Solution 1

(1) Probability Distribution of X: Poisson Distribution.
How it satisfies: Poisson distribution is generally used to model the number of successes per unit area or time (here Cars arrival is the occurrence/success)

(2)

```
> dpois(5,6.7)
[1] 0.1384904
```

(3)

```
> ppois(5,6.7)
[1] 0.3406494
```

Solution 2

(1)

```
> 1-punif(37,30,40)
[1] 0.3
```

(2)

```
> punif(32,30,40)
[1] 0.2
```

(3)

```
> punif(38,30,40) - punif(34,30,40)
[1] 0.4
```

Solution 3

Solution 4

(1) $Probability\ density\ function, f(x) = \ \lambda e^{-\frac{\lambda}{x}}, where\ \lambda = \frac{1}{5} = \ 0.2\ and\ x = oil\ changing\ time$

$$= \ \lambda e^{-\frac{0.2}{x}}$$

(2)

```
> pexp(6,rate = 1/5)
[1] 0.6988058
```

(3)

```
> pexp(5,rate = 1/5) - pexp(3,rate = 1/5)
[1] 0.1809322
```

Solution 5

(1)

```
> dbinom(25,400,.07)
[1] 0.06867971
```

(2)

```
> pbinom(24,400,.07)
[1] 0.2511457
```

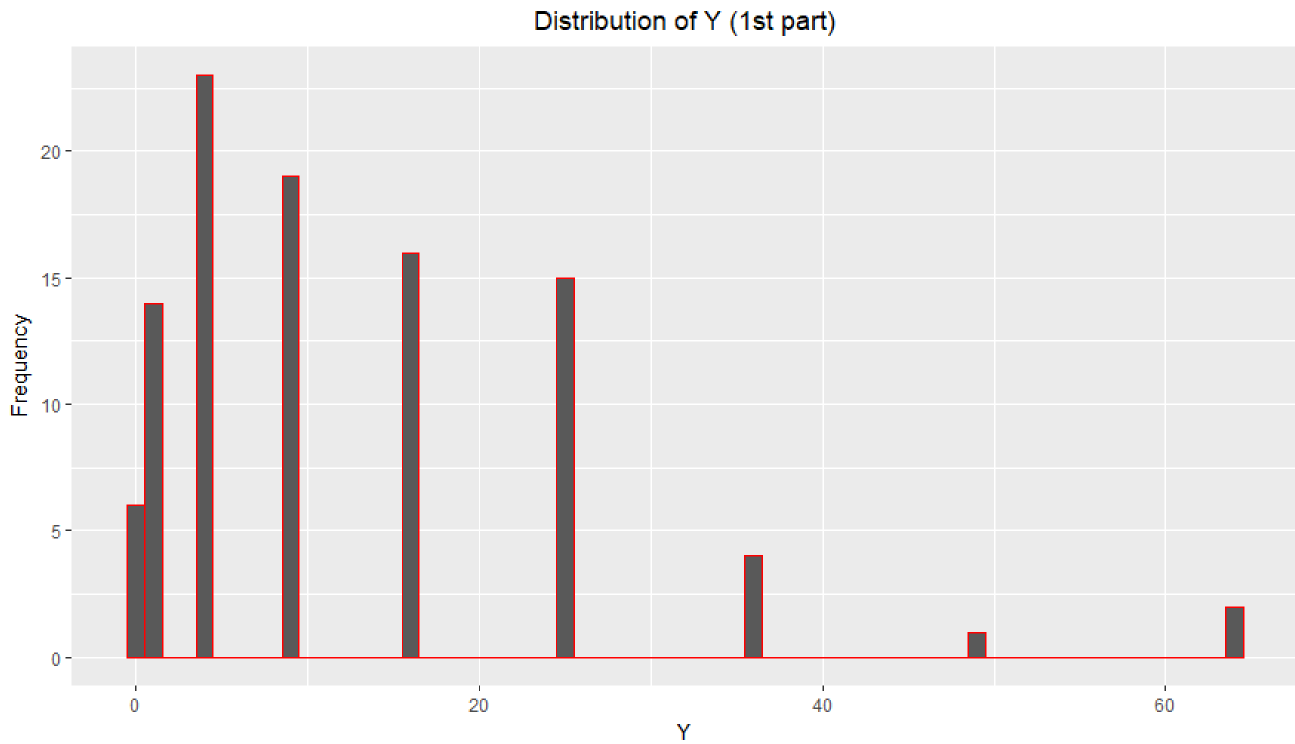
(3)

```
> pbinom(25,400,.07) - pbinom(20,400,.07)
[1] 0.2541306
```

