Facial Recognition using Eigenfaces

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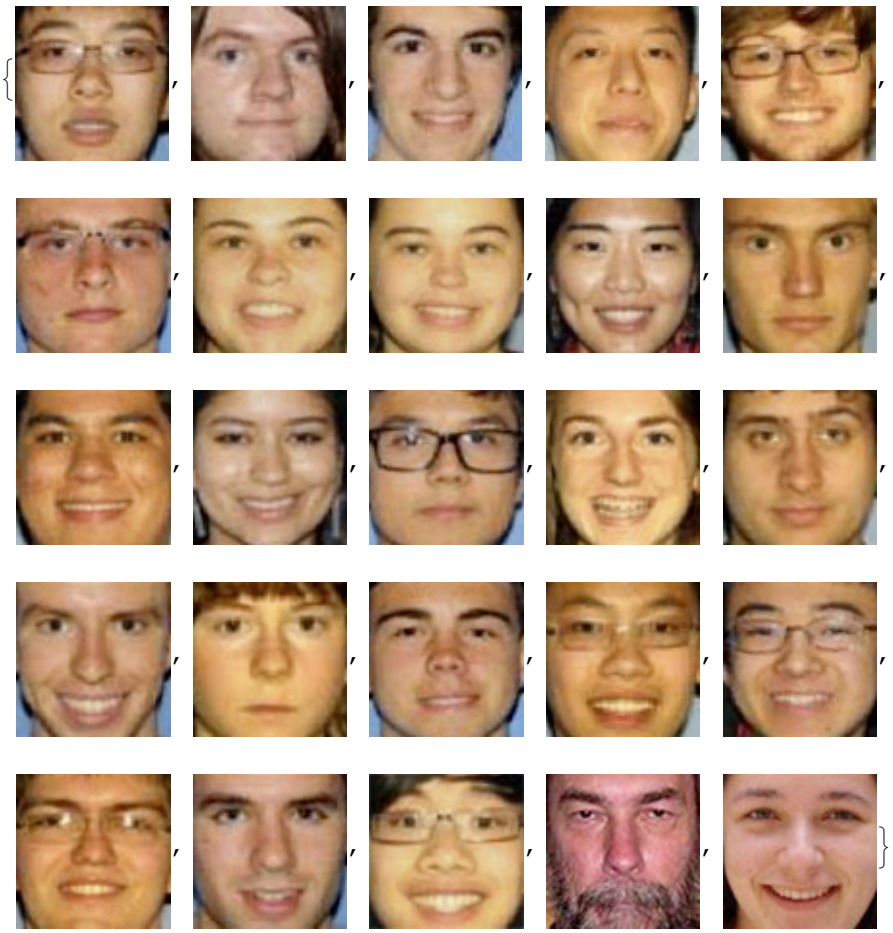
1 - Setting Up a Training Set

First import the images we want to use

```
SetDirectory[StringJoin[NotebookDirectory[], "math_images_resized"]];

photos = Map[Import, {"f1.jpg", "f2.jpg", "f3.jpg", "f4.jpg", "f5.jpg", "f6.jpg",
    "f7.jpg", "f8.jpg", "f9.jpg", "f10.jpg", "f11.jpg", "f12.jpg", "f13.jpg",
    "f14.jpg", "f15.jpg", "f16.jpg", "f17.jpg", "f18.jpg", "f19.jpg",
    "f20.jpg", "f21.jpg", "f22.jpg", "f23.jpg", "f24.jpg", "f25.jpg"}]
numPhotos = Length[photos];
```



Next convert all the photos to grayscale

```
grayPhotos = Map[ColorConvert[#, "Grayscale"] &, photos]
```

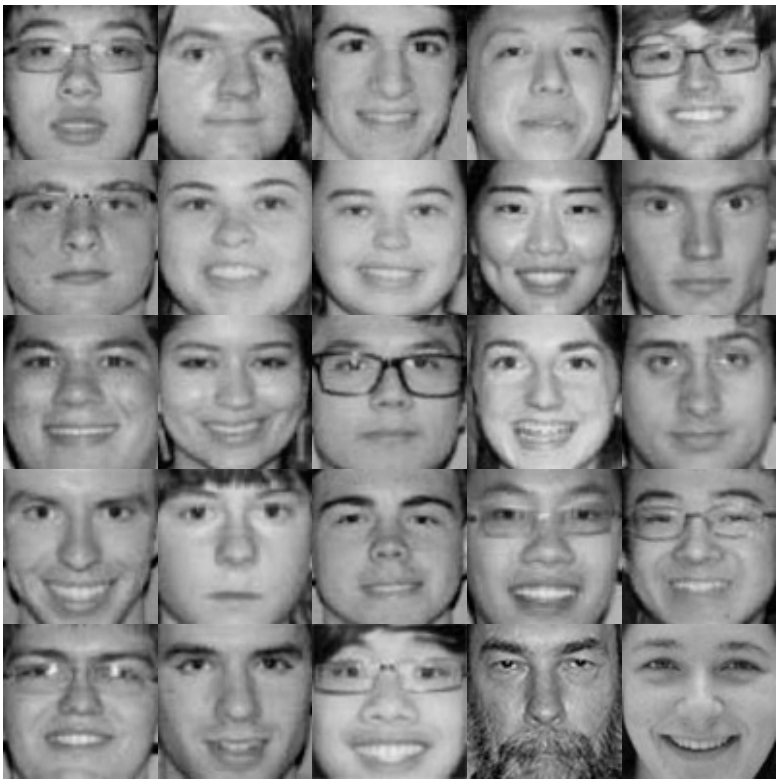
Create useful methods for converting an image to a vector vice versa

```
imageToVector[img_] := Flatten[ImageData[img]];
imageSize = Sqrt[Dimensions[imageToVector[grayPhotos[[1]]]]];
(* Assumes the photo is square *)
vectorToImage[vec_] := Image[Partition[vec, Sqrt[Dimensions[vec][[1]]]]];
grayPhotosVector = Map[imageToVector, grayPhotos];
```

(Optional) Display the images in a grid as a reference

```
grayPhotosMatrix = Transpose[Map[imageToVector, grayPhotos]];
```

```
grayPhotosGrid = Partition[grayPhotos, 5];
Grid[grayPhotosGrid, Spacings → {0, 0}]
```



