Wolfram|Alpha Step-by-Step Solution

Wolfram|Alpha Input: area between curves

STEP 1

Find the area between the following two curves:
$y = x (x - 3) (x - 1)$ and $y = 0$

STEP 2

Hint: The area between two curves, f and g, is defined as $\int_{x_0}^{x_1} |f(x) - g(x)|\, dx$, the integral of the absolute difference of the curves between their points of intersection.

Set up the integral to compute the area between $x (x - 3) (x - 1)$ and 0:

$$A = \int_{\text{lower}}^{\text{upper}} |x (x - 3) (x - 1) - 0|\, dx$$

STEP 3

Hint: To find the bounds on x for A, we need to find the intersections of $x (x - 3) (x - 1)$ and 0.

Solve the equation $(x - 3) (x - 1) x = 0$ to get the points of intersection.

$x = 0$ or $x = 1$ or $x = 3$

STEP 4

Hint: What do these solutions mean?

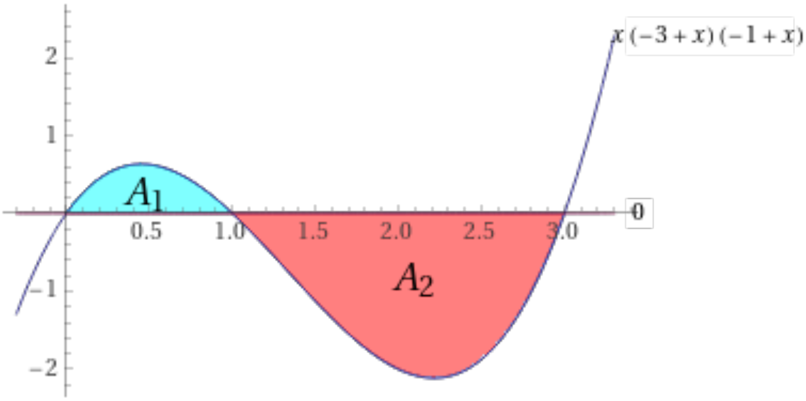
The maximum and minimum of these points of intersection form the x bounds on A, so we have:

$$A = \int_0^3 |x\,(x-3)\,(x-1) - 0|\,dx$$

STEP 5

Hint: Note that we can use geometric intuition to help compute this integral.

Plot $y = x\,(x-3)\,(x-1)$ and $y = 0$ to observe that the curves intersect to create two disjoint regions, whose areas are represented by A_1 and A_2:



STEP 6

Hint: Remember that if $[a, c] = [a, b] \cup [b, c]$,
$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx.$$

The solutions to the equation $(x-3)\,(x-1)\,x = 0$, found above, form a partition of $[0, 3]$ into the domains of integration corresponding to A_1 and A_2:

$$[0, 3] = [0, 1] \cup [1, 3]$$

STEP 7

> **Hint:** What can we do with this partition of [0, 3]?

Because A_1 and A_2 represent the areas of disjoint regions, we can write $A = A_1 + A_2$ where:

$$A_1 = \int_0^1 |x\,(x-3)\,(x-1) - 0|\,dx$$

$$A_2 = \int_1^3 |x\,(x-3)\,(x-1) - 0|\,dx$$

STEP 8

> **Hint:** Remember that on the interior of each of these domains of integration, one of $x\,(x-3)\,(x-1)$ or 0 is strictly greater than the other.

Simplify $|(x-3)\,(x-1)\,x - 0|$:

If $x\,(x-3)\,(x-1) > 0$, then $|x\,(x-3)\,(x-1) - 0| = x^3 - 4\,x^2 + 3\,x$.

If $x\,(x-3)\,(x-1) < 0$, then $|x\,(x-3)\,(x-1) - 0| = -x^3 + 4\,x^2 - 3\,x$.

STEP 9

> **Hint:** We can use visual intution to determine which of $x\,(x-3)\,(x-1)$ or 0 is greater than the other on each of the domains of integration.

Simplify each integrand by determining which of $x\,(x-3)\,(x-1)$ or 0 is greater than the other on the domain of integration and evaluate the integrals:

$$A_1 = \int_0^1 |x\,(x-3)\,(x-1) - 0|\,dx = \int_0^1 \left(x^3 - 4\,x^2 + 3\,x\right) dx = \frac{5}{12}$$

$$A_2 = \int_1^3 |x\,(x-3)\,(x-1) - 0|\,dx = \int_1^3 \left(-x^3 + 4\,x^2 - 3\,x\right) dx = \frac{8}{3}$$

STEP 10

Hint: Now that we know the areas of each of A_1 and A_2, what is left to do?

Add A_1 and A_2 to compute A:

Answer:

$$A = A_1 + A_2 = \frac{5}{12} + \frac{8}{3} = \frac{37}{12}$$