Solution 6

1)

```
> x<-rpois(100,3)
> y<-x^2
> dat<-data.frame(x,y)
> g<-ggplot(data = dat,aes(x=dat$y))+geom_histogram(binwidth = 1,col = "red")
> g<-g+ggtitle("Distribution of Y")+theme(plot.title = element_text(hjust=0.5))+xlab("Y")
+ylab("Frequency")
> g
```

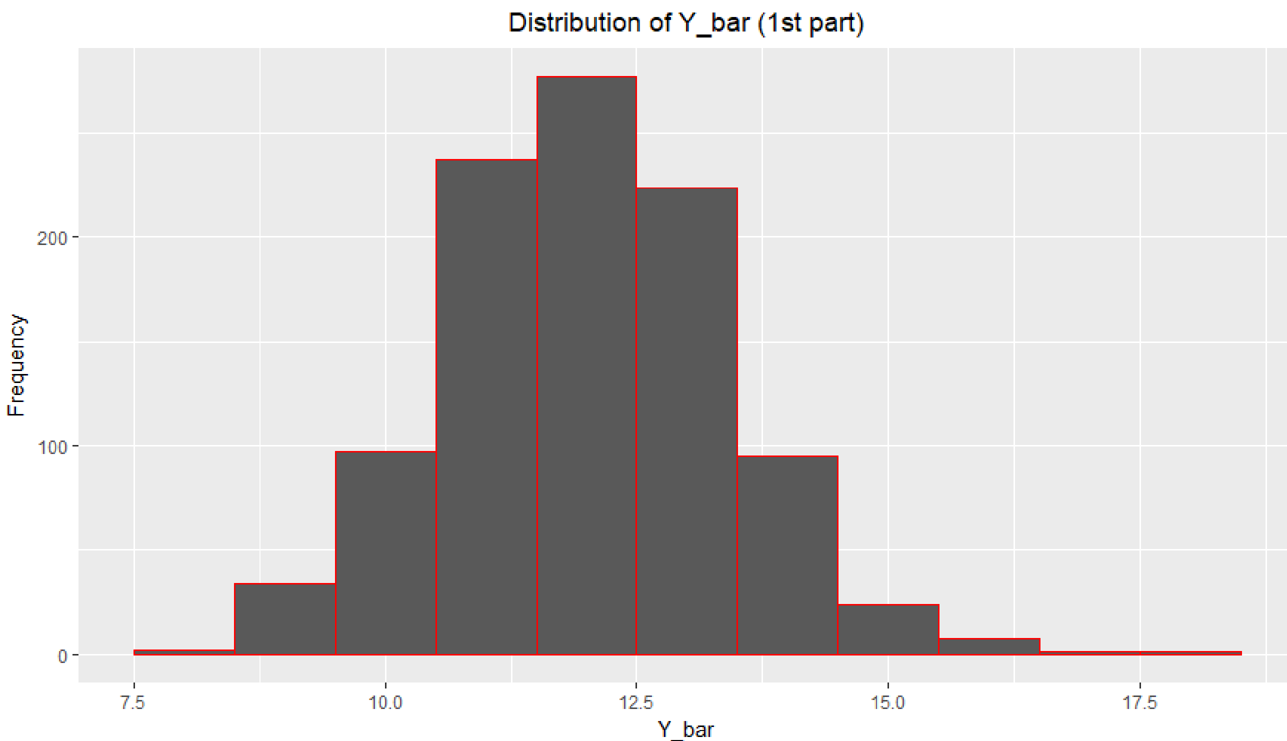


2)

```
> dt<-data.frame(y_bar=0,sd=0)  #Initializing an empty data frame
> for (i in 1:1000) {  #looping 1000 times to store y_bar in dt
+ x<-rpois(100,3)
+ y<-x^2
+ y_bar<-mean(y)
+ sd<-sd(y)
+ dt<-rbind(dt,cbind(y_bar,sd))
+ }
> dt<-data.frame(dt[2:1001,])
> g<-ggplot(data = dt,aes(x=dt$y))+geom_histogram(binwidth = 1,col = "red")
> g<-g+ggtitle("Distribution of Y_bar")+theme(plot.title = element_text(hjust=0.5))+xlab(
"Y_bar")+ylab("Frequency")
> g
```

