Adaptations on de Bruijn Sequences
Led by Anant Godbole

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What is a Ucycle?

Ucycles, also known as Universal Cycles or de Bruijn sequences, contain all words in a given alphabet and word size exactly once in a single sequence.

Ucycle for all binary 3-letter words

$111, 110, 101, 010, 100, 000, 001, 011 : 11101000$

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Ucycles, also known as Universal Cycles or de Bruijn sequences, contain all words in a given alphabet and word size exactly once in a single sequence.

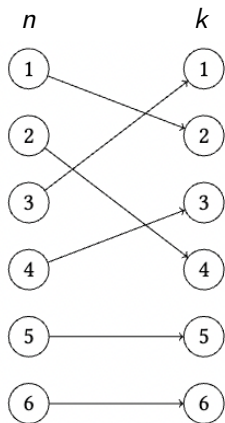Ucycle for all binary 3-letter words

$111, 110, 101, 010, 100, 000, 001, 011 : 11101000$

However we can't Ucycle everything, for example permutations:

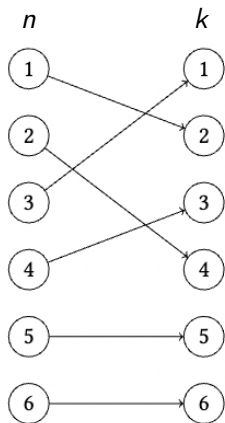$123, 231, 321, 213, 312, 132 : 123 \rightarrow 231 \rightarrow 312 \rightarrow 123$

What is a Gucycle?

Regular Case: Gucycling permutations for $k = n$, where n is the word size and k is the alphabet size

Regular Case: Gucycling permutations for $k = n$, where n is the
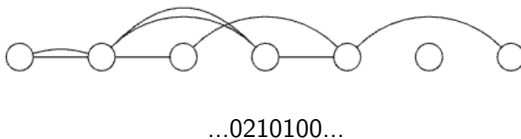word size and k is the alphabet size

...241356...

- Can we Gucycle everything we can Ucycle?

Main Questions

- Can we Gucycle everything we can Ucycle?
- What else can we Gucycle?

LS Gucycles
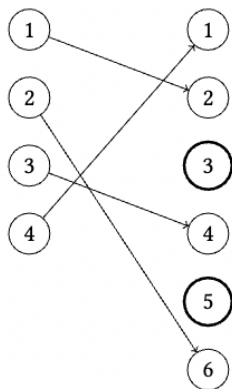
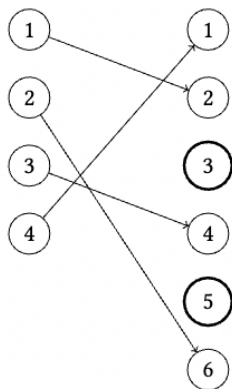- LS Gucycles can represent Ucycles
- Window of ≥ 3

...0210100...

Gucycling permutations for $k > n$

Gucycling different permutations

Gucycling permutations for $k > n$

...2641...

∄ one "loop" Gucycle

We know that there is no sink vertex to which all of these "looped" graphs can connect.

Reasoning

Assume $k \geq 4$, $n \geq 2$, and $n < k - 1$, meaning there must be at least two loops. The relationship between these loops must stay the same throughout the Gucycle, as when one loop 'leaves', another must appear to keep the same word size. Likewise, another loop can not appear until one leaves.

Possible Sink 1
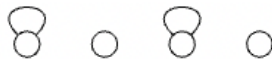
Possible Sink 2

Enumerating Distinct Gucycles

Thus, when $k > n$, many disjoint Gucycles are created to account for loop spacing. By definition, disjoint Gucycles contain distinct words.

Theorem

The total number of distinct disjoint Gucycles for $k > n$ is equal to $\lceil \binom{k-1}{n}/(k-n) \rceil$.

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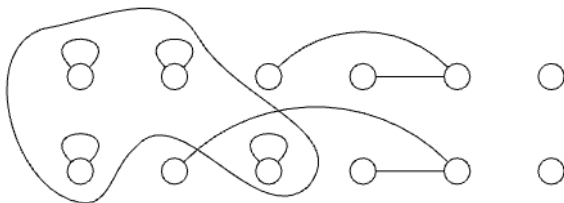
Theorem

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We get this from the spacing of sink vertices forming a partition of n, and then accounting for rotation.
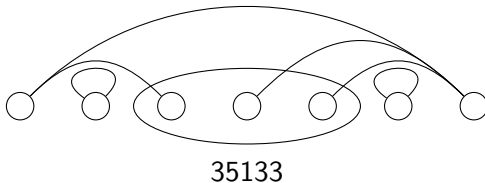
We can make hyper-graph cycles for $k > n$ words
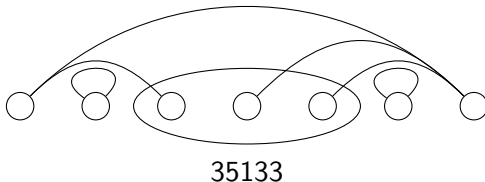

...5346... $\rightarrow$ ...5246...

Multisets

A different hypergraph strategy works $\forall$ multisets in which $\nexists$ an exact pair of any integer

35133

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A different hypergraph strategy works $\forall$ multisets in which $\nexists$ an exact pair of any integer
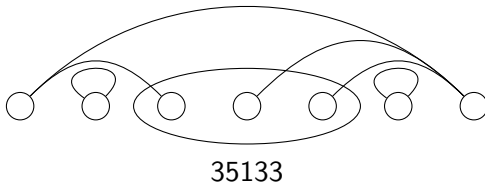

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The issue:
Must have vertex for each repeated integer as well as each omitted one. This means that words with different amounts of repeats will have different amount of vertices.

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Possible Solution: Using "hucycles"

What is a ranked perm?

What is a ranked perm?
1134447

What is a ranked perm?
1134447
3441174

Gucycling Ranked Permutations

What is a ranked perm?
1134447
3441174

3441174

Thank you!