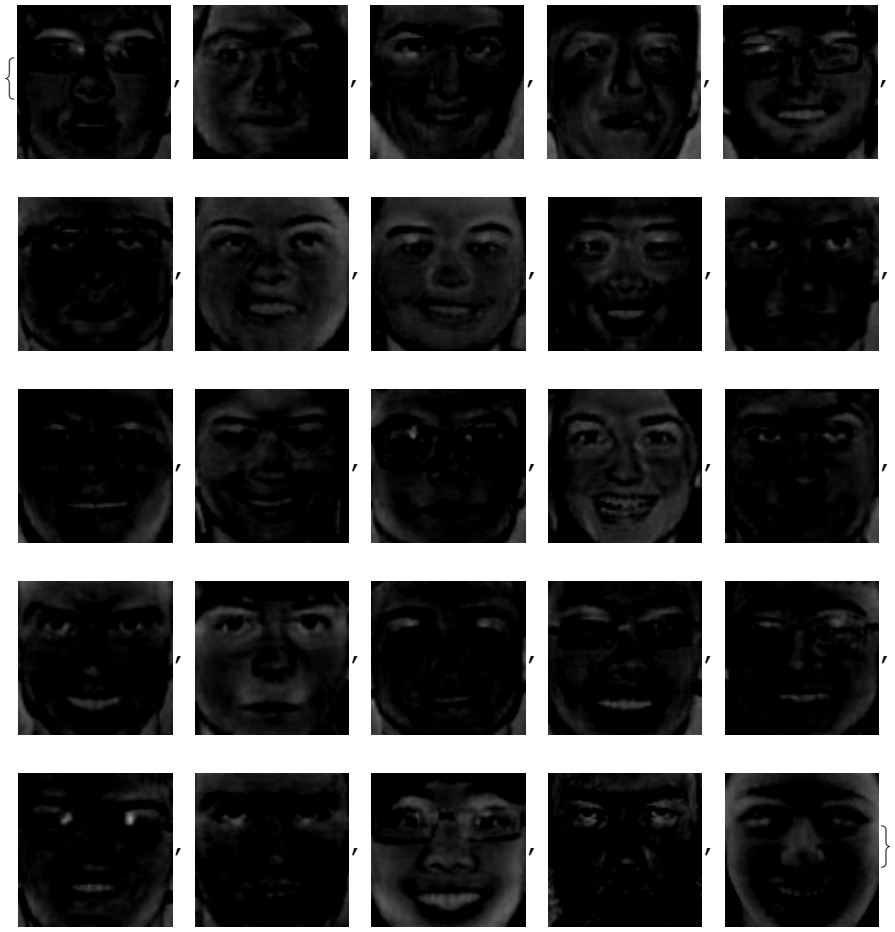
2 - Calculate the Mean and Difference Faces

Calculating the mean face

```
meanFaceMatrix = Apply[Plus, Map[ImageData, grayPhotos]] / numPhotos;
meanFaceVector = Flatten[meanFaceMatrix];
meanFaceImage = Image[meanFaceMatrix]
```

Calculating the difference face and difference vectors

```
differenceFaces = Map[ImageSubtract[#, meanFaceImage] &, grayPhotos]
differenceVectors = Map[imageToVector, differenceFaces];
```



(Optional) Display the images in a grid as a reference

```
(*displayedDifferencesFaces = 0;*)
getDisplayVector[vec_] := Module[
    {},
    Map[.5 + .5 # &, vec]
  ];
displayDifferenceVectors = Map[getDisplayVector[#] &, differenceVectors];
displayDifferenceFaces = Map[vectorToImage, displayDifferenceVectors];
differenceFacesGrid = Partition[displayDifferenceFaces, 5];
Grid[differenceFacesGrid, Spacings → {0, 0}]
```

The difference vector of the mean face:

```
vectorToImage[getDisplayVector[meanFaceVector - meanFaceVector]]
```

3 - Finding Eigenfaces

Find the small matrix

```
babyMatrix = differenceVectors.Transpose[differenceVectors];
```

Choose a fixed number of eigenfaces and diagonalize the eigenvectors of the normalized small matrix

```
numEigenFaces = numPhotos;
```

```
eigenfaceSystem = Eigensystem[babyMatrix];
eigenfaces =
  Map[Normalize[Transpose[differenceVectors].#] &, eigenfaceSystem[[2]]];
diagonalCovarianceMatrix = DiagonalMatrix[eigenfaceSystem[[1]]];
```

(Optional) Display the images in a grid as a reference

```
getEigenfaceDisplayVector[vec_] := Module[{},
    Map[.5 + 25 # &, vec]
  ];
displayEigenfaces =
  Map[vectorToImage[getEigenfaceDisplayVector[#]] &, eigenfaces];
Grid[Partition[displayEigenfaces, 5], Spacings → {0, 0}]
```

4 - Project Images into the Eigenspace

Define a function that projects an image to the eigenspace

```
projectImageToFaceSpace[imageVector_, meanFaceVector_, eigenfaces_] :=
  Module[{differenceVector},
   differenceVector = imageVector - meanFaceVector;
   Map[differenceVector.# &, eigenfaces]
  ];
```

Calculate the eigenface coefficients for each face you wish to project

```
(* A set of eigenface coefficients for each face *)
eigenfaceCoefficients =
  Map[projectImageToFaceSpace[#, meanFaceVector, eigenfaces] &, grayPhotosVector];
```

