Solution 1

(1) Probability Distribution of X: Poisson Distribution.
How it satisfies: Poisson distribution is generally used to model the number of successes per unit area or time (here Cars arrival is the occurrence/success)

(2)

```
> dpois(5,6.7)
[1] 0.1384904
```

(3)

```
> ppois(5,6.7)
[1] 0.3406494
```

Solution 2

(1)

```
> 1-punif(37,30,40)
[1] 0.3
```

(2)

```
> punif(32,30,40)
[1] 0.2
```

(3)

```
> punif(38,30,40) - punif(34,30,40)
[1] 0.4
```

Solution 3

Solution 4

(1) $Probability\ density\ function, f(x) = \ \lambda e^{-\frac{\lambda}{x}}, where\ \lambda = \frac{1}{5} = \ 0.2\ and\ x = oil\ changing\ time$

$$= \ \lambda e^{-\frac{0.2}{x}}$$

(2)

```
> pexp(6,rate = 1/5)
[1] 0.6988058
```

(3)

```
> pexp(5,rate = 1/5) - pexp(3,rate = 1/5)
[1] 0.1809322
```

Solution 5

(1)

```
> dbinom(25,400,.07)
[1] 0.06867971
```

(2)

```
> pbinom(24,400,.07)
[1] 0.2511457
```

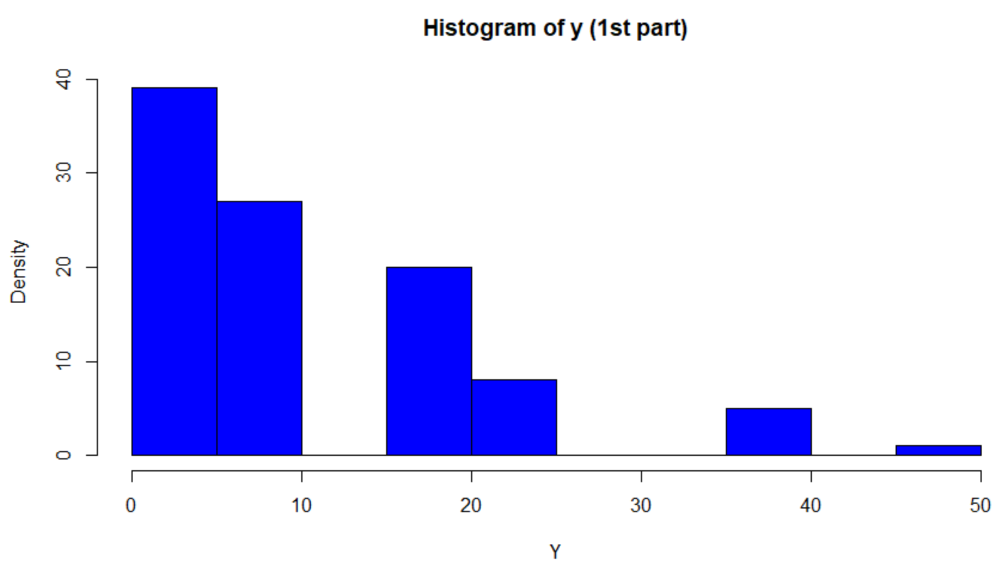
(3)

```
> pbinom(25,400,.07) - pbinom(20,400,.07)
[1] 0.2541306
```

Solution 6

1)

```
> x<-rpois(100,3)
> y<-x^2
> hist(y,main = "Histogram of y (1st part)",col = "blue",xlab = "Y", ylab = "Densit
y")
```



