ME652: Mobile Robotics
Homework #4 (10 points)

Due: June 9 (Tue), 2020

Problem 1. (2 points) Consider the following stochastic dynamical system.

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \mathbf{u} + \mathbf{w}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{x} + v$$

where process noise $\mathbf{w} \sim N(0, \mathbf{Q})$ and measurement noise $v \sim N(0, R)$ are assumed to be white and independent random processes whose noise covariance matrices are defined as:

$$\mathbf{Q} = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}, \quad R = 5.$$

Find the optimal steady-state Kalman filter as $t \to \infty$ for the given problem. Write down the value of the estimation error covariance matrix, **P**, and the optimal Kalman gain, **K**.

Problem 2. (3 points) You are asked to implement a particle filter for the following nonlinear system.

$$\mathbf{x}_k = \begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} x_k + \cos\theta_k \\ y_k + \sin\theta_k \\ \theta_k \end{bmatrix}$$

The initial state estimate and covariance matrix are:

$$\mathbf{x}_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{P}_0 = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 10000 \end{bmatrix}$$

For your initial guess, generate a randomly distributed particles around considering the above-given initial condition. Use 100 or more particles and see how the performance of your particle filter changes as you increase the number of particles.

(a) Implement a particle filter and run its prediction step with no measurement update. Compute the result and plot it.

(b) Now let us add a measurement to our estimate:

$$\mathbf{z}_k = \mathbf{x}_k + \mathbf{v}_k$$

The error covariance matrix is $\mathbf{v}_k \sim N(0, R\mathbf{I})$ where R=0.01. Compute the result and plot it. Compare this result with your intuition on particle filters.

Problem 3. (5 points) Assume that you are tracking a ground vehicle moving on a horizontal plane with constant velocity. Its initial position and the constant velocity are

$$\mathbf{x}(0) = \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \mathbf{v}(t) = \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} 2.0 \\ -0.5 \end{bmatrix}$$

At each time *t*, we take range measurements of the vehicle from 4 stationary beacons located at

$$\mathbf{p}_B^1 = \begin{bmatrix} p_{x1} \\ p_{y1} \end{bmatrix} = \begin{bmatrix} 500 \\ -100 \end{bmatrix} \qquad \mathbf{p}_B^2 = \begin{bmatrix} 500 \\ 0 \end{bmatrix} \qquad \mathbf{p}_B^3 = \begin{bmatrix} 500 \\ 100 \end{bmatrix} \qquad \mathbf{p}_B^4 = \begin{bmatrix} 500 \\ 200 \end{bmatrix}$$

Denote the range to each beacon $\mathbf{p}_B^i$ at time *t* by

$$r_i(t) = \sqrt{(x(t) - p_{xi})^2 + \left(y(t) - p_{yi}\right)^2}.$$

Each of these range measurements is corrupted by Gaussian noise $v \sim N(0,1)$, so that the measurement equation for each beacon can be written as

$$z_i = r_i(t) + v_i(t).$$

(a) Write down the continuous-time state space equations for the system, and discretize with a sampling period of 1 second to construct a discrete-time system.

(b) Find the measurement Jacobian matrix **H** (4×4 matrix) by linearizing your measurement equation.

(c) Implement the extended Kalman filter to track the position and velocity of the target over the time interval between 0 to 100 seconds. Assume that the prior mean of the initial state to be the actual initial state and the prior covariance to be 1000 **I**. Plot the estimated position trajectory of the vehicle on top of the ground-truth trajectory.

(d) Plot the time trajectory of your position estimate in *x* and *y* along with 90% confidence interval that can be determined from the corresponding error covariance term.

(e) Now, change the prior mean of the initial state for the Kalman filter to be zero and rerun the filter. Provide the resulting plots. Does the estimate change much? Discuss why or why not.