5 - Recontruct Images from Eigenspace

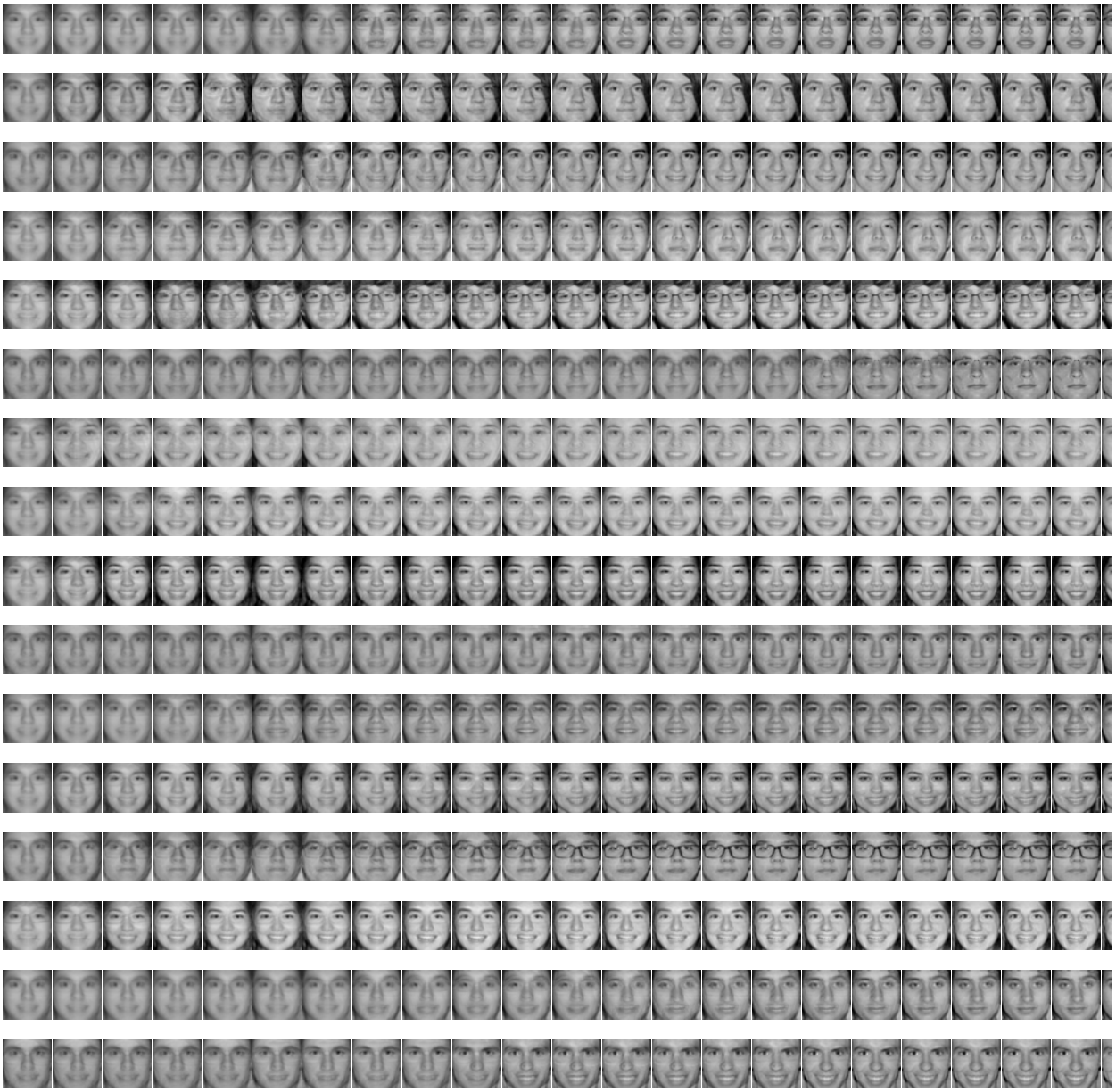
Define a function that reconstructs a face vector from eigenface coefficients

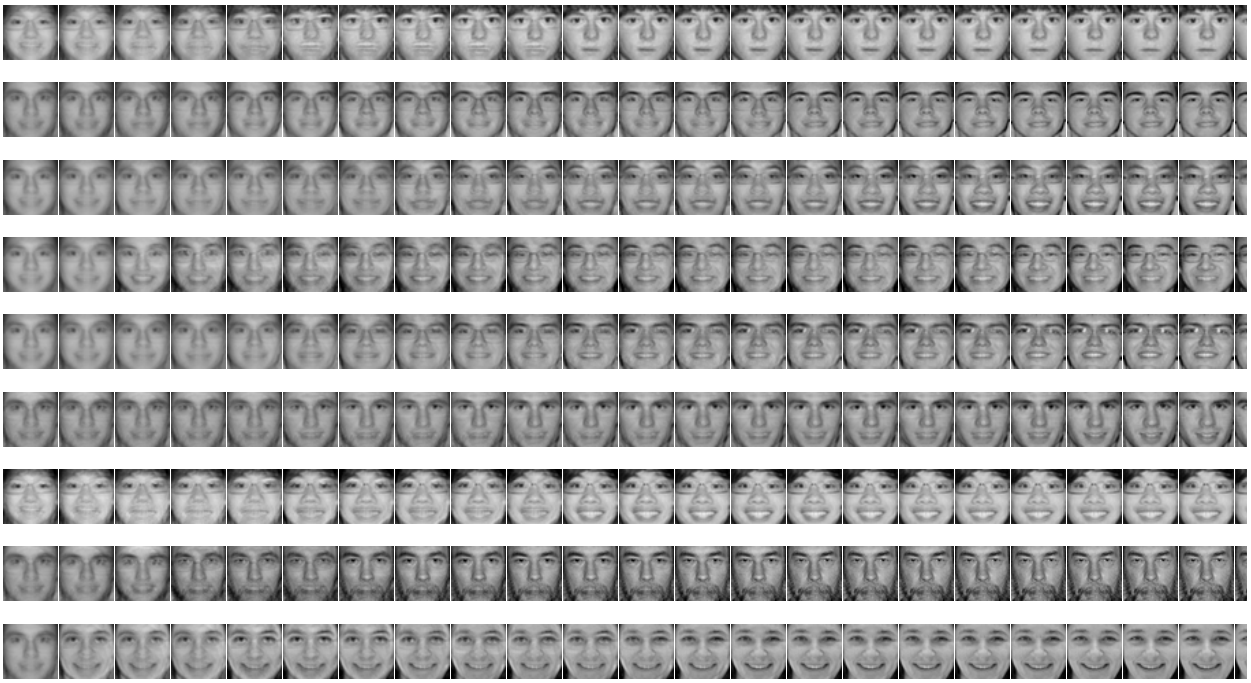
```
rebuildVectorFromEigenfaces[coefficients_,
    meanFaceVector_, eigenfaces_, numEigenfaces_] :=
  meanFaceVector + Apply[Plus, (coefficients * eigenfaces)[[ ;; numEigenfaces]]];
```

Reconstruct some training set faces

```
randomFaces =
  Map[vectorToImage, {rebuildVectorFromEigenfaces[eigenfaceCoefficients[[5]],
      meanFaceVector, eigenfaces, 3], rebuildVectorFromEigenfaces[
      eigenfaceCoefficients[[23]], meanFaceVector, eigenfaces, 24],
     rebuildVectorFromEigenfaces[eigenfaceCoefficients[[3]],
      meanFaceVector, eigenfaces, 13], rebuildVectorFromEigenfaces[
      eigenfaceCoefficients[[12]], meanFaceVector, eigenfaces, 6],
     rebuildVectorFromEigenfaces[eigenfaceCoefficients[[24]],
      meanFaceVector, eigenfaces, 1], rebuildVectorFromEigenfaces[
      eigenfaceCoefficients[[2]], meanFaceVector, eigenfaces, 22]}];
Grid[Partition[randomFaces, Length[randomFaces] / 2], Spacings → {0, 0}]
```

```
reconstructedTrainingFaces = Table[
   Table[
    ImageResize[
     vectorToImage[
      rebuildVectorFromEigenfaces[eigenfaceCoefficients[[photoNum]],
       meanFaceVector, eigenfaces, numEigenfaces]
     ], {40}],
    {numEigenfaces, numPhotos}],
   {photoNum, numPhotos}
  ];
Grid[reconstructedTrainingFaces, Spacings → {0, 0}]
```

