Hamburg

Fig. 8.27 Photo of the WFS loudspeaker system at the Seebühne Bregenz. The speakers are arranged beside and behind the audience. From Slavik and Weinzierl (2008), p. 656

and open air locations for drama create an irritating ear/eye-conflict: The actor is speaking somewhere on the stage but his or her voice will be localized at one of the few PA loudspeaker towers. This can be very confusing in a scene with many actors. A frontal, horizontal WFS line array can give additional localization cues so that the auditory event better fits the visual scene.

In early 2009, a 189-channel wave field synthesis system had been installed at the Casa del Suono in Parma, Italy. This museum is dedicated to the history of audio technology. The loudspeakers are installed behind curtains in a room that is acoustically treated. Visitors can experience the sound without seeing the actual loudspeaker system.[81] Another WFS system had been installed in the Tresor club in Berlin; a famous techno club. Here, the conditions for a wave field synthesis setup are challenging: Standing waves emerge between the solid concrete floor and ceiling. Furthermore, the ceiling is so low above the loudspeaker array that very early reflections superimpose with the desired wave field. Furthermore, the techno music has to be produced specifically for the system so that single tracks can receive their individual virtual source locations or paths. On the other hand, wave field synthesis offers new possibilities for spatial mixing. At the moment, conventional stereophonic dance music tends to make only little use of hard panning.[82] The reason for that is simply that the constellation of loudspeakers and listeners in night clubs is typically far away from the ideal stereo triangle discussed in Sect. 7.2.2. Loudspeakers may be positioned far apart, so a sound hard-panned to either speaker might be inaudible over a large area of the dancefloor. In many night clubs the two stereo channels are mixed together to get rid of hard panning and incoherent loudspeaker signals. Hence, producers of electronic dance music have an eye on mono compatibility. Wave field synthesis systems in night clubs would give music producers and disc jockeys the opportunity to use space as creative and dramaturgical tool instead of trying to stay mono-compatible.

Automotives are a real challenge for both conventional stereophonic audio and wave front synthesis systems. Due to the five seats there is a distribution of listeners, so at least several sweet spots are desired. The exact location of the driver's and passengers' heads may be unknown and may neither be in plane nor static. So sweet spots may even be insufficient as they do not account for head- and torso movement. There is limited space inside a car, so it is not easy to install line arrays of loudspeakers or to surround the interior completely with loudspeakers in one height. Curved loudspeaker arrays on the other hand are challenging in terms of computational effort and synthesis error compensation. Standing waves occur due to the rather small dimensions of cars compared to audible wavelengths. Seats are obstacles that create absorption, deflection and reflections. Their positions are readjusted to the individual needs of the driver and the passengers, so it is challenging to include them in a sound field synthesis calculation. Despite these issues, Audi decided to install Fraunhofer IDMTs wave front synthesis system in the Q7. The system is depicted in Fig. 8.28.

Another commercial system for TV-soundbars is developed and distributed by Sonic Emotion. It promises an enlargement of the sweet spot to a sweet area and an extension of the loudspeaker base by synthesizing plane waves. Figure 8.29 shows a sound bar including multiple speakers that can be used to synthesize wavefronts.[83]

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[81] A detailed description and a photo can be found in Adriaensen (2010).

[82] See e.g. Owsinski (2014), p. 51 and Stirnat and Ziemer (2017).

[83] More information and advertisements can be found in on the Sonic Emotion website Sonic Emotion (2017) and on their Youtube-channel Sonic Emotion (2012).

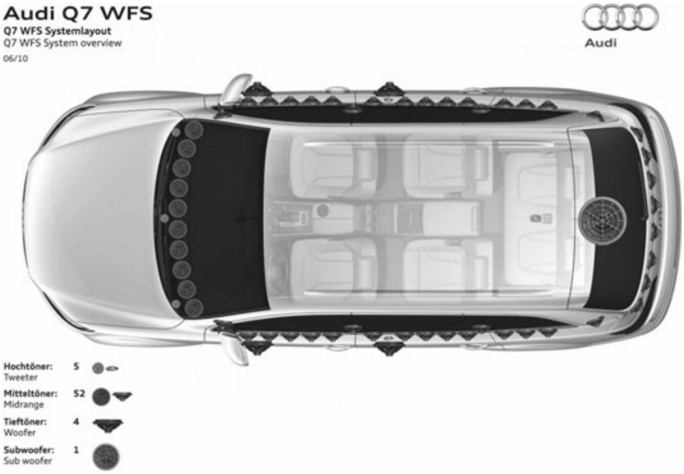

Fig. 8.28 Wave front synthesis installation in a car. Photo from Audi Technology Portal (2011), © Audi

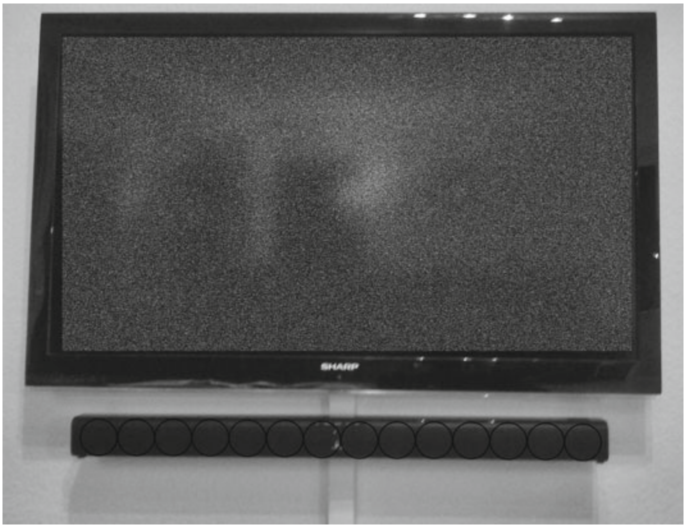

Fig. 8.29 Synthesizing plane waves with multiple loudspeakers in a sound bar enlarges the sweet spot for stereo source signals

Most of these sound field systems aim at reconstructing the spatio-temporal properties of sound waves in terms of wave front synthesis. Psychoacoustic sound field synthesis as installed at the University of Hamburg is an alternative approach which takes auditory perception into account in its derivation. This approach is discussed in Chap. 9.

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