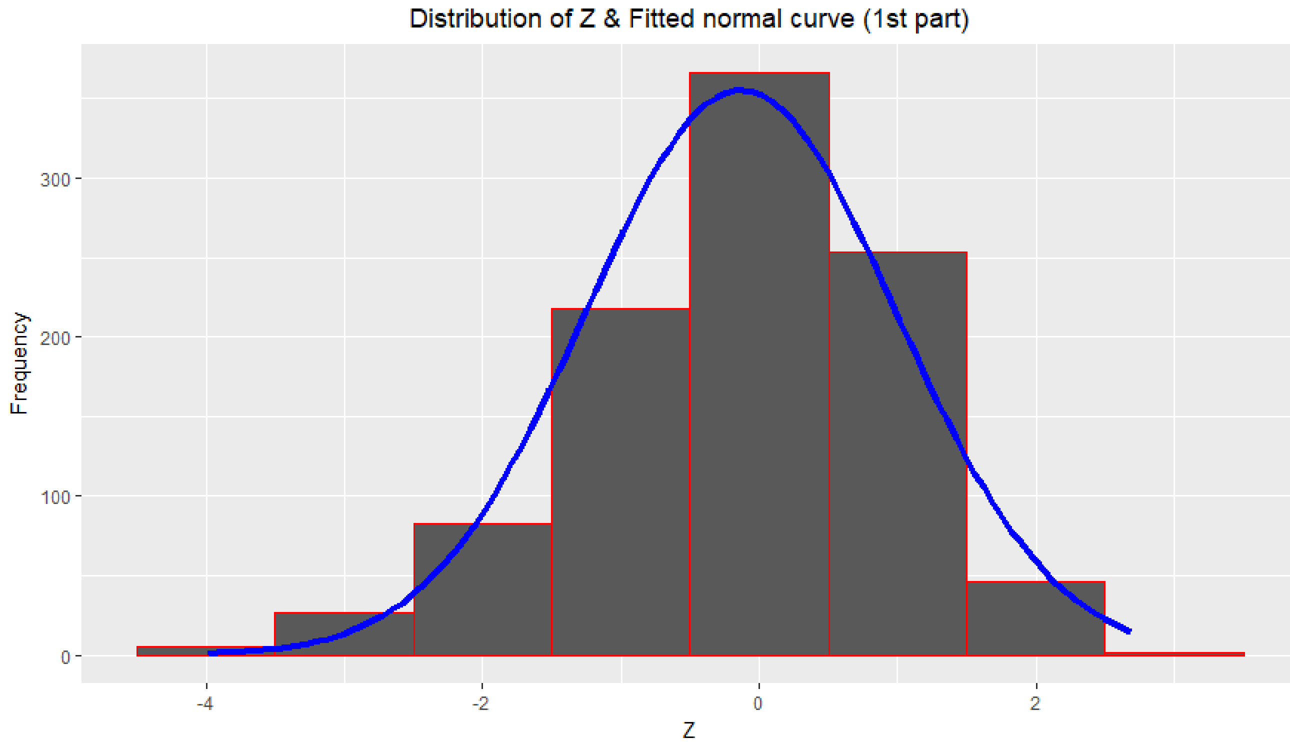
Histogram below (normally shaped) is generated from random variable X (poisson distributed)