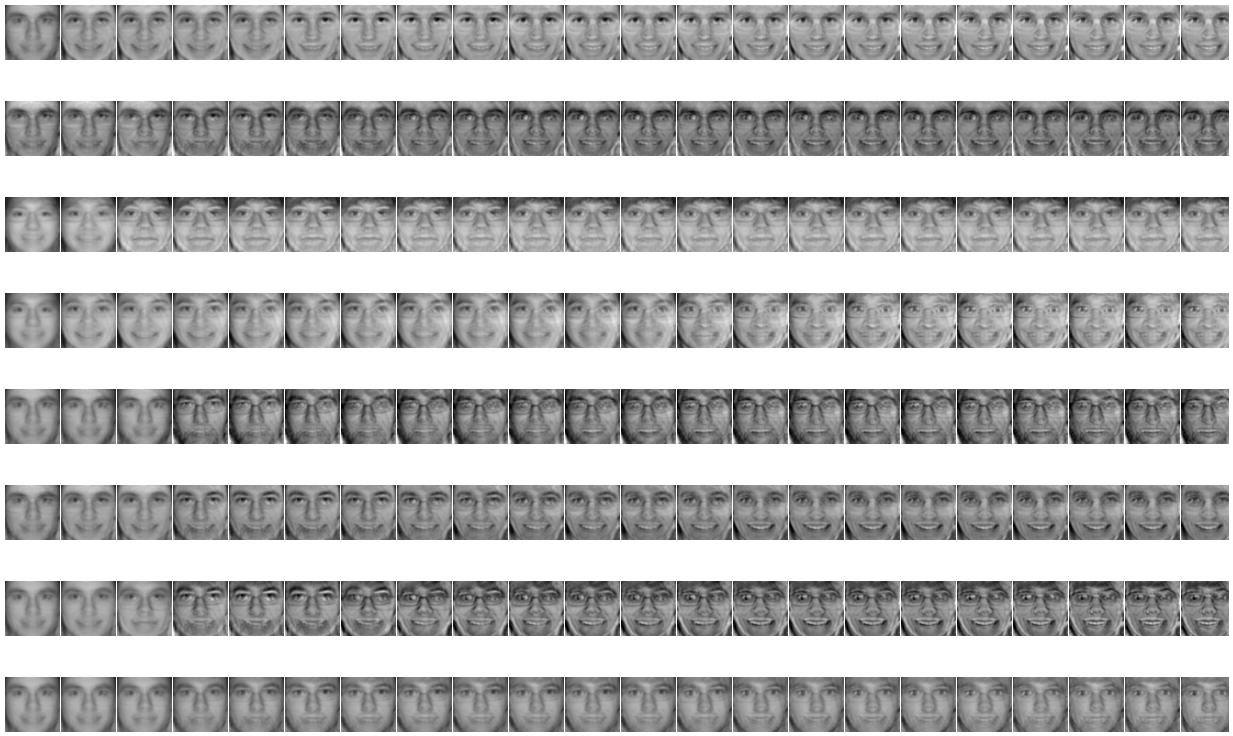




Reconstruct some faces not in training set

```
otherPhotos = Map[Import, {"fb1.jpg", "fb2.jpg", "fb3.jpg", "fb4.jpg",
    "orangutan.jpg", "steven.jpg", "kirby.jpg", "flower.jpg"}];
numOtherPhotos = Length[otherPhotos];
otherGrayPhotos = Map[ColorConvert[#, "Grayscale"] &, otherPhotos];
Grid[Partition[otherGrayPhotos, Length[otherPhotos]], Spacings → {0, 0}]
otherGrayPhotosVector = Map[imageToVector, otherGrayPhotos];
```

```
otherEigenfaceCoefficients =
  Map[projectImageToFaceSpace[#, meanFaceVector, eigenfaces] &,
   otherGrayPhotosVector];
imageSize = 50;
otherReconstructedFaces =
  Table[
   Table[
    ImageResize[
     vectorToImage[
      rebuildVectorFromEigenfaces[otherEigenfaceCoefficients[[photoNum]],
       meanFaceVector, eigenfaces, numEigenfaces]
     ], {imageSize}],
    {numEigenfaces, numPhotos}],
   {photoNum, numOtherPhotos}
  ];
{Grid[otherReconstructedFaces, Spacings → {0, 0}],
 Grid[Partition[Map[ImageResize[#, {imageSize}] &, otherGrayPhotos], 1],
  Spacings → {0, 0}]}
```



```
Grid[{otherGrayPhotos,
  Table[vectorToImage[rebuildVectorFromEigenfaces[otherEigenfaceCoefficients[[
      photoNum]], meanFaceVector, eigenfaces, numPhotos]],
   {photoNum, numOtherPhotos}]}, Spacings → {0, 0}]
```

6 - Measure Reconstruction Accuracy

Define a function that gets the difference between two image vectors

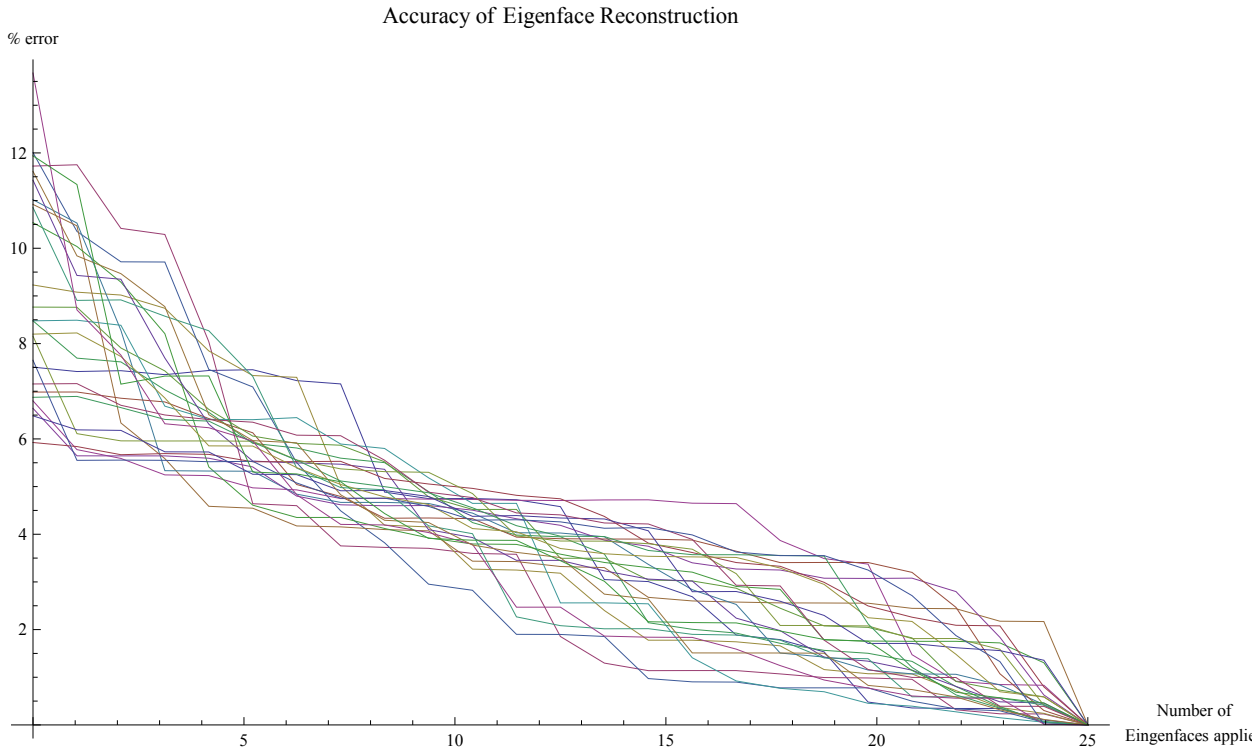
```
getImageVectorDifference[imageVector1_, imageVector2_] :=
  Module[{differenceVector},
   differenceVector = imageVector1 - imageVector2;
   Total@Abs@differenceVector / Times @@ Dimensions@imageVector1 * 100
  ];
```