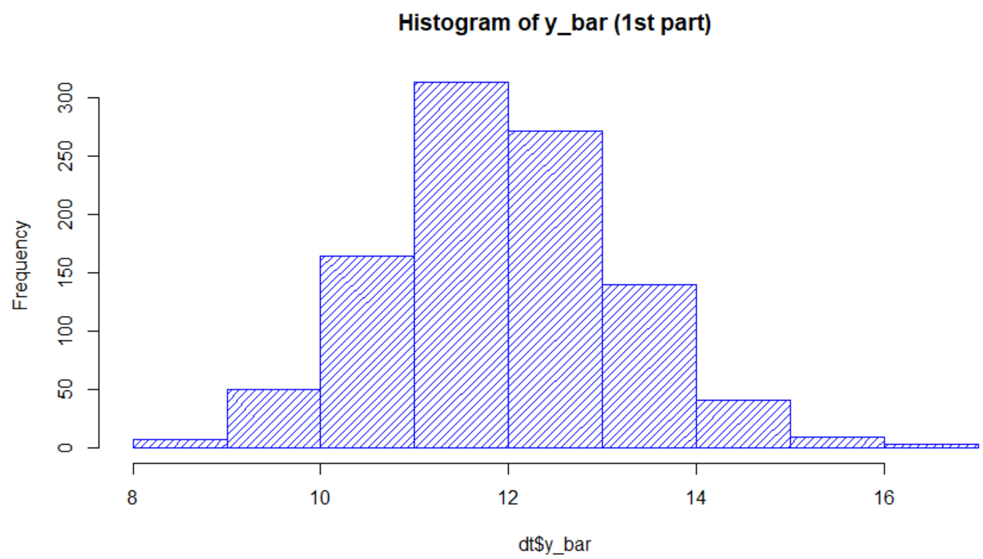
2)

```
> dt<-data.frame(y_bar=0,sd=0)  #Initializing an empty data frame
> for (i in 1:1000) {  #looping 1000 times to store y_bar in dt
+ x<-rpois(100,3)
+ y<-x^2
+ y_bar<-mean(y)
```

```
+ sd<-sd(y)
+ dt<-rbind(dt,cbind(y_bar,sd))
+ }
> dt<-data.frame(dt[2:1001,])
> hist(dt$y_bar,col = "blue",density = 20, main = "Histogram of y_bar (1st part)")
```

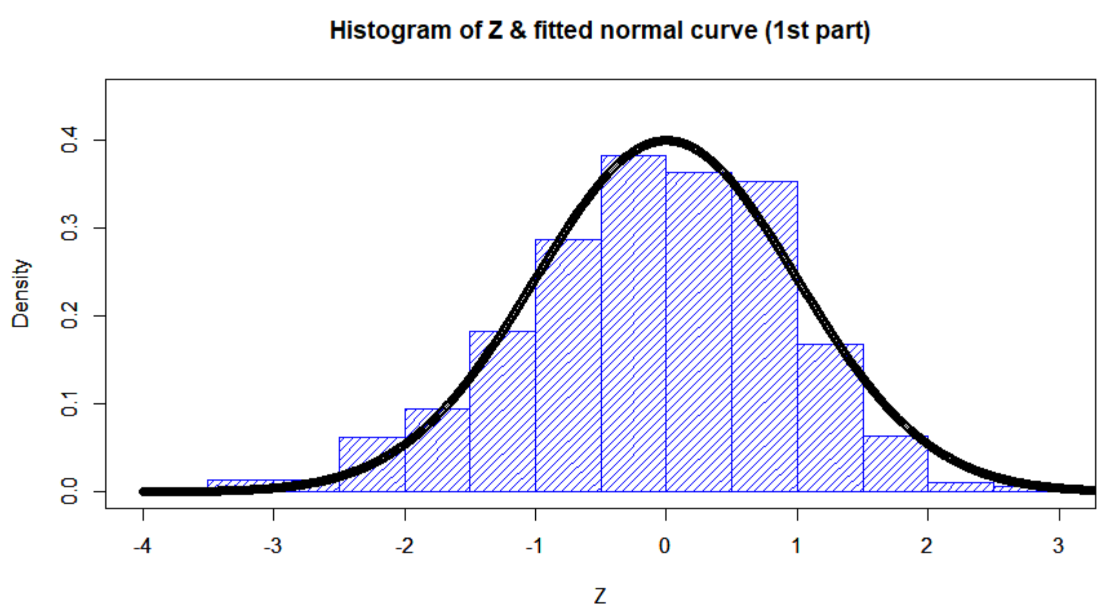
Histogram below (normally shaped) is generated from random variable X (poisson distributed)

