ECSE 446/546

Image Synthesis

Direct Illumination II

image credit: feelgrafix

Prof. Derek Nowrouzezahrai

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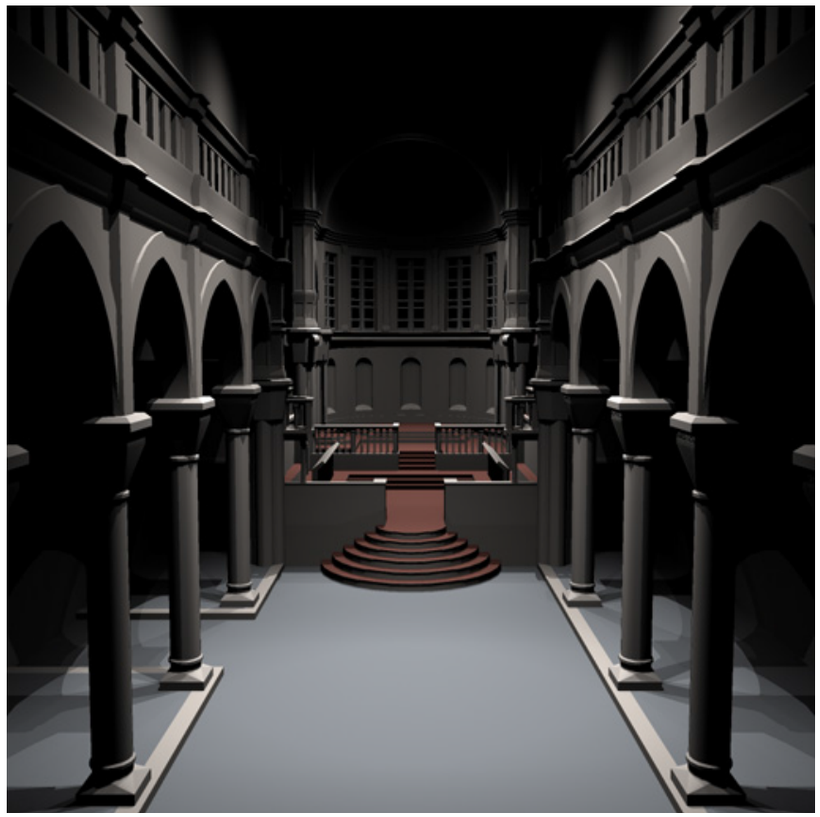
(with some slides adapted from Drs. Novák, Jarosz, and Debevec)

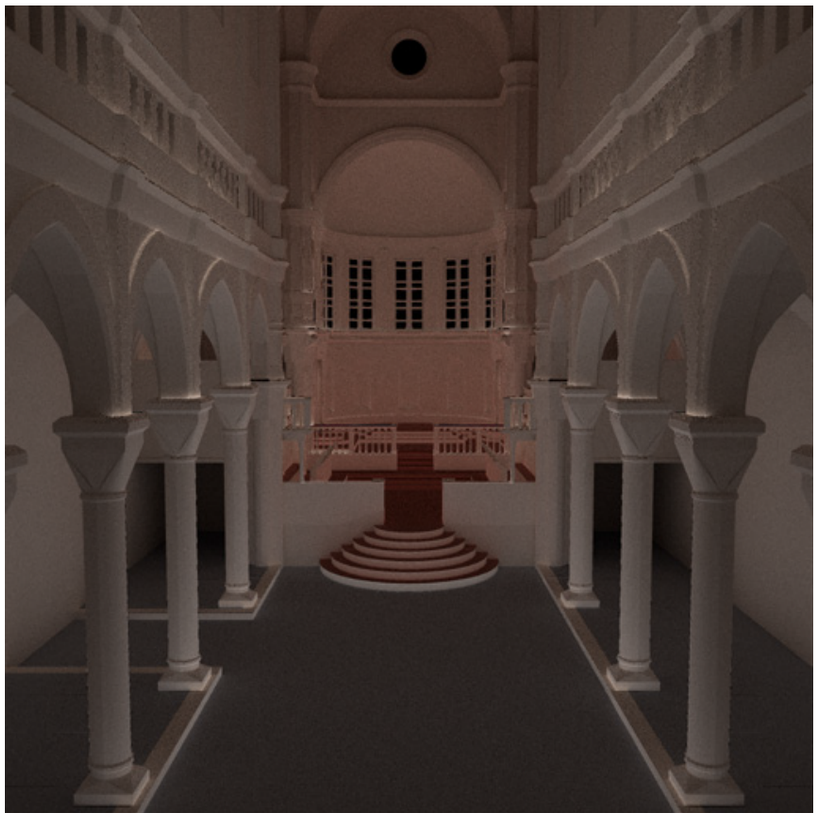
McGill

Direct vs. Indirect Illumination

Direct illumination



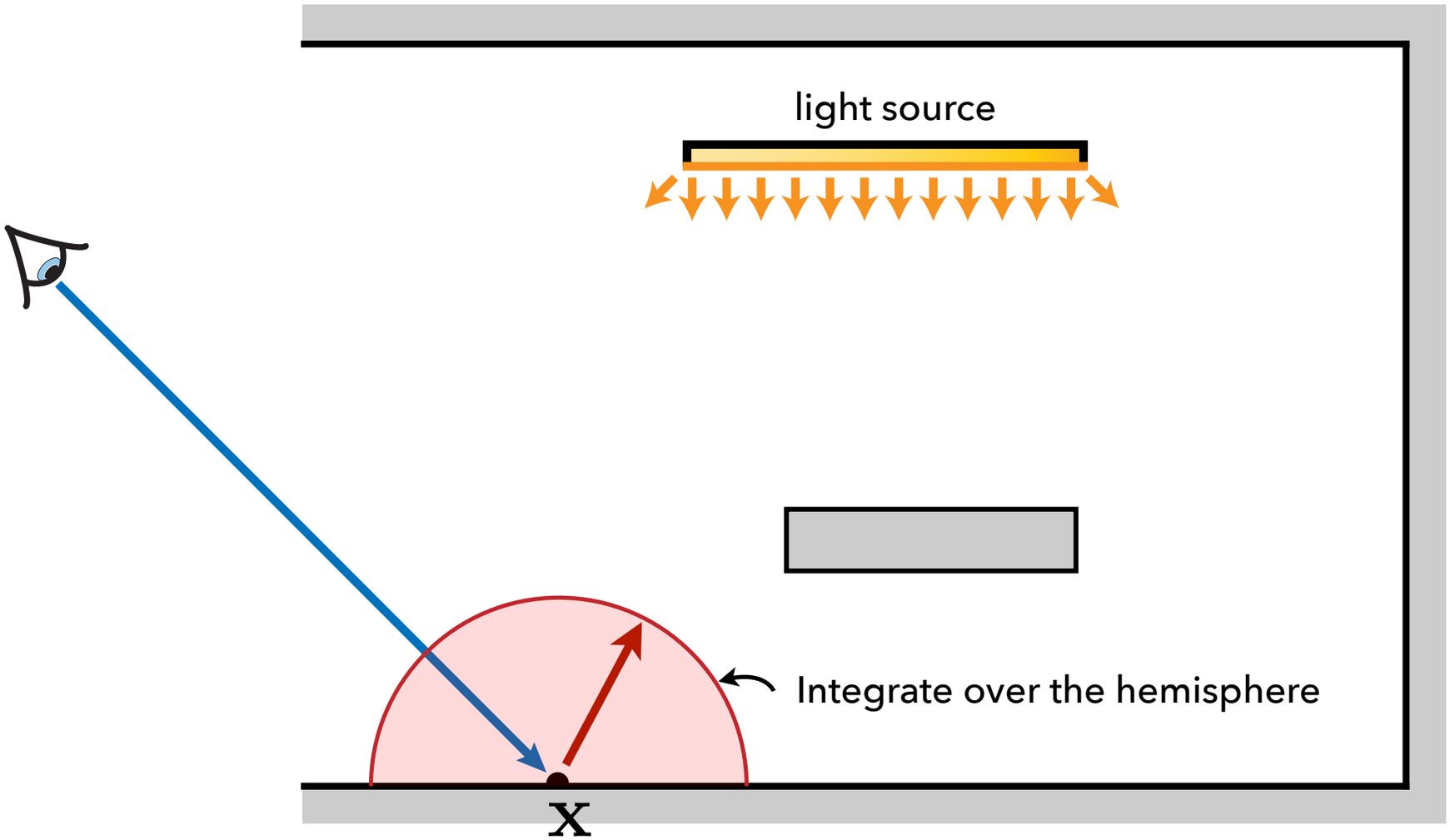
Indirect illumination



Direct + indirect illumination

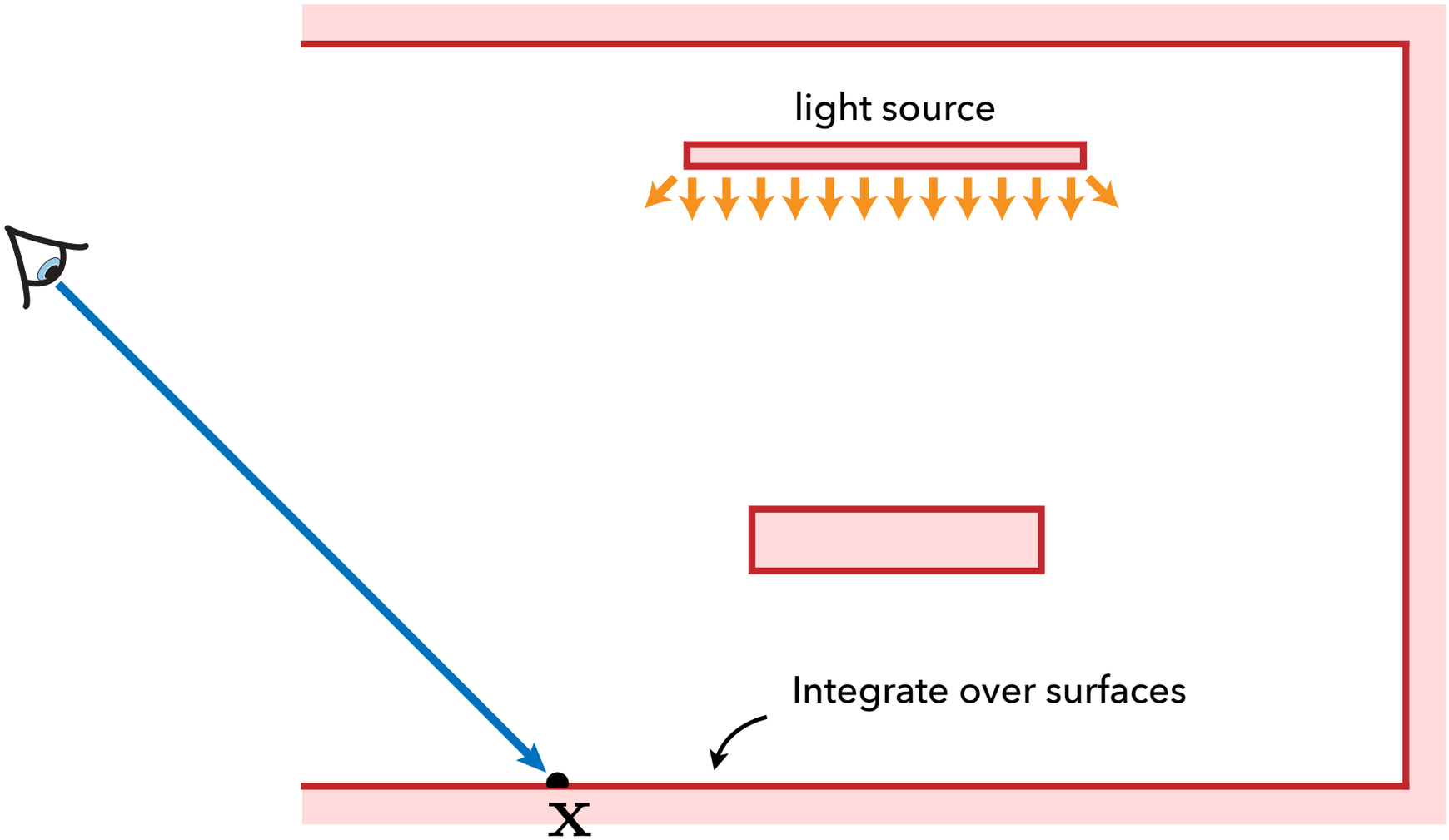
Reflection Eq. – Hemispherical Form

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_{H^2} f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_e(r(\mathbf{x}, \vec{\omega}_i), -\vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$