Calculate the differences between the normal images and the reconstructed images

```
imgNum = 21;
imgSize = 50;
accuracyTable = {
   Table[ImageResize[grayPhotos[[imgNum]], imgSize], {i, 0, 25}],
   Table[ImageResize[vectorToImage[rebuildVectorFromEigenfaces[
        eigenfaceCoefficients[[imgNum]], meanFaceVector, eigenfaces,
        numEigenfaces]], imgSize], {numEigenfaces, 0, 25}],
   Table[StringJoin[
     "\n",
     ToString[
      NumberForm[getImageVectorDifference[
        grayPhotosVector[[imgNum]], rebuildVectorFromEigenfaces[
         eigenfaceCoefficients[[imgNum]], meanFaceVector, eigenfaces, i]], 2]
     ],
     "%"], {i, 0, 25}]
  };
Grid[accuracyTable, Spacings → {0, 0}];
getAccuracyTable[imageVectors1_, imageVectors2_] := Module[
   {},
   {
    Map[vectorToImage, imageVectors1],
    Map[vectorToImage, imageVectors2],
    Table[
     StringJoin[
      "\n",
      ToString[
       NumberForm[
        getImageVectorDifference[imageVectors1[[1]], imageVectors2[[i]]], 2]
      ],
      "%"], {i, 1, 25}
    ]
   }
  ];
```

Graph the accuracy over number of applied eigenfaces

```
diffs = Table[Table[getImageVectorDifference[grayPhotosVector[[faceNum]],
      rebuildVectorFromEigenfaces[eigenfaceCoefficients[[faceNum]],
       meanFaceVector, eigenfaces, numEigenfaces]],
     {numEigenfaces, 0, 24}], {faceNum, 1, 25}];
faceNum = 2;
ListPlot[diffs, Joined → True, InterpolationOrder → 1,
 DataRange → {0, 25}, PlotLabel → "Accuracy of Eigenface Reconstruction",
 AxesLabel → {"Number of\nEingenfaces applied", "% error"}]
```



7 - Facial Recognition

To recognize a face in the training set, we seek the training set image with the minimum variance with the reconstructed image

```
getPercentMatches[imgNum_, numEigenfaces_] := Module[{},
    inputVector = rebuildVectorFromEigenfaces[eigenfaceCoefficients[[imgNum]],
      meanFaceVector, eigenfaces, numEigenfaces];
    differencePercentagesBetweenFaces = Table[
      getImageVectorDifference[grayPhotosVector[[i]], inputVector], {i, 25}];
    sortedOrder = Sort[differencePercentagesBetweenFaces]
  ];
getMatchedPhotos[matchOrder_] := Module[{},
    Table[grayPhotos[[Position[
        differencePercentagesBetweenFaces, sortedOrder[[i]]][[1, 1]]]], {i, 25}]
  ];
getPercentMatchesOther[imgNum_, numEigenfaces_] := Module[{},
    inputVector = rebuildVectorFromEigenfaces[otherEigenfaceCoefficients[[
       imgNum]], meanFaceVector, eigenfaces, numEigenfaces];
    differencePercentagesBetweenFaces = Table[
      getImageVectorDifference[grayPhotosVector[[i]], inputVector], {i, 25}];
    sortedOrder = Sort[differencePercentagesBetweenFaces]
  ];
```

(Optional) Create a grid that shows the order of the face matches

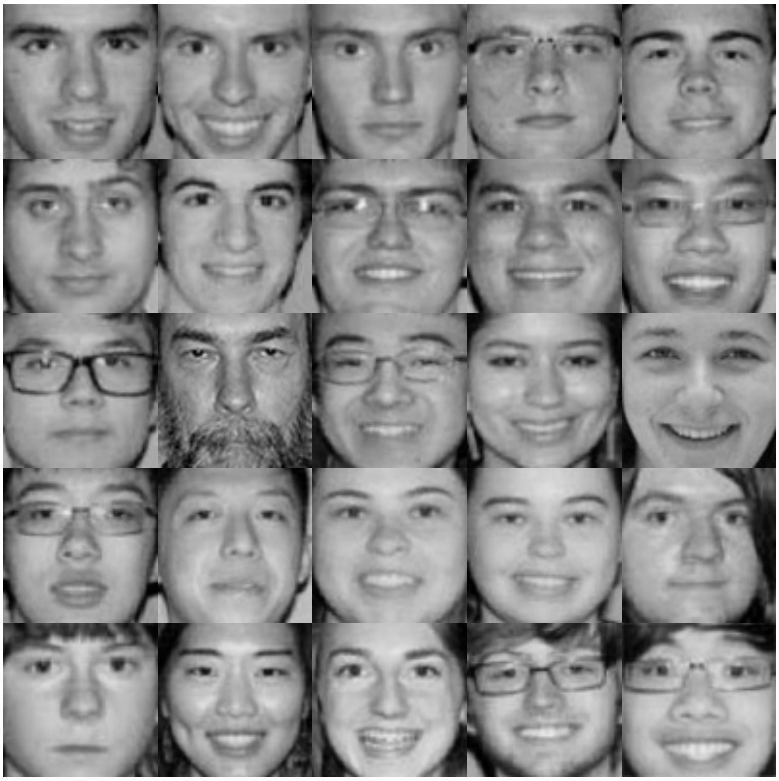
```
Style[Grid[Partition[{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
    17, 18, 19, 20, 21, 22, 23, 24, 25}, 5], Frame → All], FontSize → 60]
```