$$E(Y) = approximated\ by\ the\ mean\ of\ \bar{Y} \cong 11.94$$



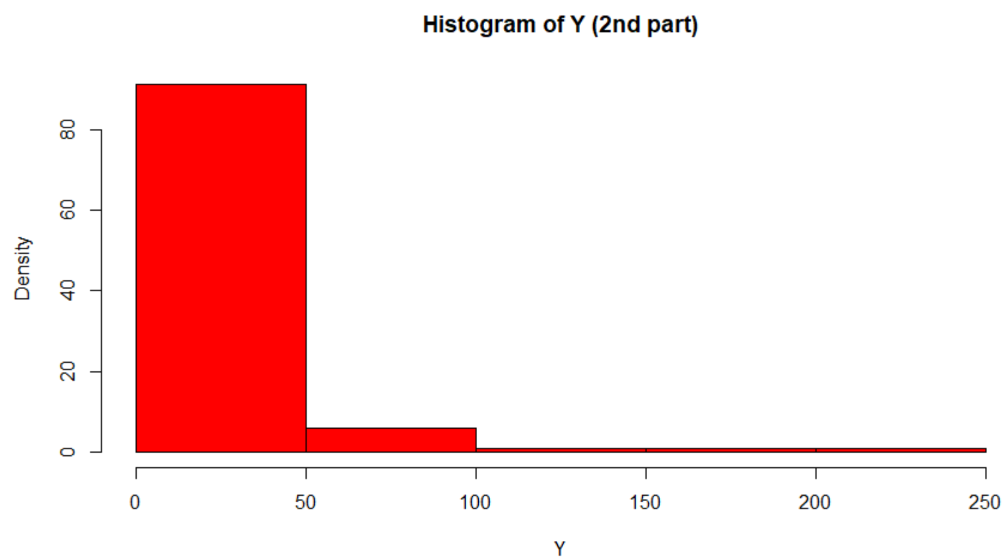
3)

```
> u_bar<-mean(dt$y_bar)
> dt$z<-with(dt, (dt$y_bar - u_bar)/(dt$sd/10))
> hist(dt$z,density = 20,col = "blue", main = "Histogram of Z & fitted normal curve
(1st part)",xlim = c(-4,3),ylim = c(0,0.45),probability = TRUE,xlab = "Z",ylab = "D
ensity")
> par(new = TRUE)
```

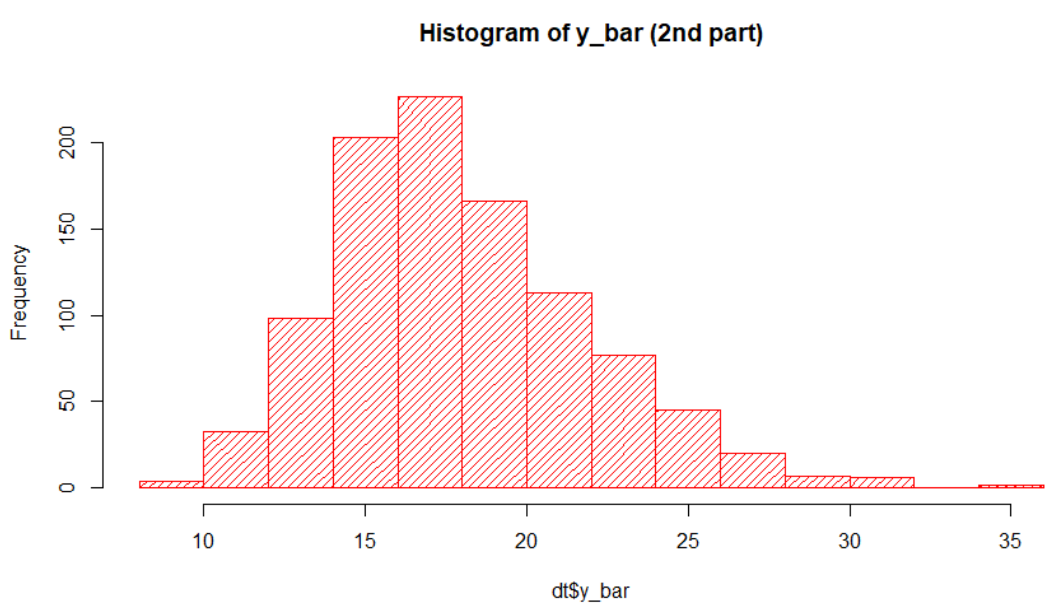


4)