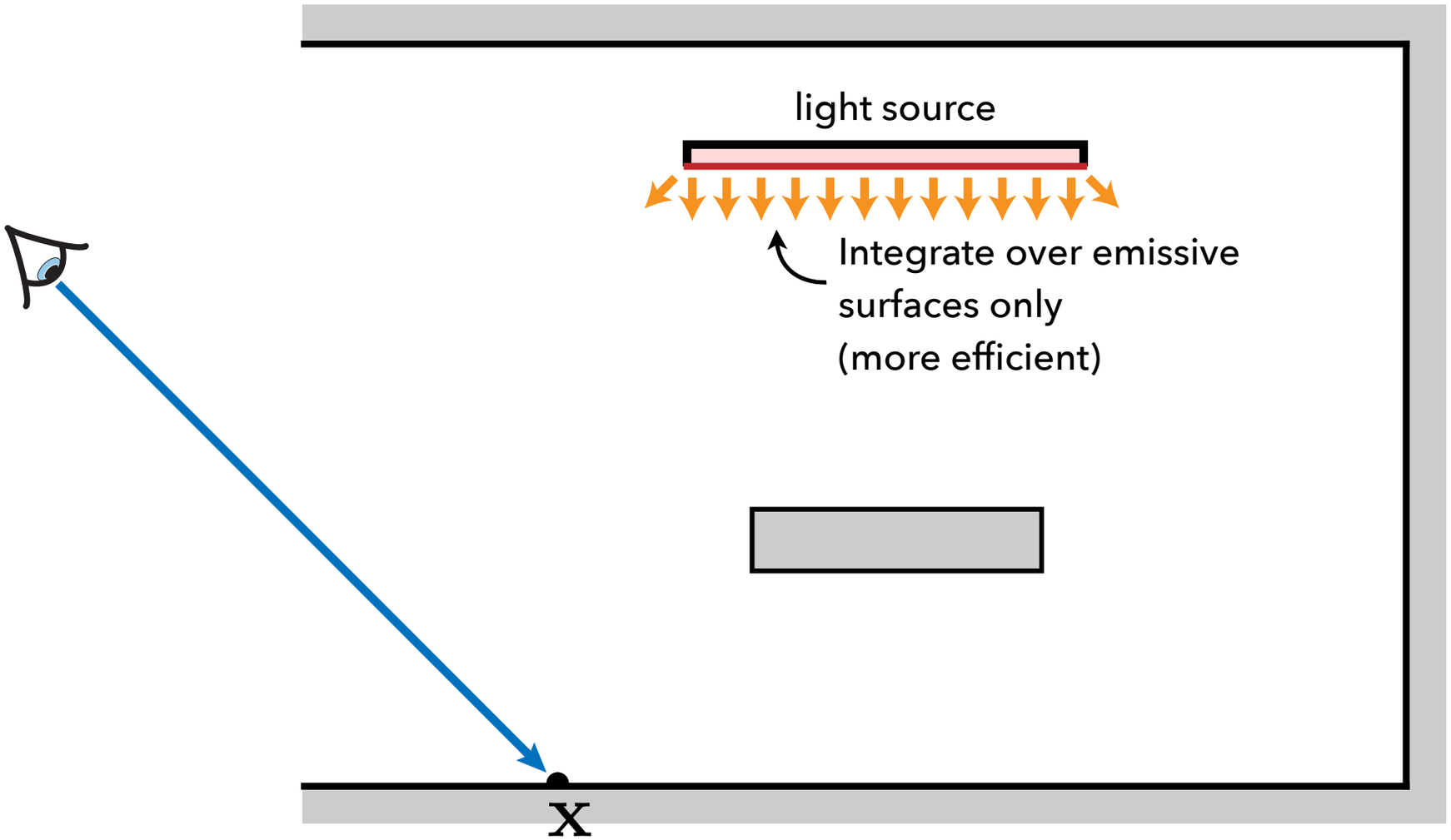
Reflection Eq. – Surface Area Form

$$L_r(\mathbf{x}, \mathbf{z}) = \int_A f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_e(\mathbf{y}, \mathbf{x}) V(\mathbf{x}, \mathbf{y}) \frac{|\cos\theta_i| \, |\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA(\mathbf{y})$$



Reflection Eq. – Surface Area Form

$$L_r(\mathbf{x}, \mathbf{z}) = \int_{A_e} f_r(\mathbf{x}, \mathbf{y}, \mathbf{z}) L_e(\mathbf{y}, \mathbf{x}) V(\mathbf{x}, \mathbf{y}) \frac{|\cos\theta_i| \, |\cos\theta_o|}{\|\mathbf{x} - \mathbf{y}\|^2} dA(\mathbf{y})$$



Light Sources

Point light

Directional light

Quad light

Sphere light

Mesh light

Delta lights

(create hard shadows)

Area/Shape lights

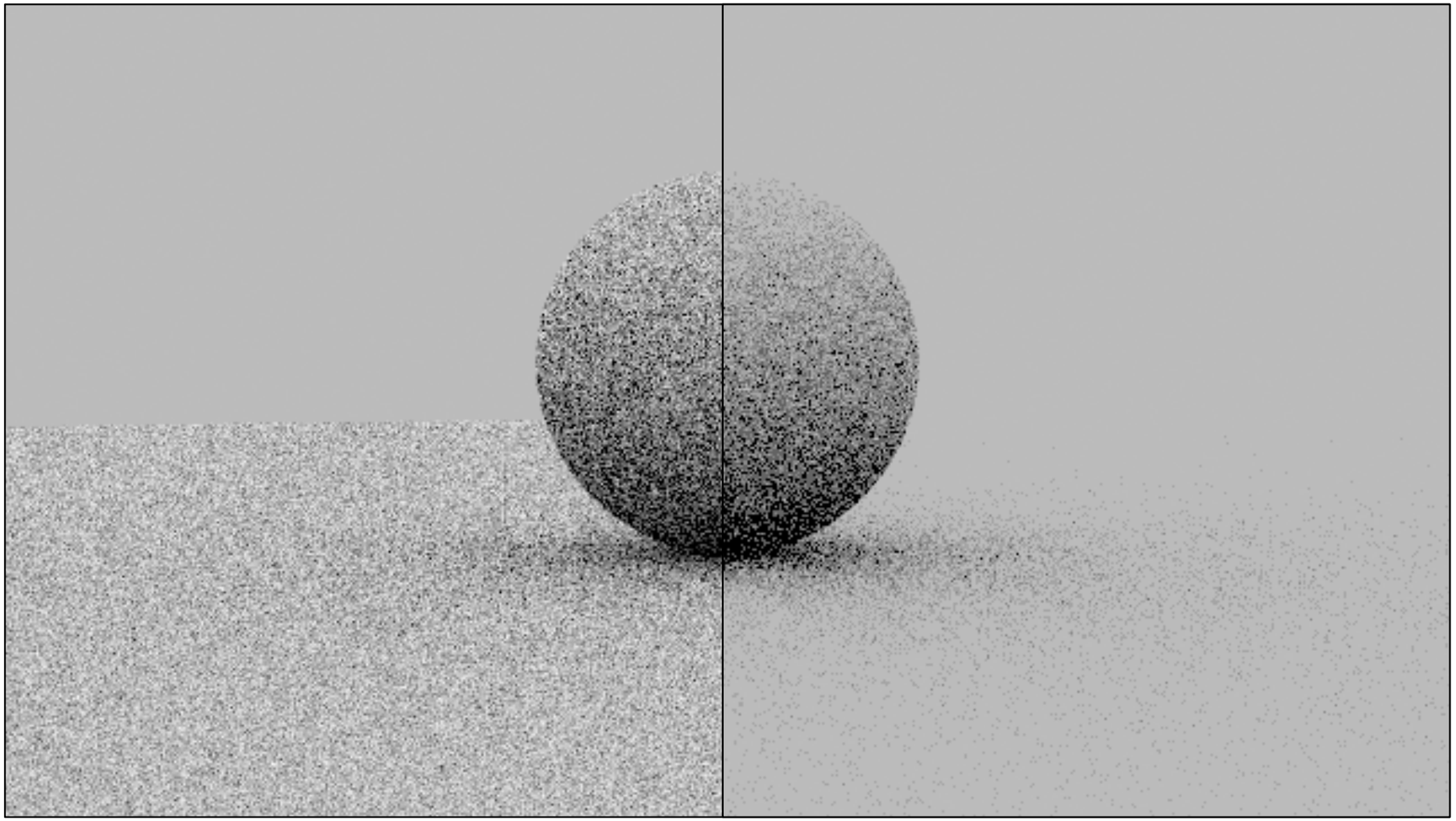
(create soft shadows)