$$E(Y) = approximated\ by\ the\ mean\ of\ \bar{Y}\ \cong 11.94$$



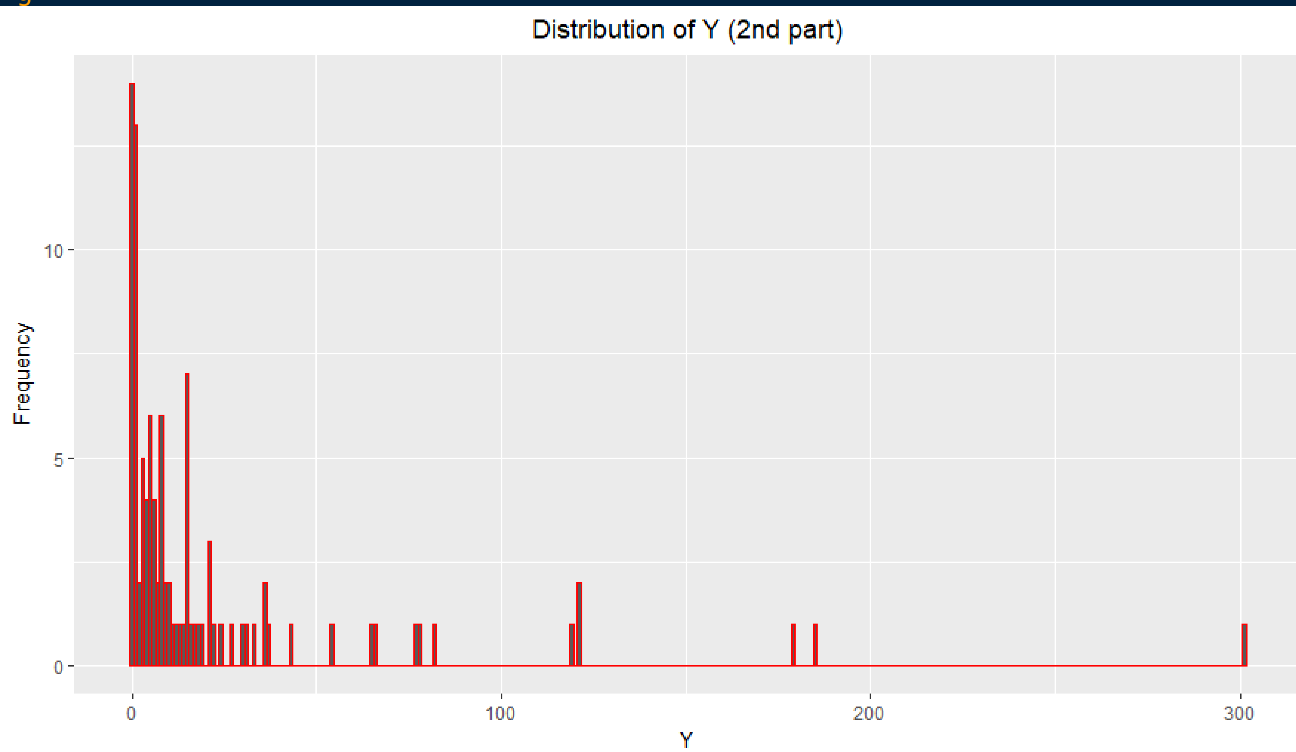
3)

```
> u_bar<-mean(dt$y_bar)
> dt$z<-with(dt, (dt$y_bar - u_bar)/(dt$sd/10))
>
> mean_z<-mean(dt$z)
> sd_z<-sd(dt$z)
> g<-ggplot(data = dt,aes(x=dt$z))+geom_histogram(binwidth = 1,col = "red")
> g<-g+ggtitle("Distribution of Z & Fitted normal curve")+theme(plot.title = element_t
ext(hjust=0.5))+xlab("Z")+ylab("Frequency")
> g<-g+stat_function(fun = function(x,mean_z,sd_z){
+ dnorm(x=x,mean_z,sd_z)*1000},
+ args = c(mean = mean_z, sd = sd_z)
+ ,size = 1.5,col = "blue")
> g
```