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Recognize Training Faces

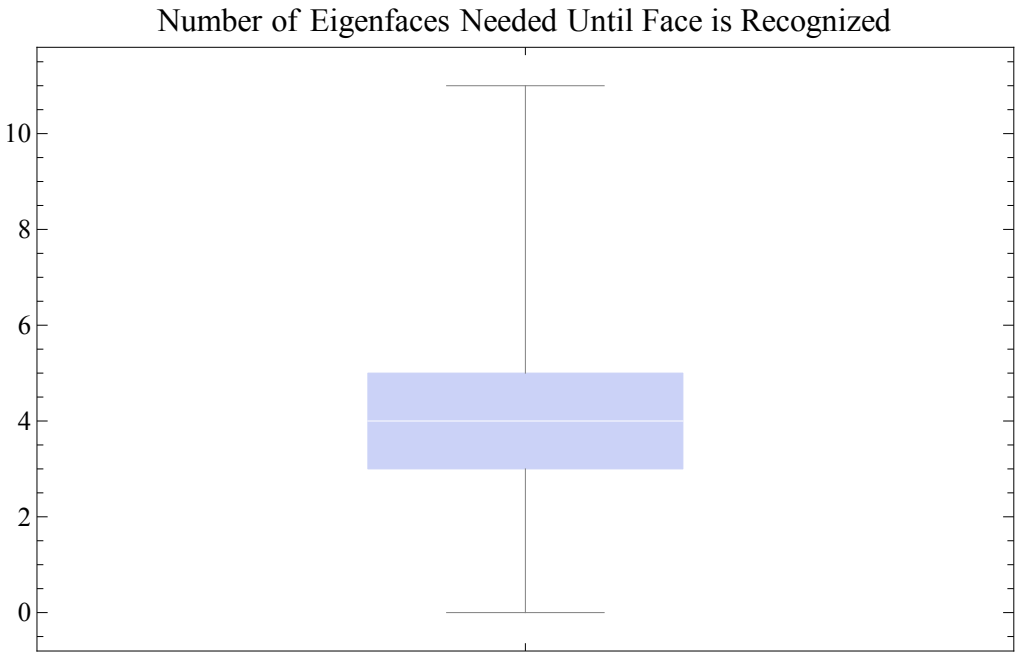
Choose a face number (faceNum) from the training faces set and number of eigenfaces to apply (numEigen) to recreate the face

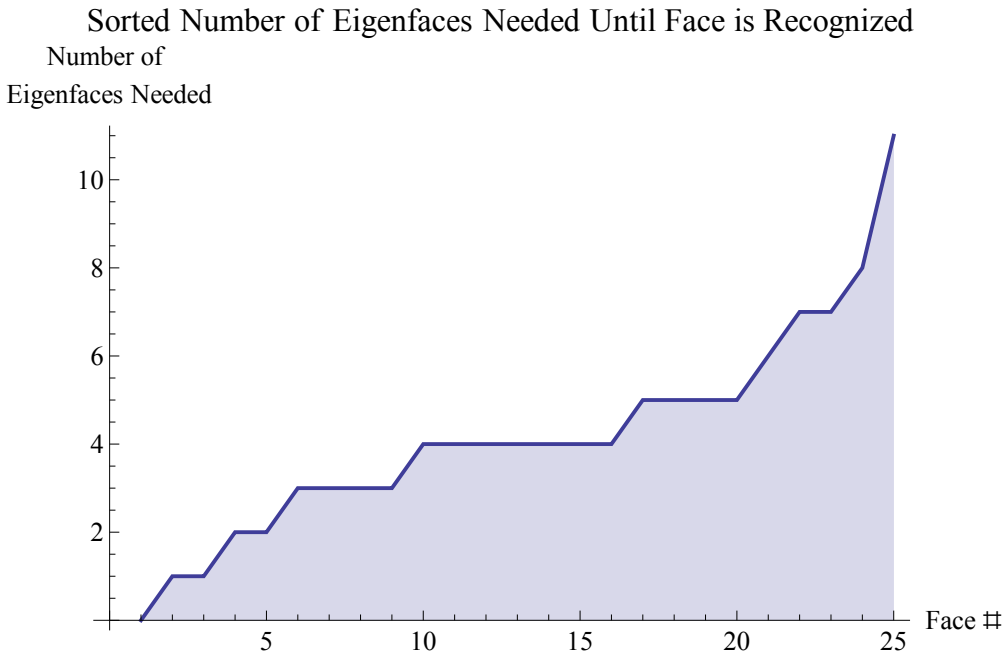
```
faceNum = 16;
numEigen = 8;
recreatedPhoto = vectorToImage[rebuildVectorFromEigenfaces[
    eigenfaceCoefficients[[faceNum]], meanFaceVector, eigenfaces, numEigen]];
Grid[{{grayPhotos[[faceNum]], recreatedPhoto}}, Spacings → {0, 0}]
order = getPercentMatches[faceNum, numEigen];
Grid[Partition[getMatchedPhotos[order], 5], Spacings → {0, 0}]
SetOptions[BarChart, BaseStyle → {FontFamily → "Times", FontSize → 14}];
BarChart[order, ChartStyle → "Pastel", AxesLabel → {"Faces", "% error"},
 PlotLabel → "Recognizing a Reconstructed Training Set Face",
 ChartLabels -> {"1", "2", "3", "4", "5", "6", "7", "8", "9", "10", "11", "12",
   "13", "14", "15", "16", "17", "18", "19", "20", "21", "22", "23", "24", "25"}]
```



Results of Recognizing Training Faces

```
numEigenToRecognize =
  {8, 5, 7, 3, 4, 3, 2, 4, 2, 5, 6, 4, 4, 4, 0, 11, 4, 3, 5, 3, 7, 1, 1, 4, 5};
SetOptions[ListPlot, BaseStyle → {FontFamily → "Times", FontSize → 14}];
SetOptions[BoxWhiskerChart, BaseStyle → {FontFamily → "Times", FontSize → 14}];
GraphicsGrid[{{BoxWhiskerChart[numEigenToRecognize,
    PlotLabel → "Number of Eigenfaces Needed Until Face is Recognized"]},
  {ListPlot[Sort[numEigenToRecognize], Filling → Axis,
    PlotStyle → {Thick, PointSize[Large]}, Joined → True,
    PlotLabel → "Sorted Number of Eigenfaces Needed Until Face is Recognized",
    AxesLabel → {"Face #", "Number of \nEigenfaces Needed"}]}}]
```