Importance Sampling the Cosine

Uniform hemispherical sampling

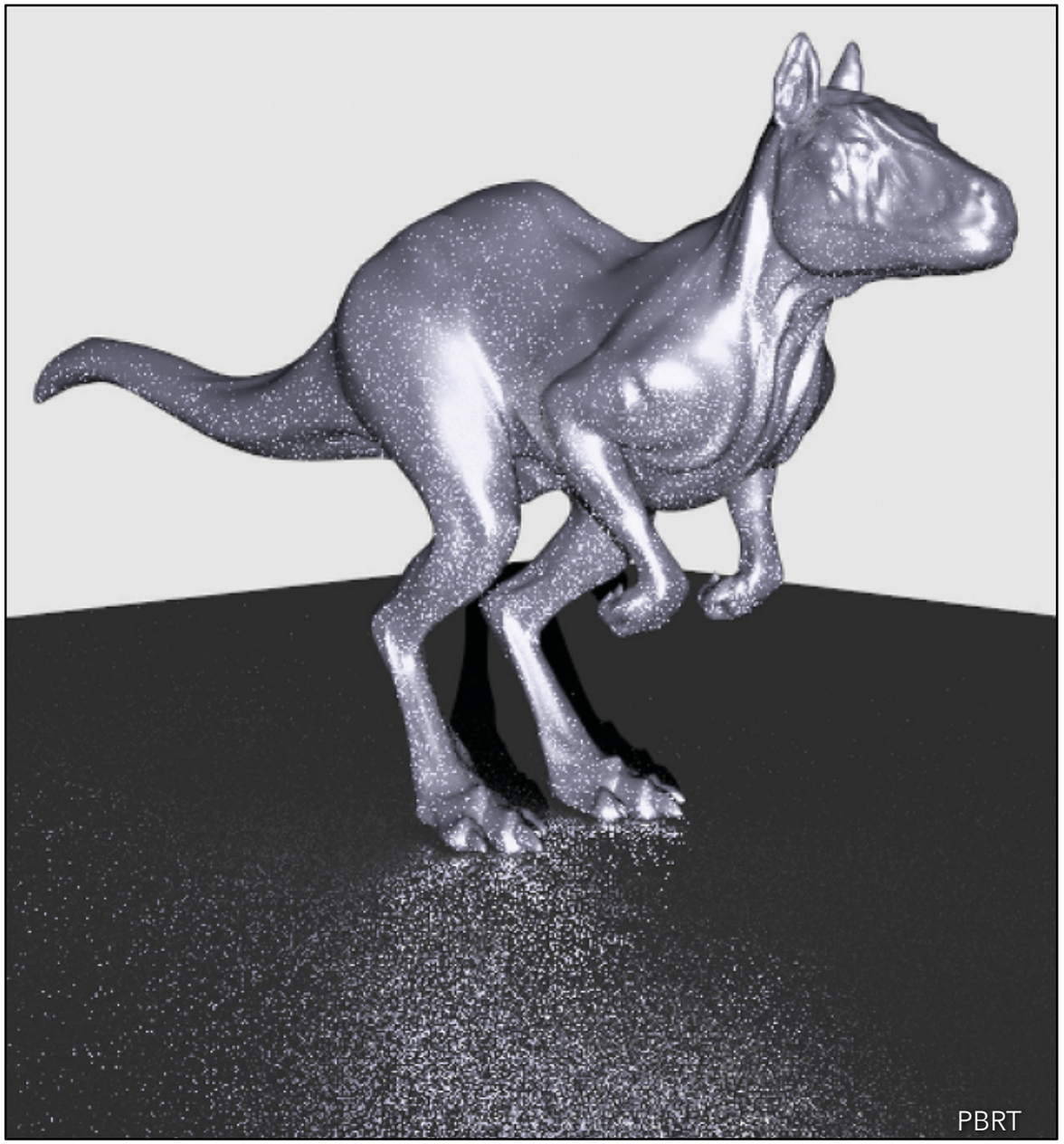
Cosine-weighted importance sampling

4 samples/pixel

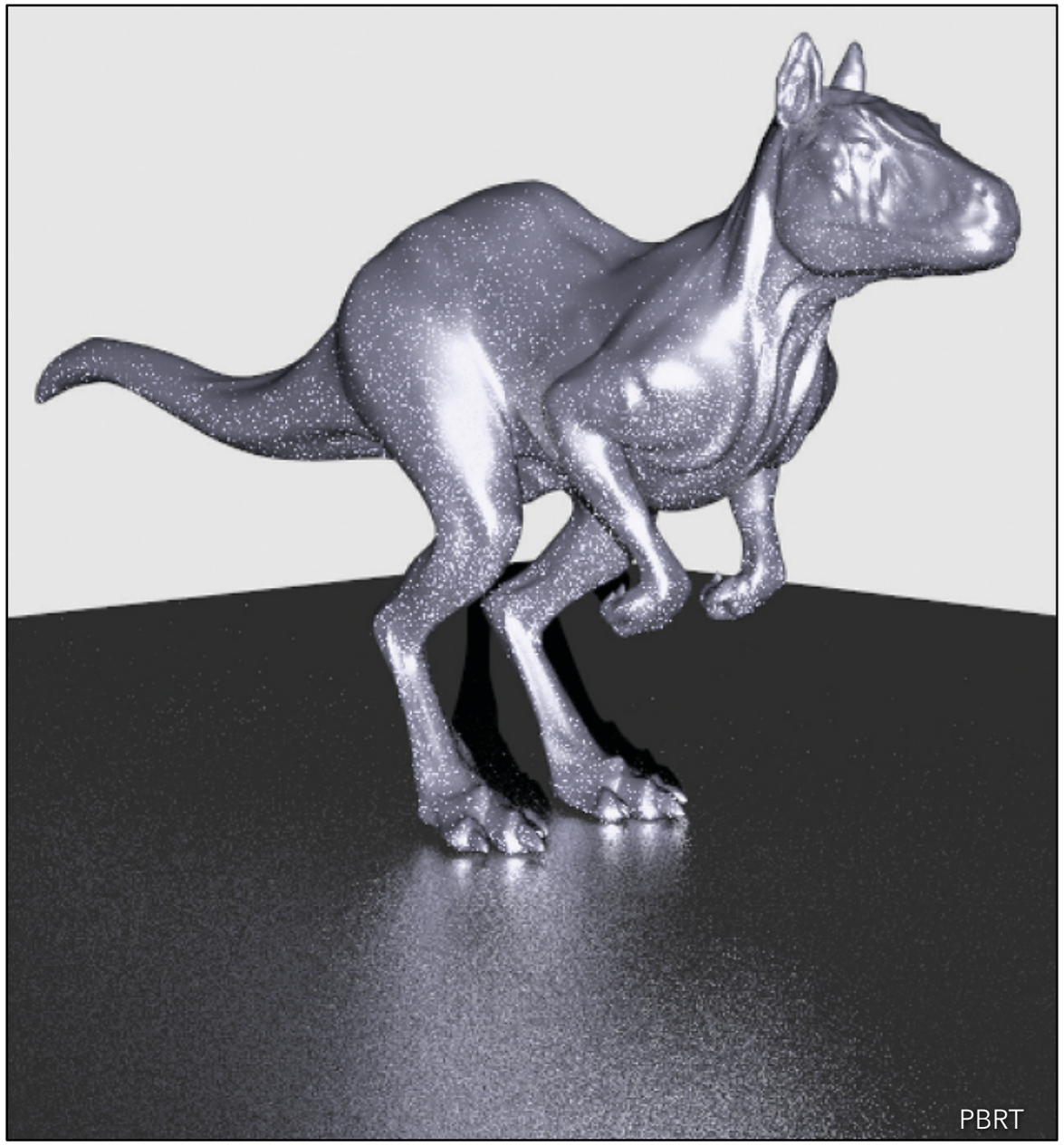


Importance Sampling the BRDF

Uniform hemispherical sampling



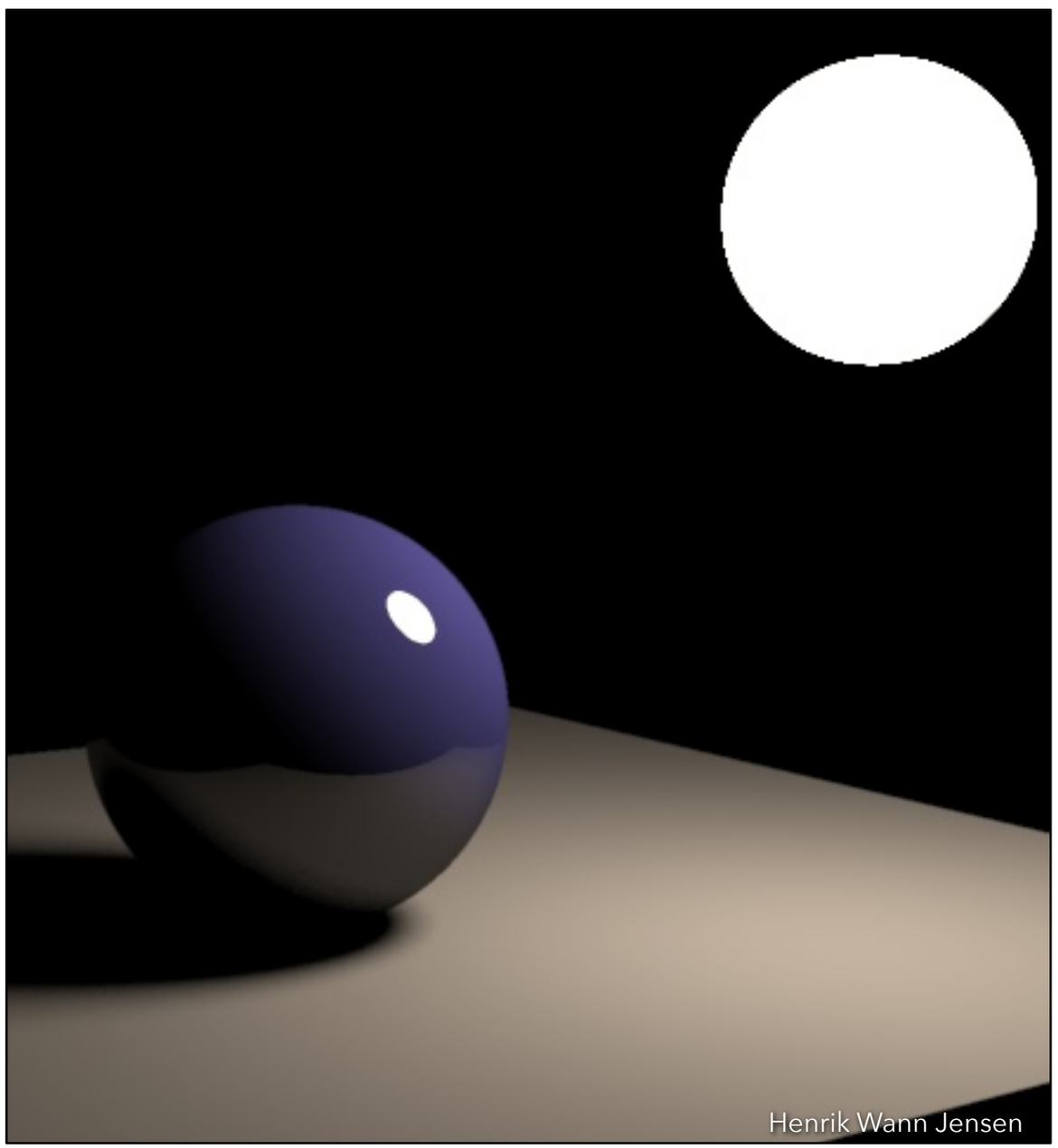
BRDF importance sampling



Importance Sampling the Light

Uniform hemispherical sampling

Uniform surface area sampling



Multiple Importance Sampling

- *Multi-sample* model

$$\langle F^{\sum N_s} \rangle = \sum_{s=1}^{M} \frac{1}{N_s} \sum_{i=1}^{N_s} w_s(x_i) \frac{f(x_i)}{p_s(x_i)}$$

- *Balance* heuristic:

