What math should you know before you start your undergraduate studies at BME?

Review for prospective students

Algebra

Number sets

You should know the basic concepts and notations of set theory, as well as the properties of different number sets.

Natural numbers:	$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$
Positive natural numbers:	$\mathbb{N}^+ = \{n \mid n \in \mathbb{N} \text{ és } n > 0\} = \{1, 2, 3, \ldots\}$
Integers:	$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
Positive integers:	$\mathbb{Z}^+ = \{n \mid n \in \mathbb{Z} \text{ and } n > 0\} = \{1, 2, 3, \ldots\} = \mathbb{N}^+$
Rational numbers:	$\mathbb{Q} = \left\{ q \;\middle|\; \text{if there are such } n \in \mathbb{Z} \text{ and } m \in \mathbb{Z} \text{ that } q = \frac{n}{m} \right\}$
Real numbers:	$\mathbb{R}$
Positive real numbers	$\mathbb{R}^+ = \{x \mid x \in \mathbb{R} \text{ and } x > 0\}$
Relationship:	$\mathbb{N}^+ \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

Elementary algebraic operations

You should know how to use brackets, how to operate with fractions (sum, difference, common denominator, multiplication, division, multiplication property, reduction, rationalizing the denominator), and what absolute value is:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Factorials and binomial coefficients

$$n! = n(n-1)(n-2)\ldots 2 \cdot 1 \qquad 0! = 1$$

$$\binom{n}{k} = \frac{n!}{k!\ (n-k)!}$$

The most important algebraic identities

You should know the following identities by heart and use them as a matter of routine.

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Operations with powers and roots

You should know the definition and generalization of powers.

For $n \in \mathbb{N}^+, a \in \mathbb{R}$ $\quad a^n = a \cdot a \cdot \ldots \cdot a$ (n times)

For $a \in \mathbb{R},\ a \neq 0$ $\quad a^0 = 1$

For $n \in \mathbb{N}^+, a \in \mathbb{R},\ a \neq 0$ $\quad a^{-n} = \frac{1}{a^n}$

For $n, m \in \mathbb{N}^+,\ \mathrm{m} \neq 1,\ a > 0$ $\quad a^{\frac{n}{m}} = \sqrt[m]{a^n}$

You should know the following identities by heart. (We assume that each of the following expressions is finite.)

Powers of products and fractions $\quad (ab)^n = a^n \cdot b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Roots of products and fractions $\quad \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Powers with the same base $\quad a^n \cdot a^m = a^{n+m} \qquad \frac{a^n}{a^m} = a^{n-m}$

Multiple powers $\quad \left(a^m\right)^n = a^{m \cdot n}$

Operations with base a logarithms

You should know the definition and usage of general base a logarithms by heart. (We assume that each of the following expressions is finite.)

Definition: the exponent to which a, the base, must be raised to produce x. $a^{\log_a x} = x \quad (x>0,\ a>0,\ a\neq 1)$

Consequences	$\log_a 1 = 0$ and $\log_a a = 1$
Logarithm of products	$\log_a (xy) = \log_a x + \log_a y$
Logarithm of fractions	$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
Logarithm of powers	$\log_a x^n = n \log_a x$
Change base	$\log_a x = \frac{\log_b x}{\log_b a}$

Sample problems in algebra

Rationalize the denominator: $\dfrac{\sqrt{8}}{\sqrt{18}-\sqrt{2}}$

Simplify the following expression $(c>0)$: $\sqrt{\dfrac{c}{\sqrt[3]{c^2\cdot\sqrt{c}}}}$

Evaluate $\dfrac{4^{-9}+4^{-6}}{4^{-9}+4^{-7}}$.

Express B from $t=\dfrac{\lg A-\lg B}{\lg 2}$ in terms of the other variables.

Evaluate $\left(\dfrac{\sqrt{10}}{10}\right)^{-2+\lg 9}$.