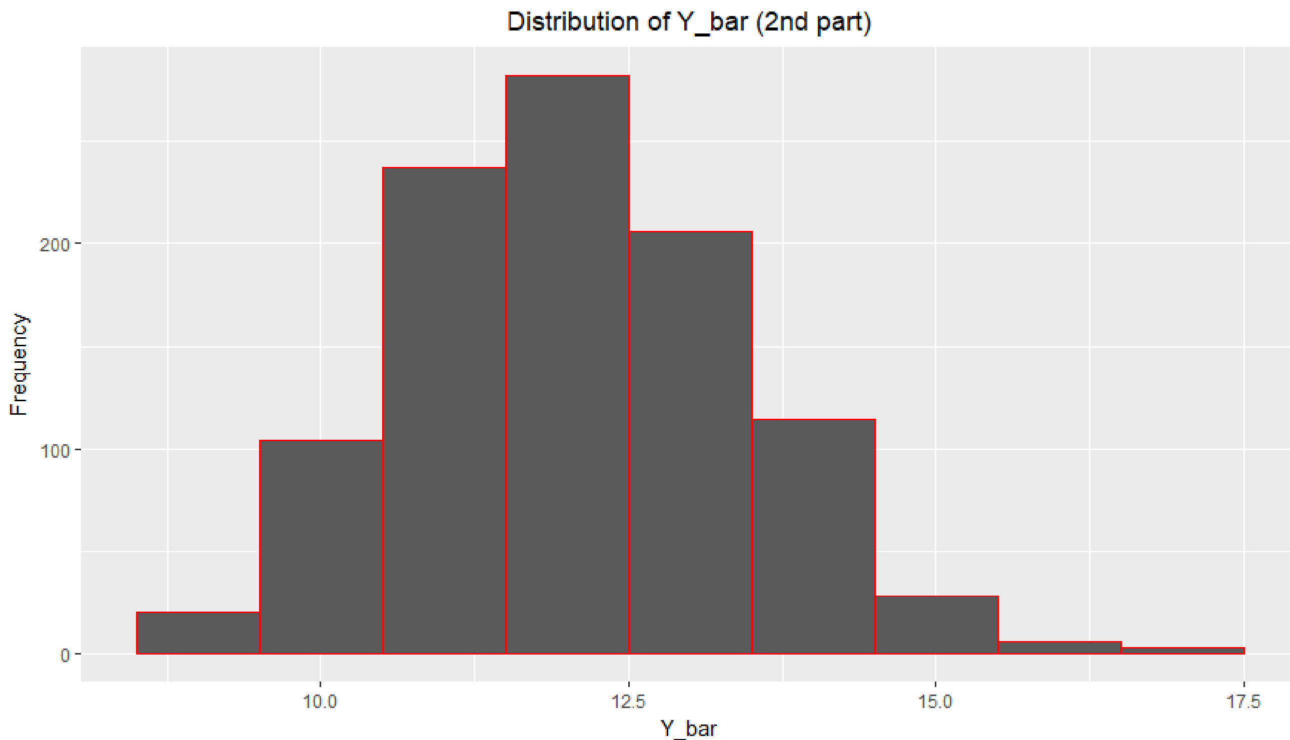
4)

```
> x<-rexp(100,1/3)
> y<-x^2
> dat<-data.frame(x,y)
> g<-ggplot(data = dat,aes(x=dat$y))+geom_histogram(binwidth = 1,col = "red")
> g<-g+ggtitle("Distribution of Y (2nd part)")+theme(plot.title = element_text(hjust=0
.5))+xlab("Y")+ylab("Frequency")
> g
```