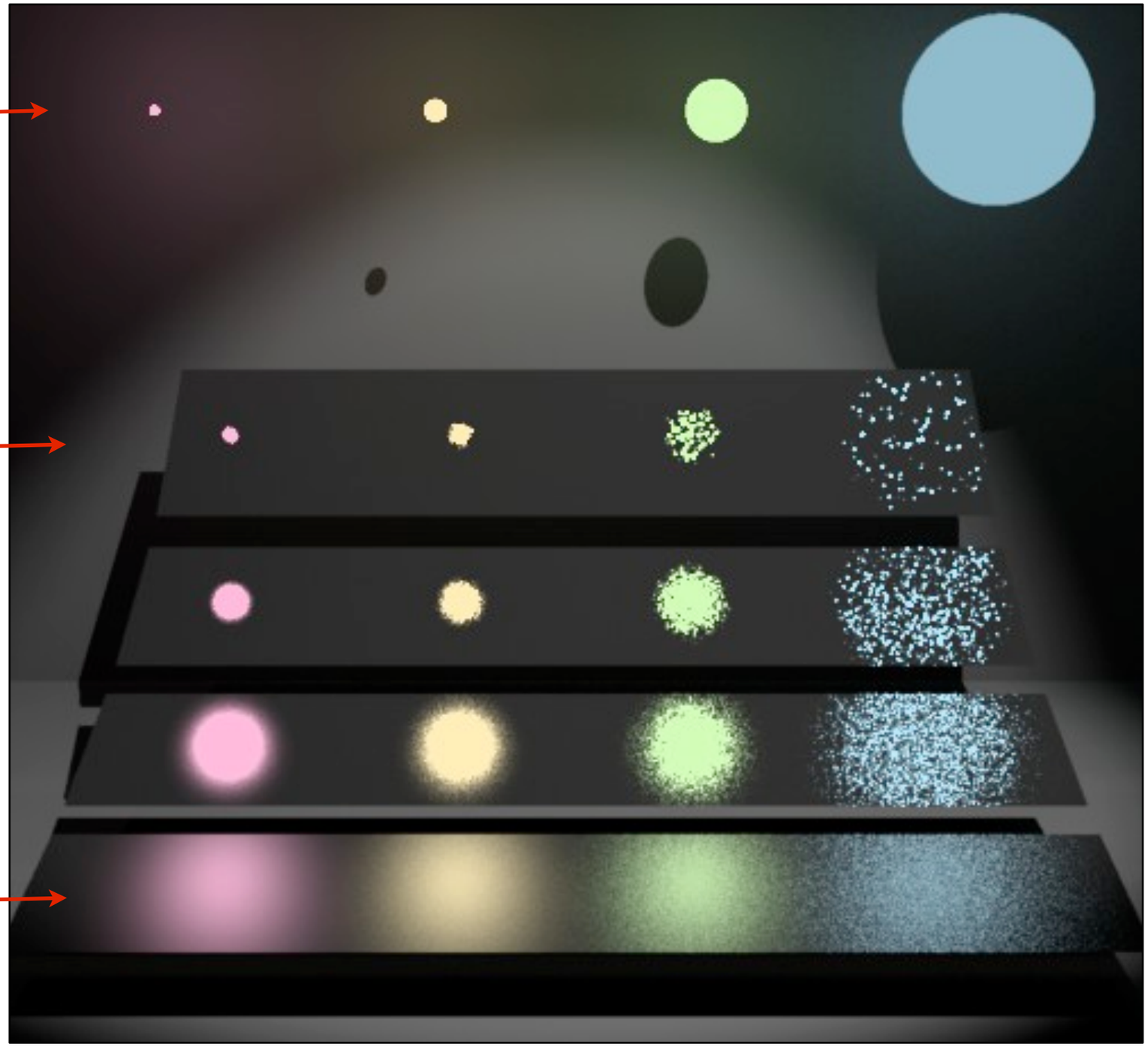
$$w_s(x) = \frac{N_s \, p_s(x)}{\sum_j N_j \, p_j(x)}$$

Sampling the Light

sphere lights

highly glossy surface

moderately glossy surface



Eric Veach and Leonidas J. Guibas 1995.

Sampling the BRDF

sphere lights

highly glossy surface

moderately glossy surface

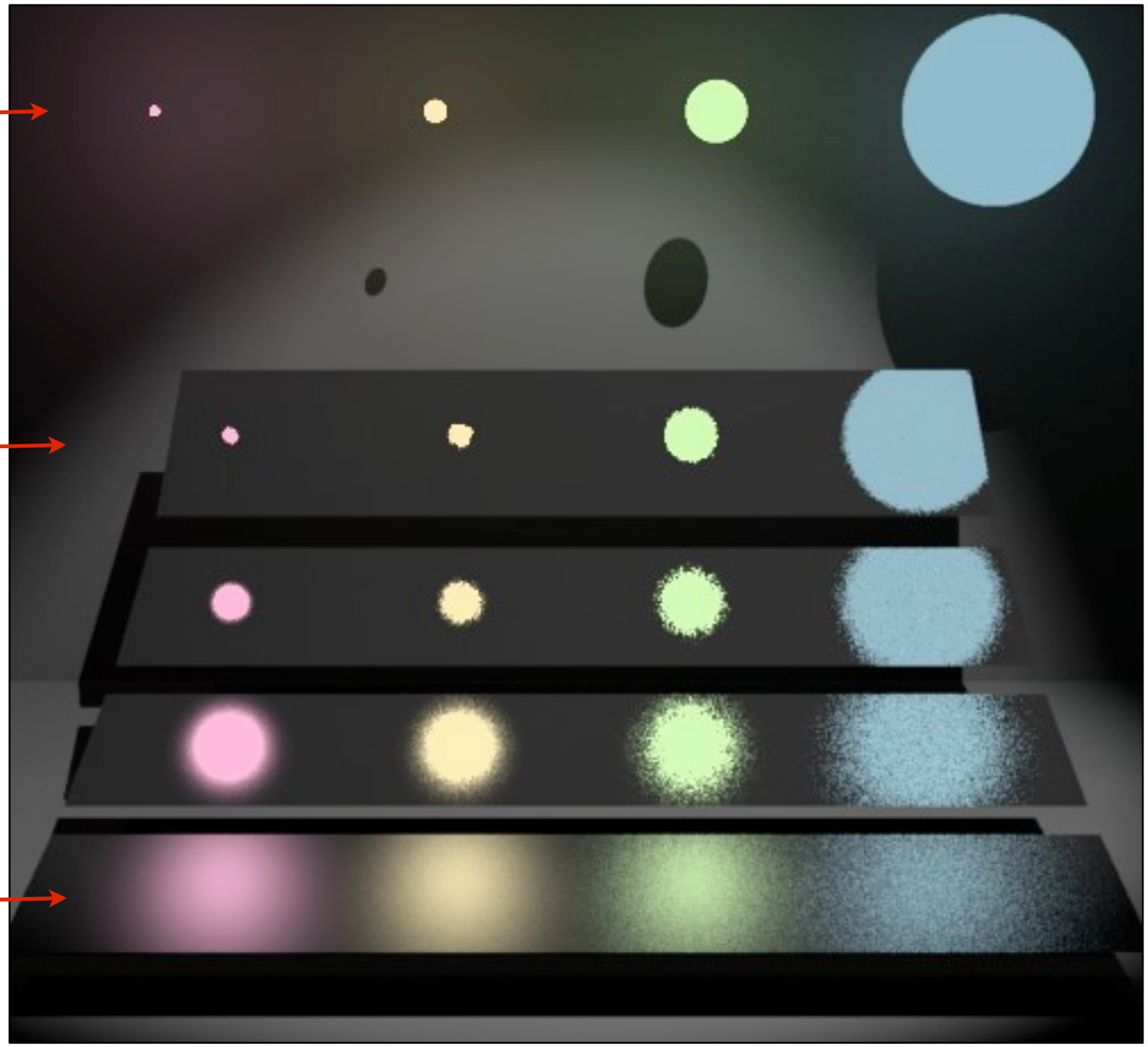
Eric Veach and Leonidas J. Guibas 1995.

Multiple Importance Sampling

sphere lights

highly glossy surface

moderately glossy surface

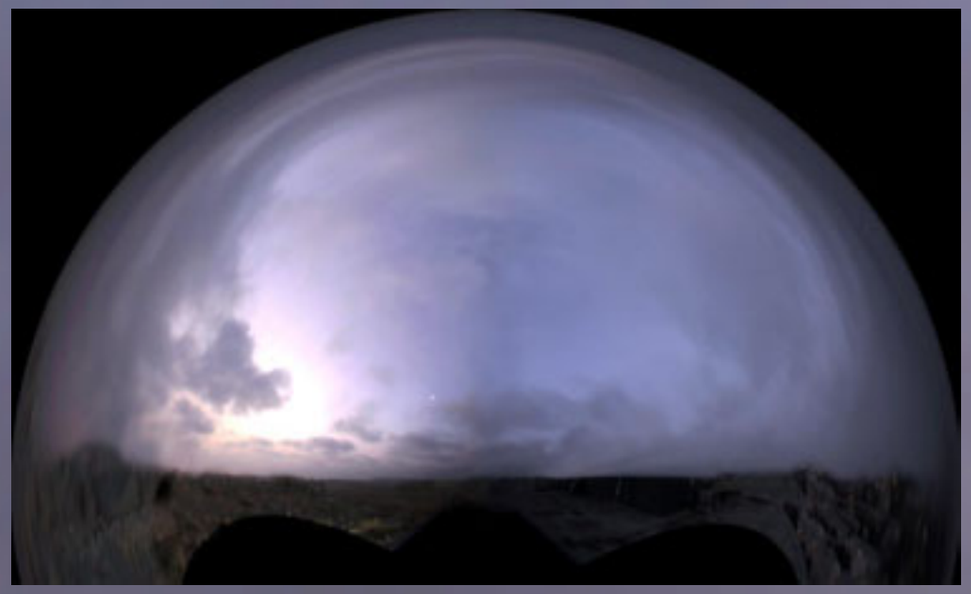


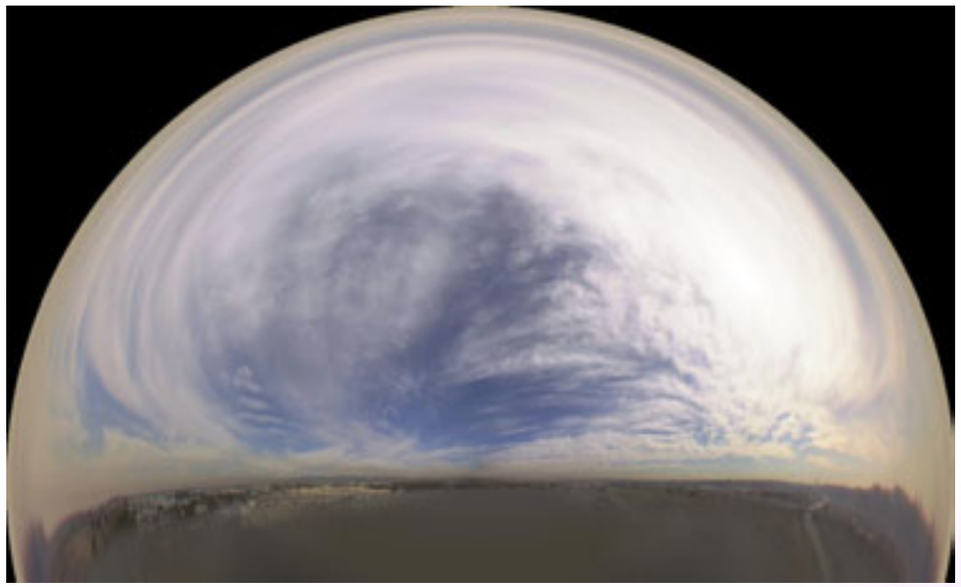
Eric Veach and Leonidas J. Guibas 1995.