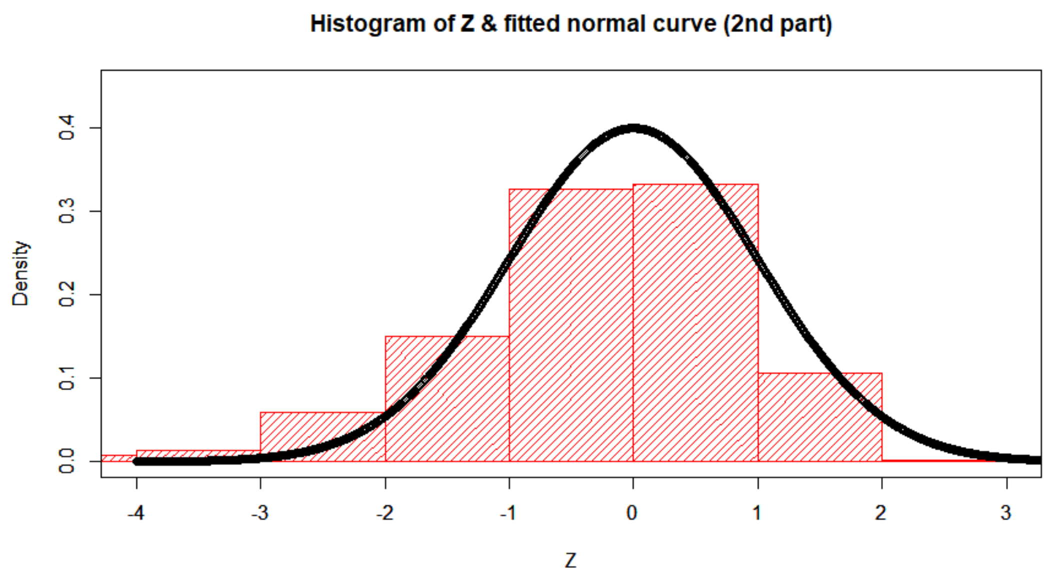
```
> x<-rexp(100,1/3)
> y<-x^2
> hist(y,main = "Histogram of Y (2nd part)", col = "red",xlab = "Y", ylab = "Densit
y")
```




```
> dt<-data.frame(y_bar=0,sd=0)  #Initializing an empty data frame
> for (i in 1:1000) {  #looping 1000 times to store y_bar in dt
+   x<-rexp(100,1/3)
+   y<-x^2
+   y_bar<-mean(y)
+   sd<-sd(y)
+   dt<-rbind(dt,cbind(y_bar,sd))
+ }
> dt<-data.frame(dt[2:1001,])
> hist(dt$y_bar,col = "red",density = 20, main = "Histogram of y_bar (2nd part)")
```



```
> u_bar<-mean(dt$y_bar)
> dt$z<-with(dt, (dt$y_bar - u_bar)/(dt$sd/10))
> hist(dt$z,density = 20,col = "red", main = "Histogram of Z & fitted normal curve
(2nd part)",xlim = c(-4,3),ylim = c(0,0.45),probability = TRUE,xlab = "Z",ylab = "D
ensity")
> par(new = TRUE)
> plot(seq(-4,4,by = 0.005),dnorm(seq(-4,4,by = 0.005)),xlim = c(-4,3),ylim = c(0,0
.45),xlab = "Z",ylab = "Density")
```



<u>Comment:</u> Normal curve fit very good on both the distribution in part 1 { poison: (1), (2) ,(3) } & part 2 { (4) - exponential}. It implies that whatever was our X (exponential or poison), the distribution of the mean of their samples $\bar{Y}$ is normally distributed.