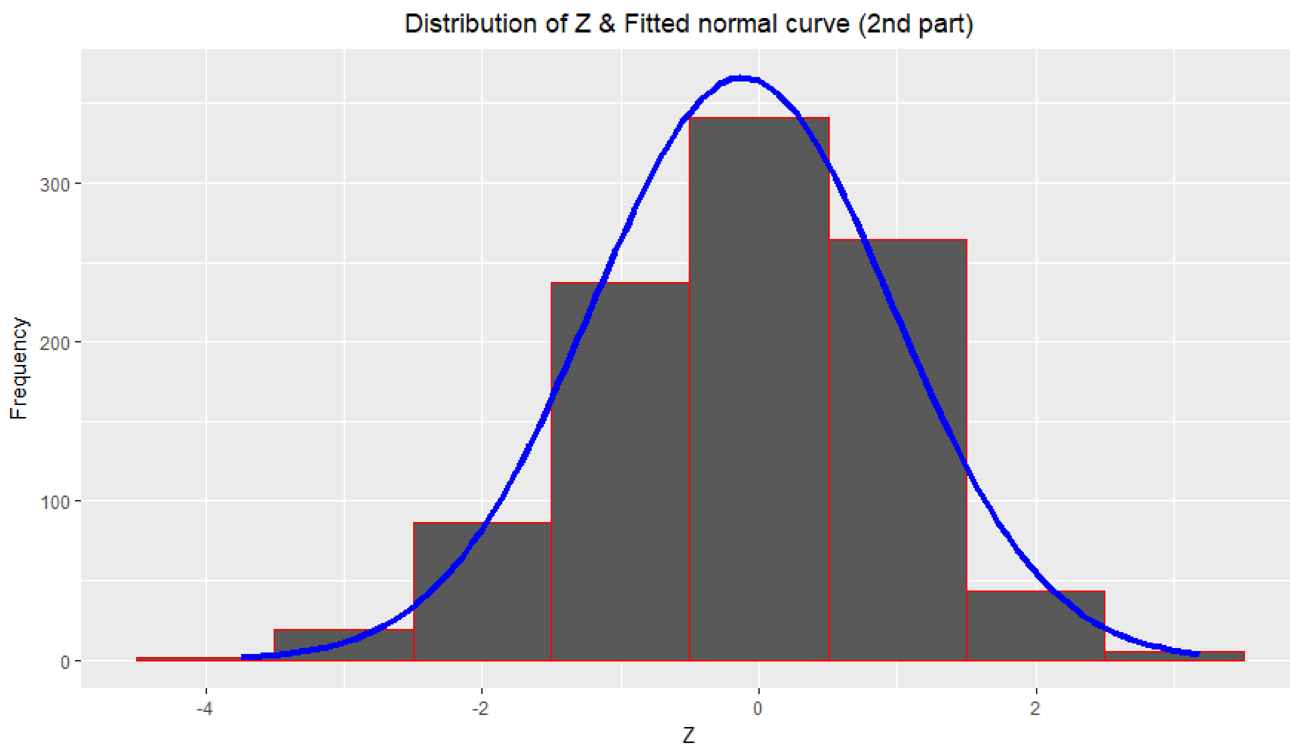



```
> dt<-data.frame(y_bar=0,sd=0)  #Initializing an empty data frame
> for (i in 1:1000) {  #looping 1000 times to store y_bar in dt
+   x<-rpois(100,3)
```

```
+   y<-x^2
+   y_bar<-mean(y)
+   sd<-sd(y)
+   dt<-rbind(dt,cbind(y_bar,sd))
+ }
> dt<-data.frame(dt[2:1001,])
> g<-ggplot(data = dt,aes(x=dt$y))+geom_histogram(binwidth = 1,col = "red")
> g<-g+ggtitle("Distribution of Y_bar (2nd part)")+theme(plot.title = element_text(hjust=
0.5))+xlab("Y_bar")+ylab("Frequency")
> g
```




```
> u_bar<-mean(dt$y_bar)
> dt$z<-with(dt, (dt$y_bar - u_bar)/(dt$sd/10))
> mean_z<-mean(dt$z)
> sd_z<-sd(dt$z)
> g<-ggplot(data = dt,aes(x=dt$z))+geom_histogram(binwidth = 1,col = "red")
> g<-g+ggtitle("Distribution of Z & Fitted normal curve (2nd part)")+theme(plot.title = e
lement_text(hjust=0.5))+xlab("Z")+ylab("Frequency")
> g<-g+stat_function(fun = function(x,mean_z,sd_z){
+   dnorm(x=x,mean_z,sd_z)*1000},
+   args = c(mean = mean_z, sd = sd_z)
+   ,size = 1.5,col = "blue")
> g
```



<u>Comment:</u>

1. Normal curve fit very good on both the distribution in part 1 & part 2
   a. It implies that whatever is our X (exponential or poison), the distribution of the mean of $\bar{Y}$ (where $Y = X^2$) is normally distributed.
2. Mean of $\bar{Y}$ for both the distributions is roughly 12