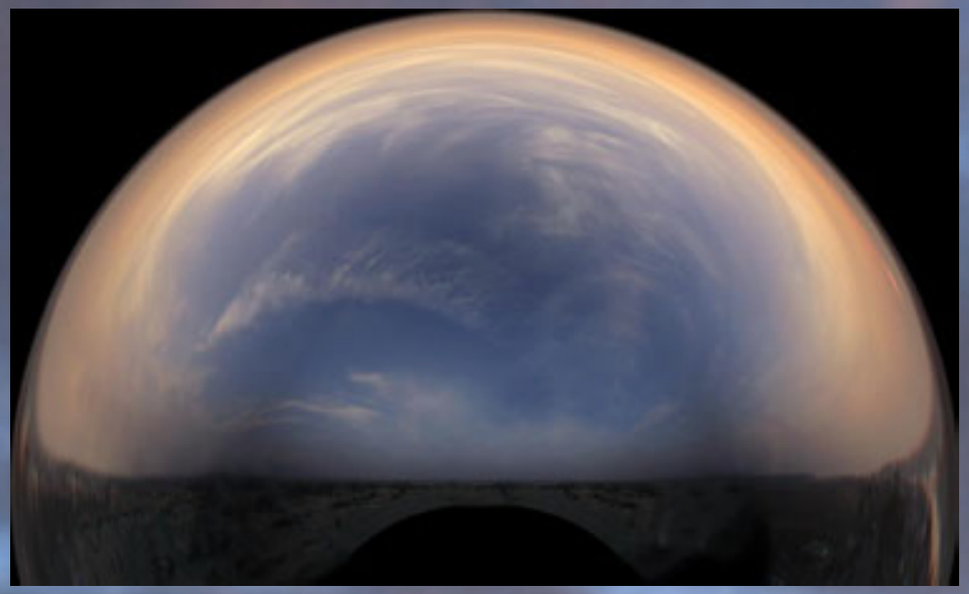
Today's Menu

- Environment lighting (image-based lighting)
- Marginal/conditional inversion sampling
- Hierarchical sample warping
- Product sampling

Environment Lighting







Environment Lighting

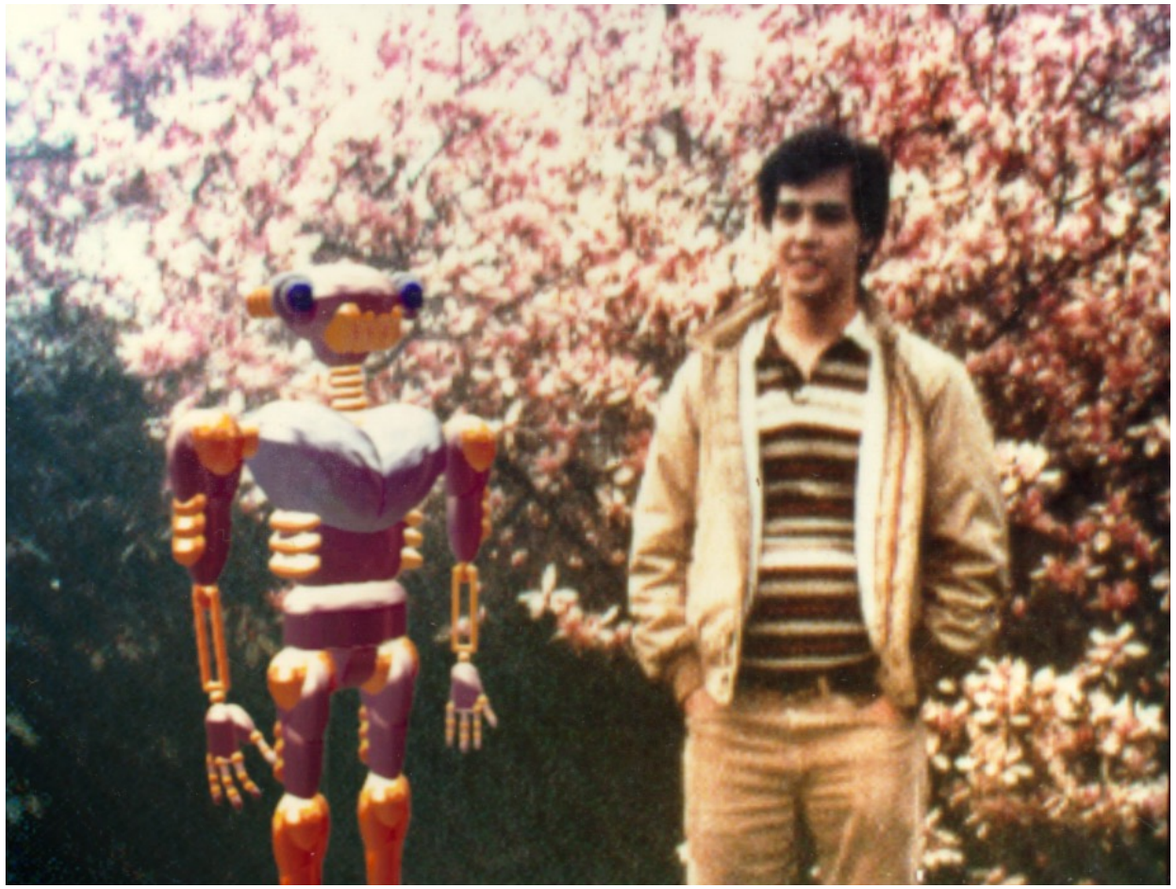
- Sidesteps tedious modelling of the environment by representing it using *one or more images*
- The image "wraps" around the virtual scene, serving as a *distant* source of illumination

source: 3delight

Related Techniques

- Reflection mapping
- Environment mapping/lighting
- Image-based lighting (IBL)

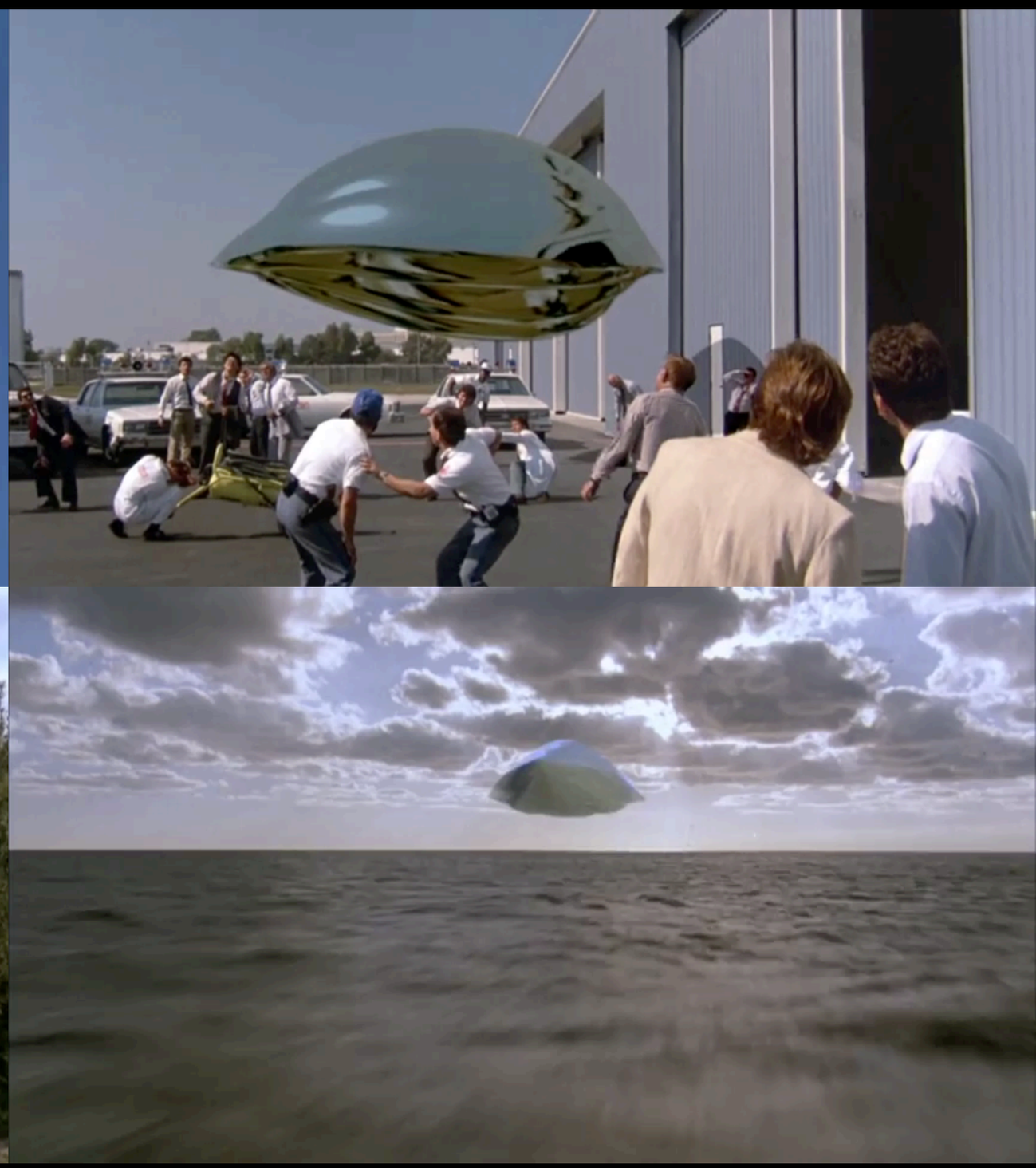
Reflection Mapping (1982)



Mike Chou and Lance Williams

Gene Miller and Ken Perlin

Flight of the Navigator (1986)