Number of Eigenfaces Needed Until Face is Recognized
10
8
6
4
2
0

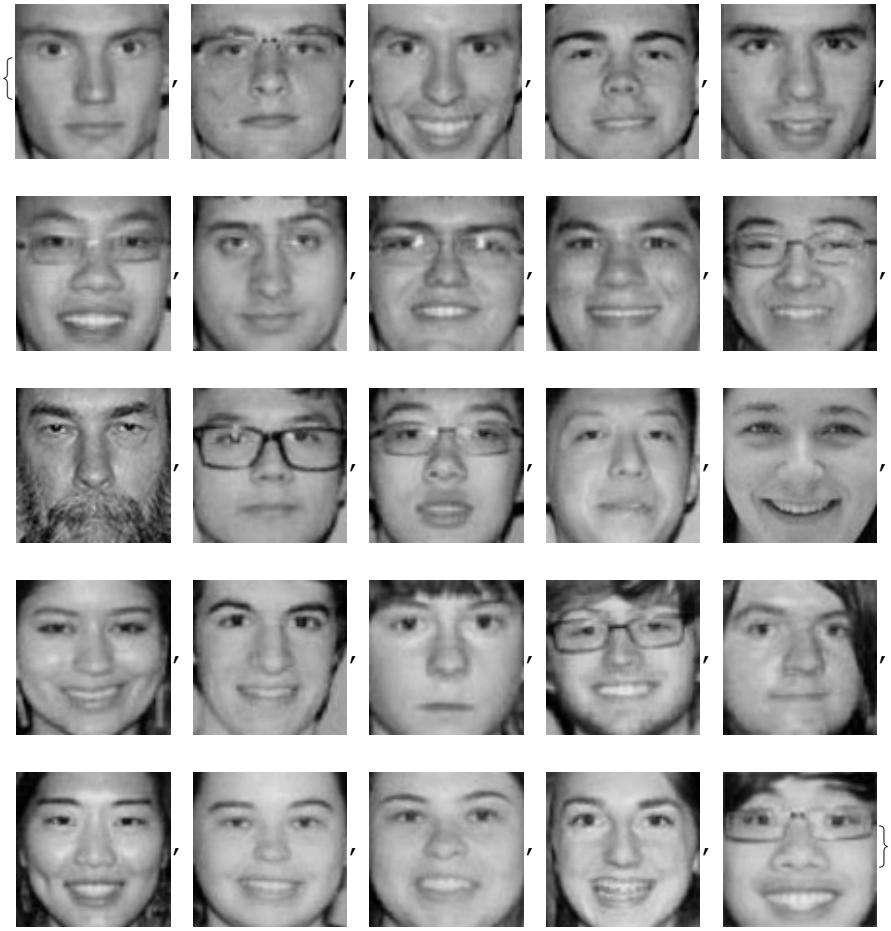

Sorted Number of Eigenfaces Needed Until Face is Recognized
Number of
Eigenfaces Needed
10
8
6
4
2
5
10
15
20
25
Face #

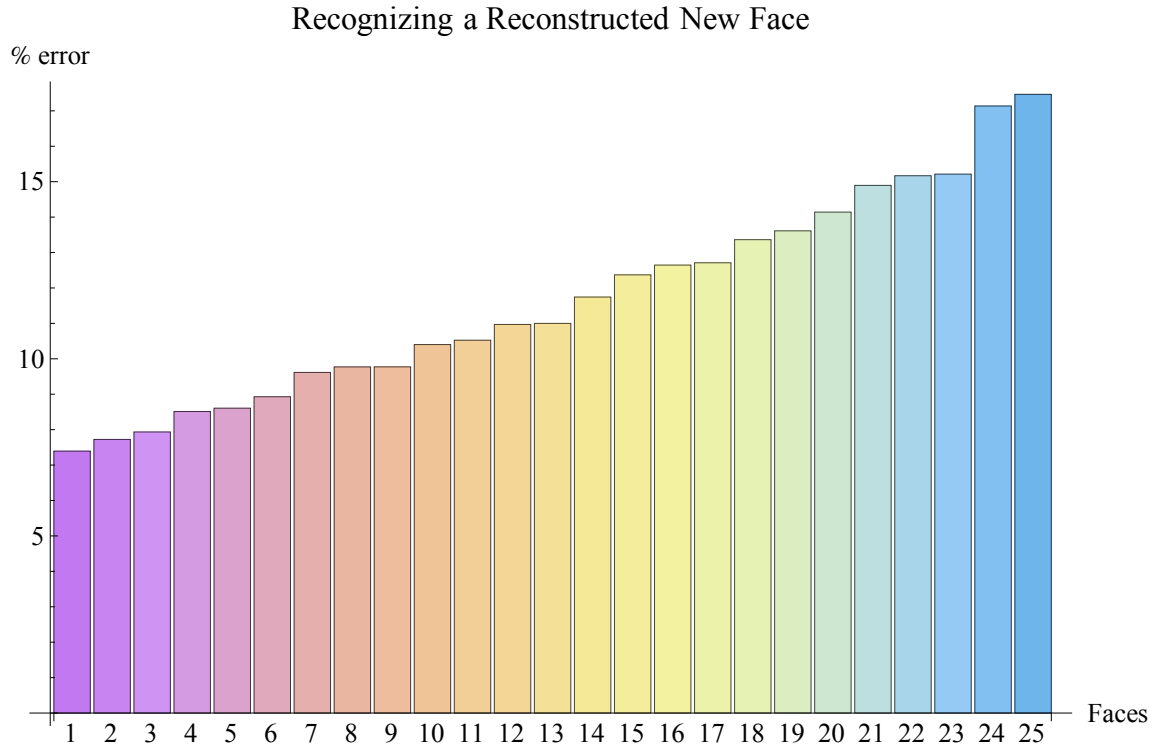
Recognizing Foreign Faces

Choose a face number (faceNum) from the foreign faces set and number of eigenfaces to apply (numEigen) to recreate the face

```
faceNum = 8;
numEigen = 24;
recreatedPhotos = Table[
   vectorToImage[rebuildVectorFromEigenfaces[otherEigenfaceCoefficients[[i]],
     meanFaceVector, eigenfaces, numEigen]], {i, numOtherPhotos}];
{otherGrayPhotos[[faceNum]], recreatedPhotos[[faceNum]]};
order = getPercentMatchesOther[faceNum, numEigen];
getMatchedPhotos[order]

BarChart[order, ChartStyle → "Pastel", AxesLabel → {"Faces", "% error"},
 PlotLabel → "Recognizing a Reconstructed New Face",
 ChartLabels -> {"1", "2", "3", "4", "5", "6", "7", "8", "9", "10", "11", "12",
   "13", "14", "15", "16", "17", "18", "19", "20", "21", "22", "23", "24", "25"}]
```