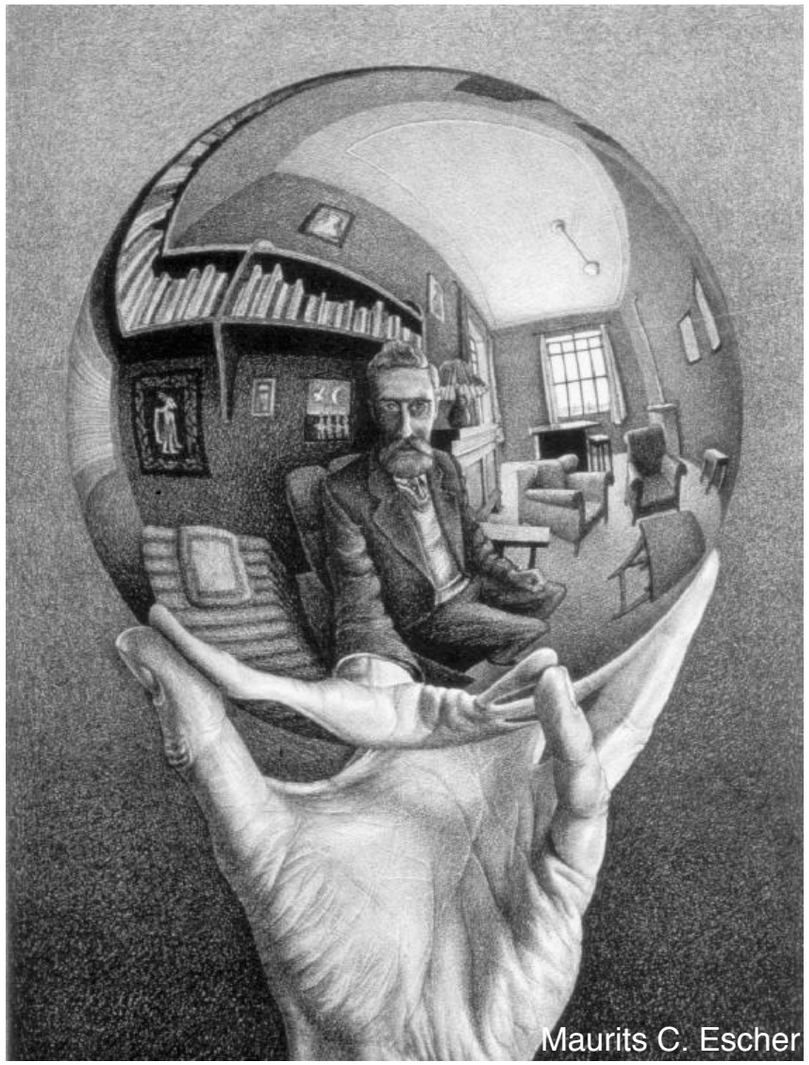
The Abyss (1989)

Terminator 2 (1991)

Acquiring Environment Lighting

- Mirrored ball + camera
- Fisheye lens images
- Stitching images together
- Panoramic camera

Acquisition – Low Tech.



Lightprobe

omnidirectional,
360° panoramic,
HDR image

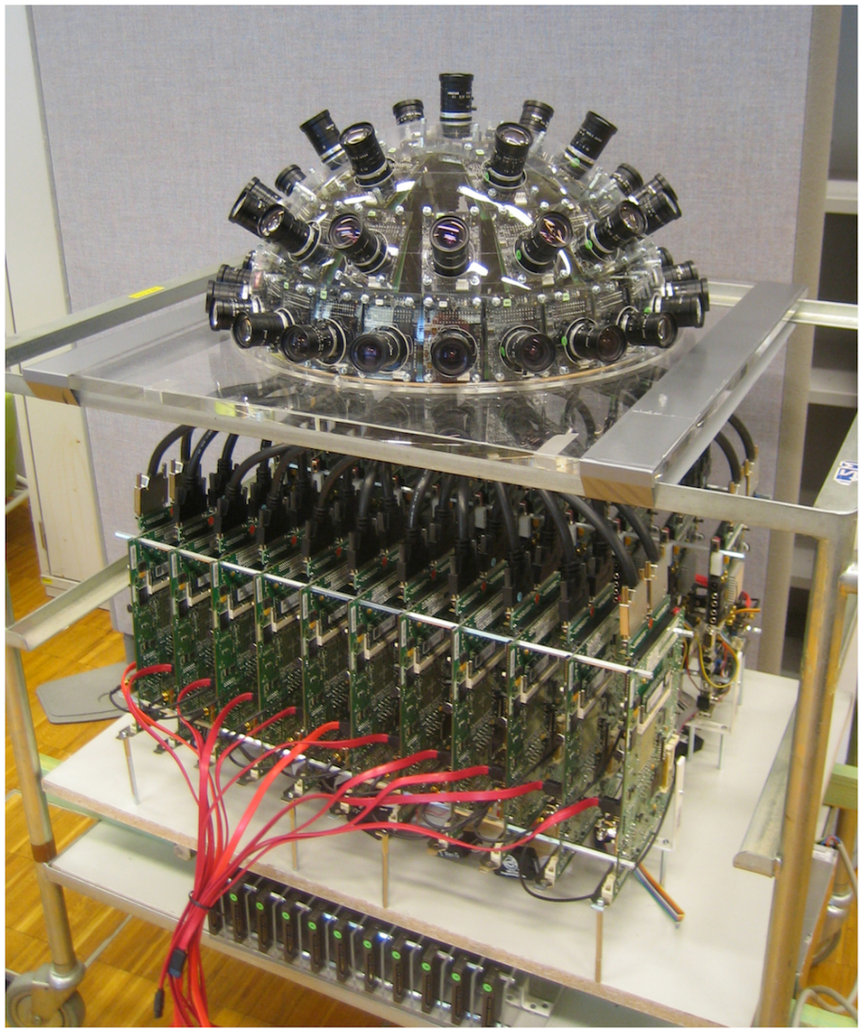
Acquisition – Even Lower Tech.

Gene Miller and Ken Perlin

Acquisition – Stitched Panorama

source: photoanswers.co.uk

Acquisition – High Tech.



lsm.epfl.ch

immersivemedia.com

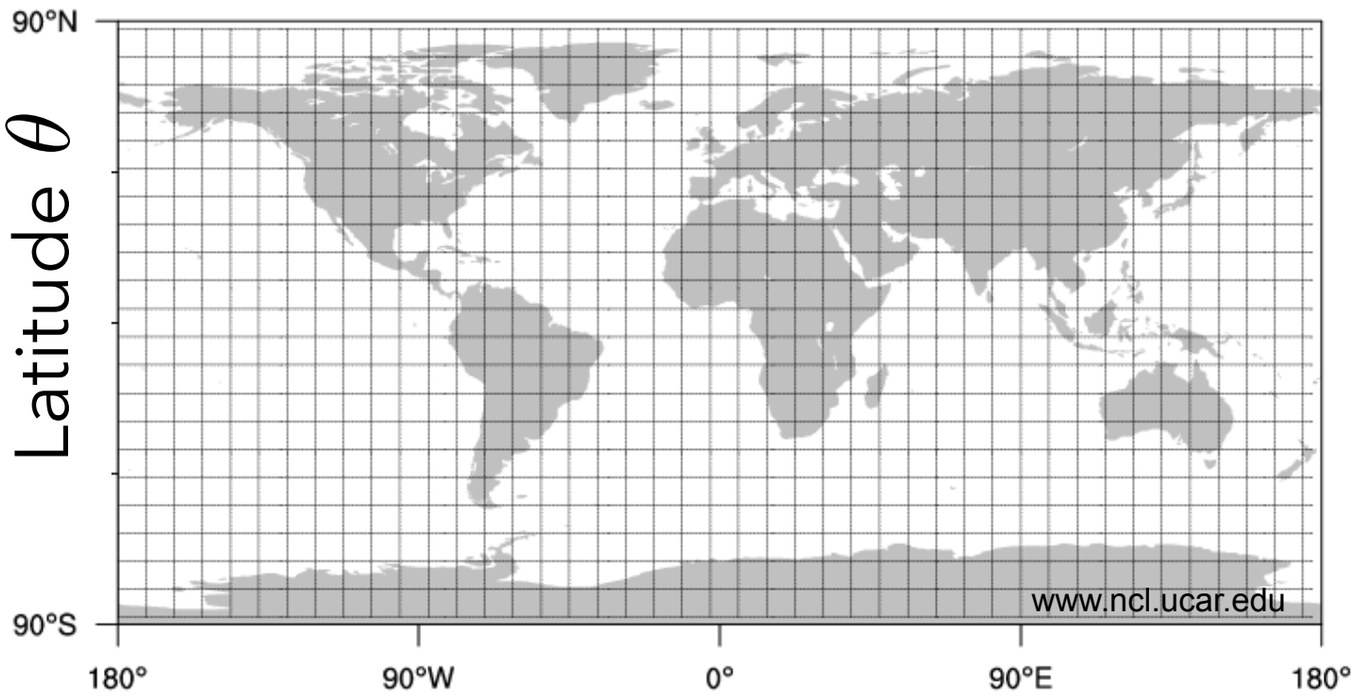
Storing Environment Maps

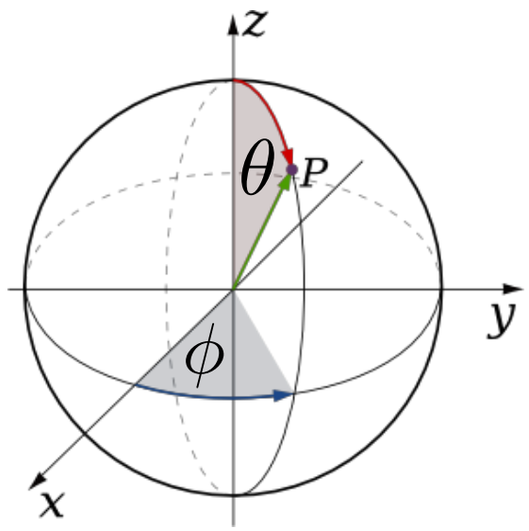
- Various ways to represent/parametrize environment maps
- Related to cartography
  - Projecting the earth (sphere) onto a plane

Mirror Ball

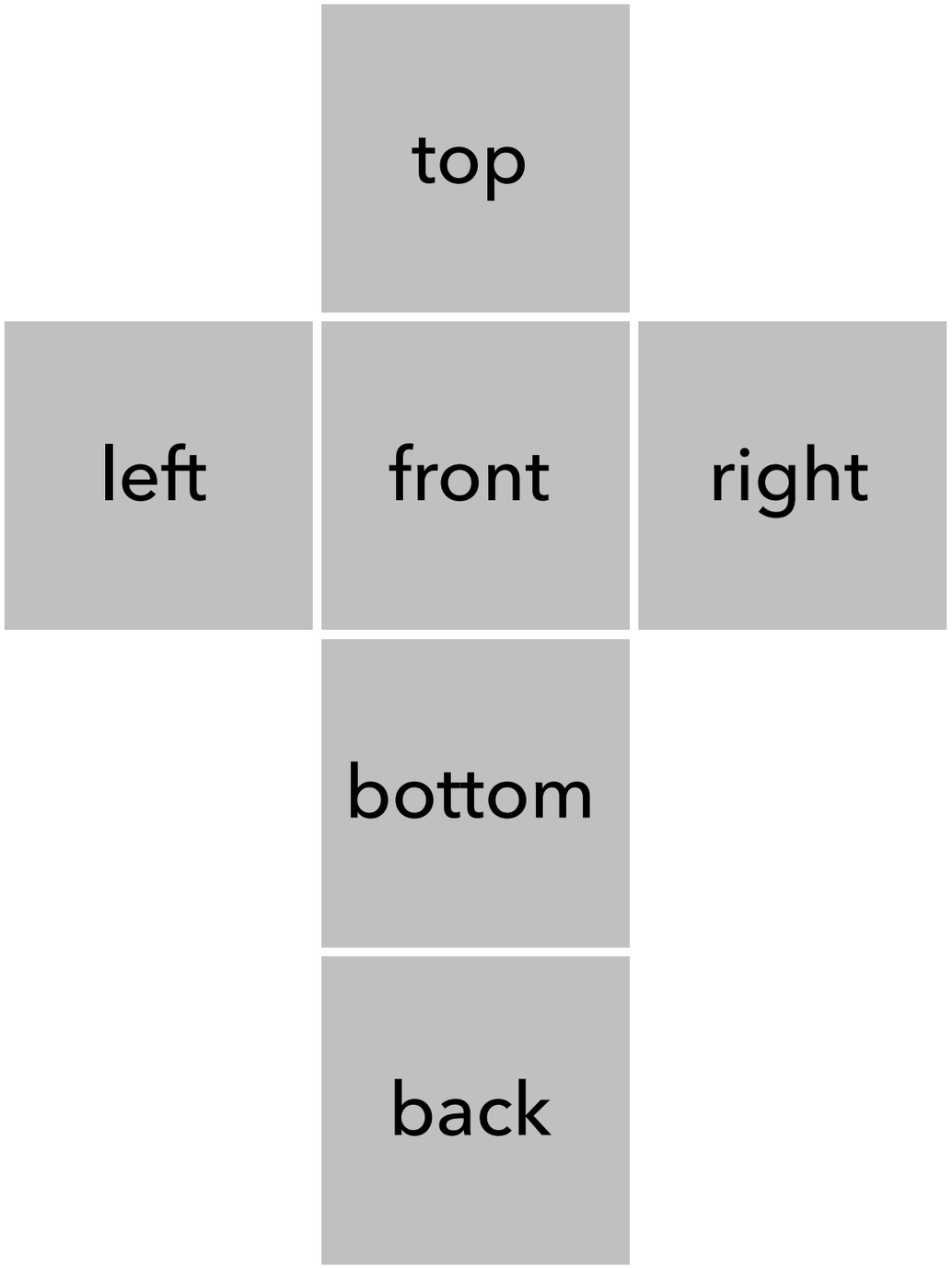
Angular Map

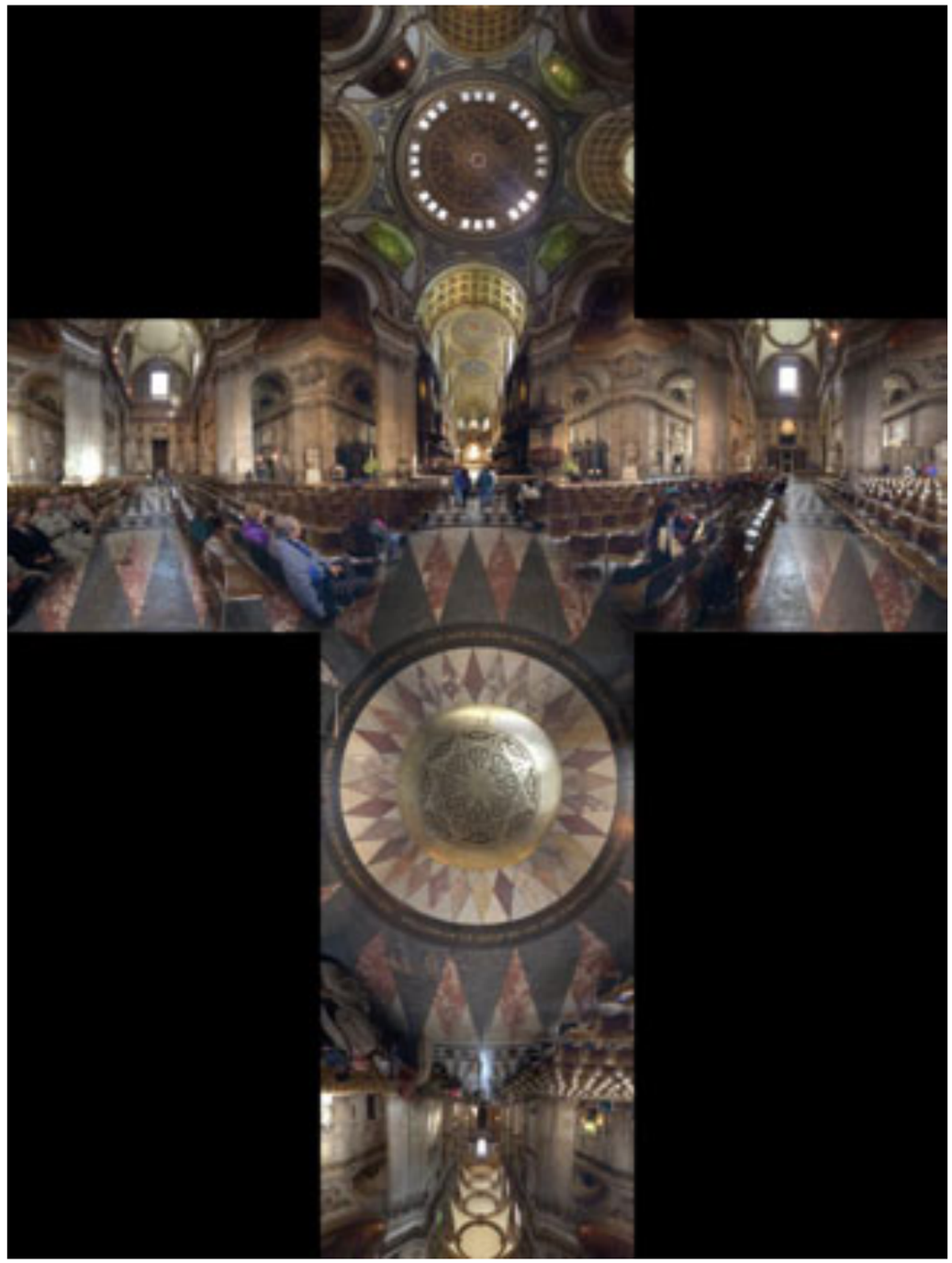
Latitude/Longitude Map





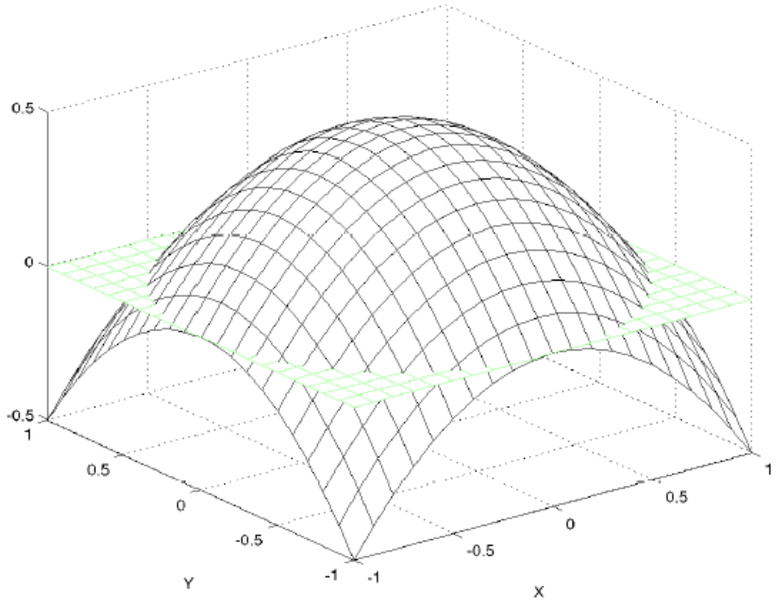
Cube Map (Skybox)