Recognizing a Reconstructed New Face
% error
15
10
5
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Faces

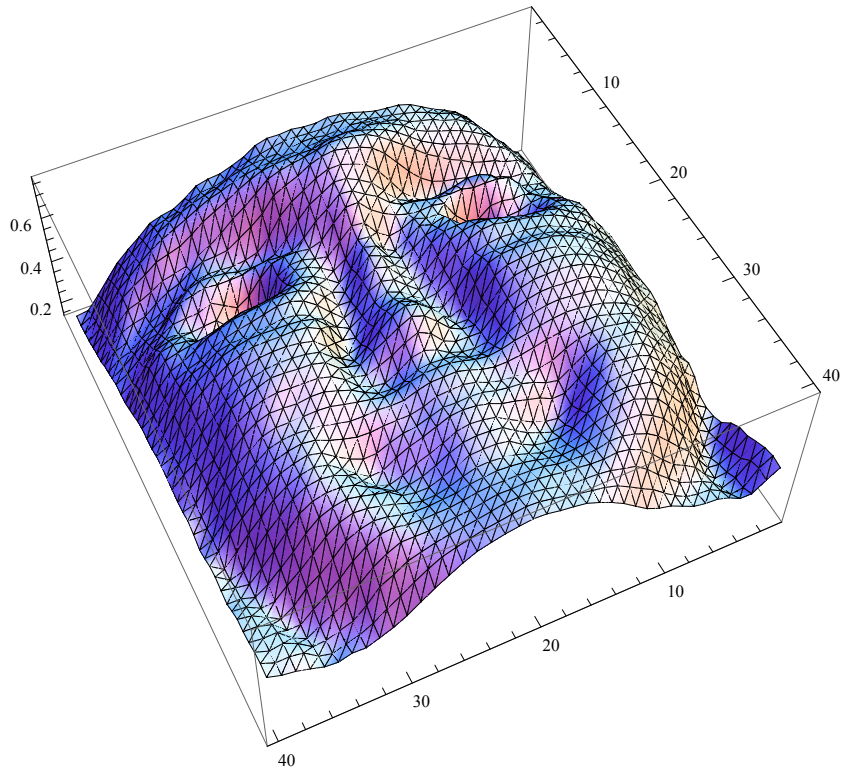
Results of Recognizing Foreign Faces

```
t = Table[{Grid[{{otherGrayPhotos[[i]], recreatedPhotos[[i]]}}, Spacings → {0, 0}],
    Grid[{getMatchedPhotos[getPercentMatchesOther[i, 24]][[1 ;; 5]]},
     Spacings → {0, 0}]}, {i, 8}];
Grid[t, Spacings → {2, 0}]
```

(Optional) Face Visualizations

Mean Face Luminosity Visualization

```
imgSize = 40;
ListPlot3D[Partition[imageToVector[
    ImageResize[meanFaceImage, {imgSize, imgSize}]], imgSize], Mesh → All]
```



Face Luminosity Visualization

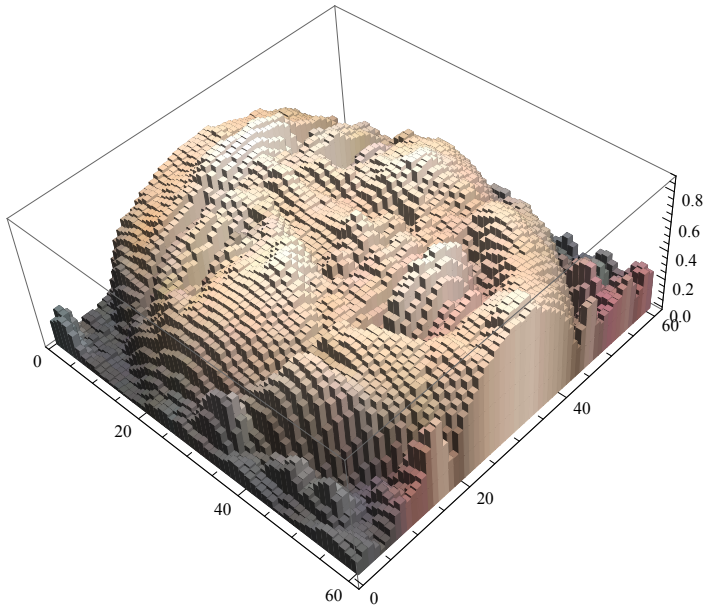
```
density = 60;
picNum = 9;
alpha = 1;
blurValue = 0;
colorData =
  ImageData[Blur[ImageResize[photos[[picNum]], {density, density}], blurValue]];

getColor[x0_, y0_] := Module[
   {x = x0, y = y0},
   y *= density;
   x *= density;
   If[y < 1, y = 1,];
   If[x < 1, x = 1,];
   If[y > density, RGBColor[0, 0, 0, alpha],
    If[x > density , RGBColor[0, 0, 0, alpha], pictureColor[Round[x], Round[y]]]]
  ];

pictureColor[x_, y_] := RGBColor[colorData[[x]][[y]][[1]],
   colorData[[x]][[y]][[2]], colorData[[x]][[y]][[3]], alpha];
resizeImage[i_] :=  Blur[ImageResize[grayPhotos[[i]], {density, density}],
   blurValue];
getImageVector[i_] := Transpose[
   Partition[imageToVector[resizeImage[i]], density]];

v1[i_] := ListPlot3D[{getImageVector[i]}, Mesh → None,
   InterpolationOrder → 1, ColorFunction → Function[{x, y, z}, getColor[x, y]]];
v2[i_] := ListPointPlot3D[{getImageVector[i]},
   ColorFunction → Function[{x, y, z}, getColor[x, y]],
   BoxRatios → {1, 1, .5}, PlotStyle → PointSize[Large]];
v3[i_] := ListPlot3D[{getImageVector[i]}, InterpolationOrder → 0, ColorFunction →
    Function[{x, y, z}, getColor[x, y]], Mesh → None, BoundaryStyle → None];
v4[i_] := DiscretePlot3D[getImageVector[i][[k]][[j]], {j, 1, density},
  {k, 1, density}, ColorFunction → Function[{x, y, z}, getColor[x, y]],
  ExtentSize → Full, PlotStyle → EdgeForm[], ExtentElementFunction → "Cube"]
```

```
v4[picNum]
```



Animations

```
(*Export["/animation.gif",Table[rebuildFromEigenfaces[
    coefficients, meanImage, eigenfaces, k], {k, 1, 25}],"GIF"];*)
```