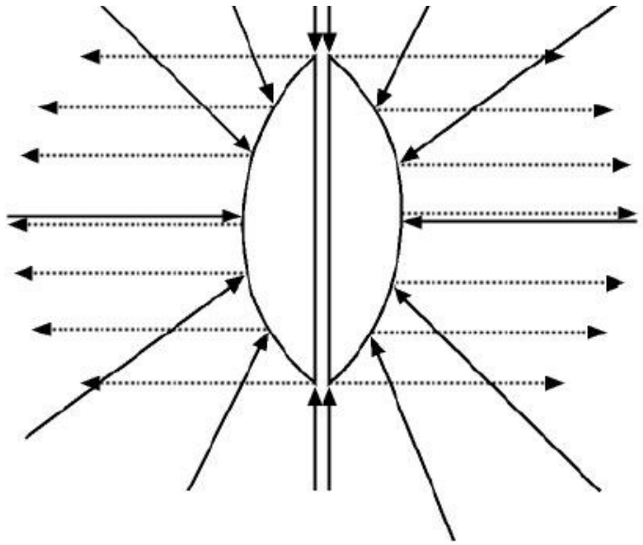




Other Parameterizations

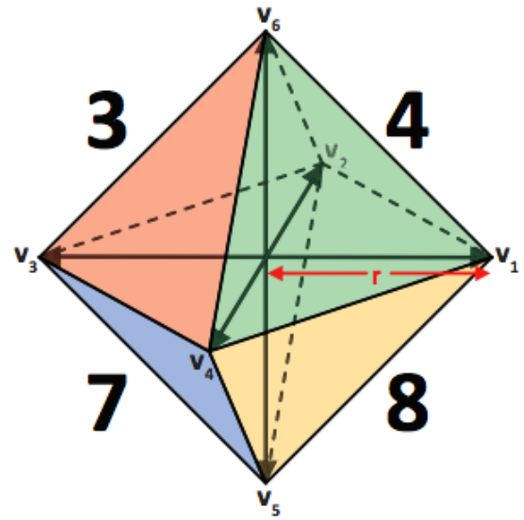
Dual-paraboloid

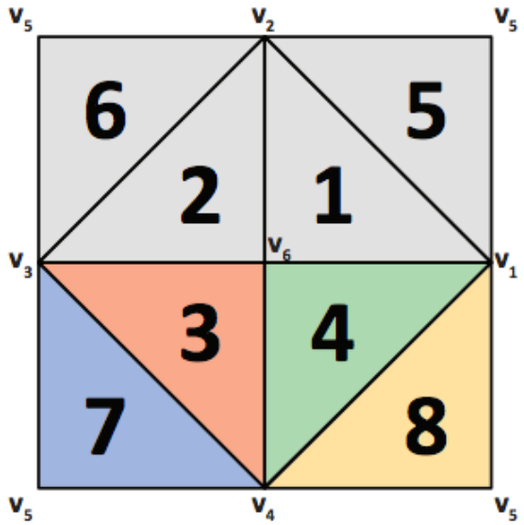




Courtesy of Brabec

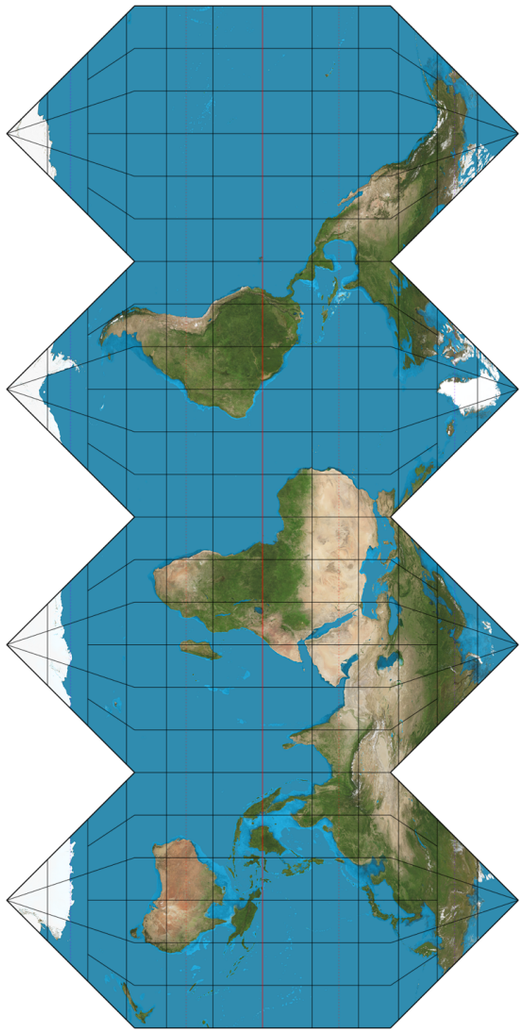
Octahedron map





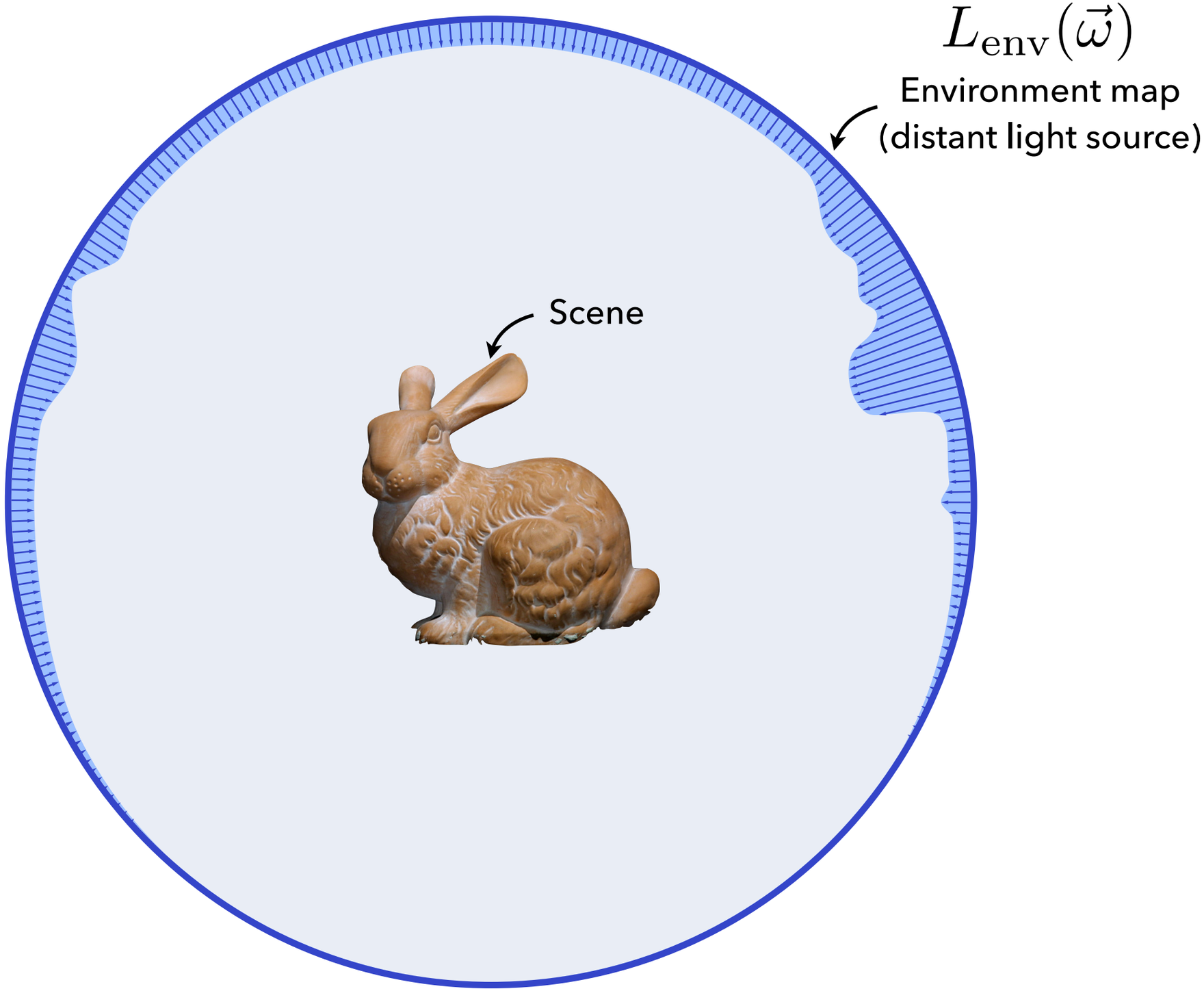
Courtesy of Engelhardt and Dachsbacher

HEALPix



Courtesy of Ryazanov

Environment Lighting



Environment Lighting

- The image "wraps" around the virtual scene, serving as a *distant* source of illumination

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_\Omega f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

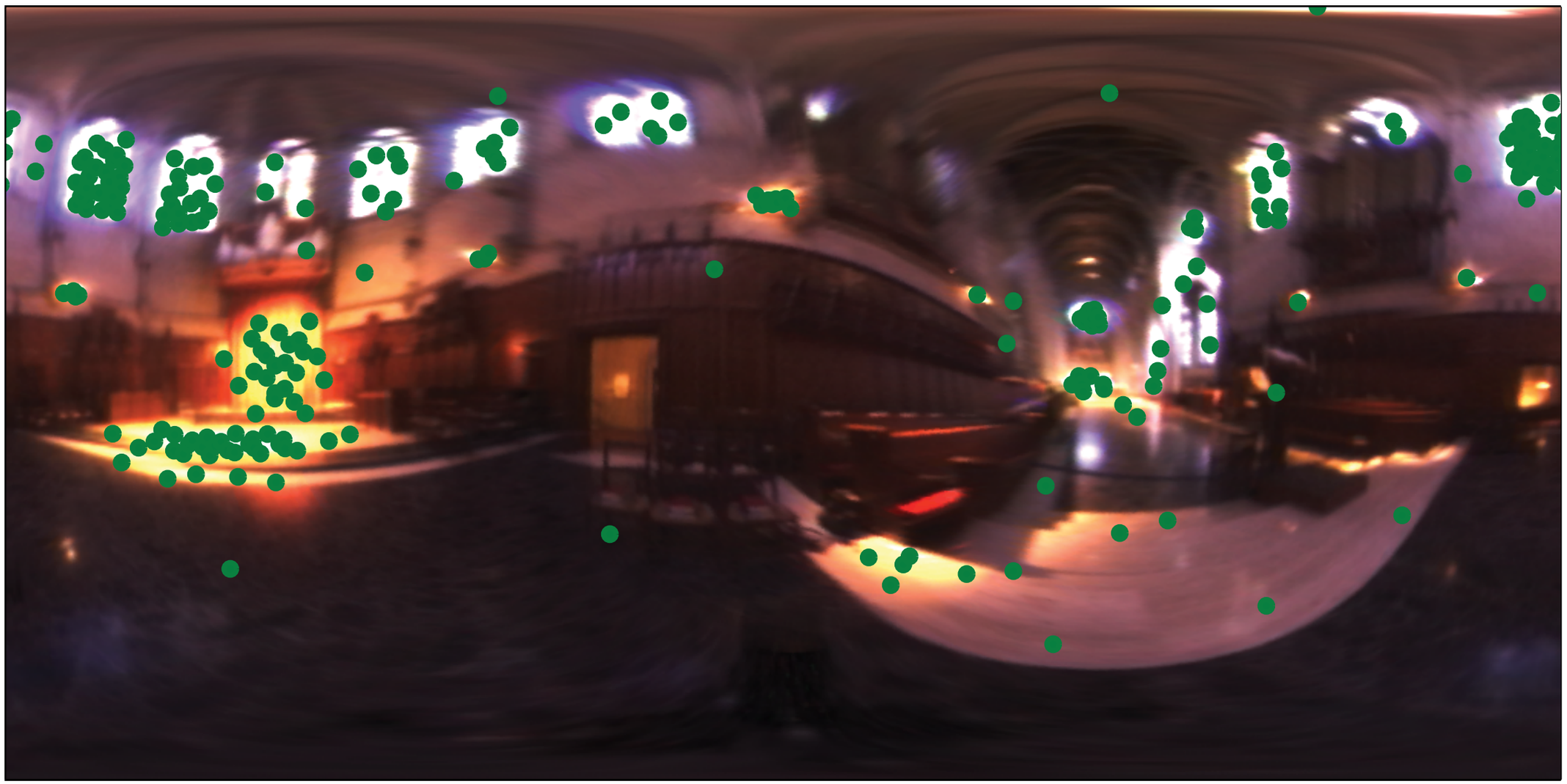
$$= \int_\Omega f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_{\text{env}}(\vec{\omega}_i) V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Environment Lighting

Environment Lighting

$$L_r(\mathbf{x}, \vec{\omega}_r) = \int_\Omega f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_{\text{env}}(\vec{\omega}_i) V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i$$

Importance Sampling L_{env}



$$p(\vec{\omega}_i) \propto L_{\text{env}}(\vec{\omega}_i)$$

Importance Sampling L_{env}

$$p(\vec{\omega}_i) \propto L_{\text{env}}(\vec{\omega}_i)$$

- Several strategies exist
- We'll discuss:
  - Marginal/Conditional CDF method
  - Hierarchical warping method

Visual Break

openfootage.net

Marginal/Conditional CDF

- Assume the lat/long parameterization
- Draw samples from joint $p(\theta, \phi) \propto L_{\text{env}}(\theta, \phi) \sin \theta$

Why the Sine?

- General case of integrating some $f(\vec{\omega})$ over S^2

$$d\vec{\omega} = \sin\theta d\theta d\phi$$

Comes from the Jacobian

$$\int_{S^2} f(\vec{\omega})\, d\vec{\omega} = \int_0^{2\pi}\int_0^{\pi} f(\theta,\phi)\sin\theta\, d\theta d\phi$$

$$\approx \frac{1}{N}\sum_{i=1}^{N}\frac{f(\theta_i,\phi_i)\sin\theta_i}{p(\theta_i,\phi_i)}$$

- If we set $p(\theta,\phi) \propto f(\theta,\phi)\sin\theta$ then the sines will effectively cancel out

Marginal/Conditional CDF

- Assume the lat/long parameterization
- Draw samples from joint $p(\theta, \phi) \propto L_{\text{env}}(\theta, \phi) \sin\theta$
  - Step 1: create scalar version $L'(\theta, \phi)$ of $L_{\text{env}}(\theta, \phi) \sin\theta$
  - Step 2: compute marginal PDF

$$p(\theta) = \int_0^{2\pi} L'(\theta, \phi)\, d\phi$$

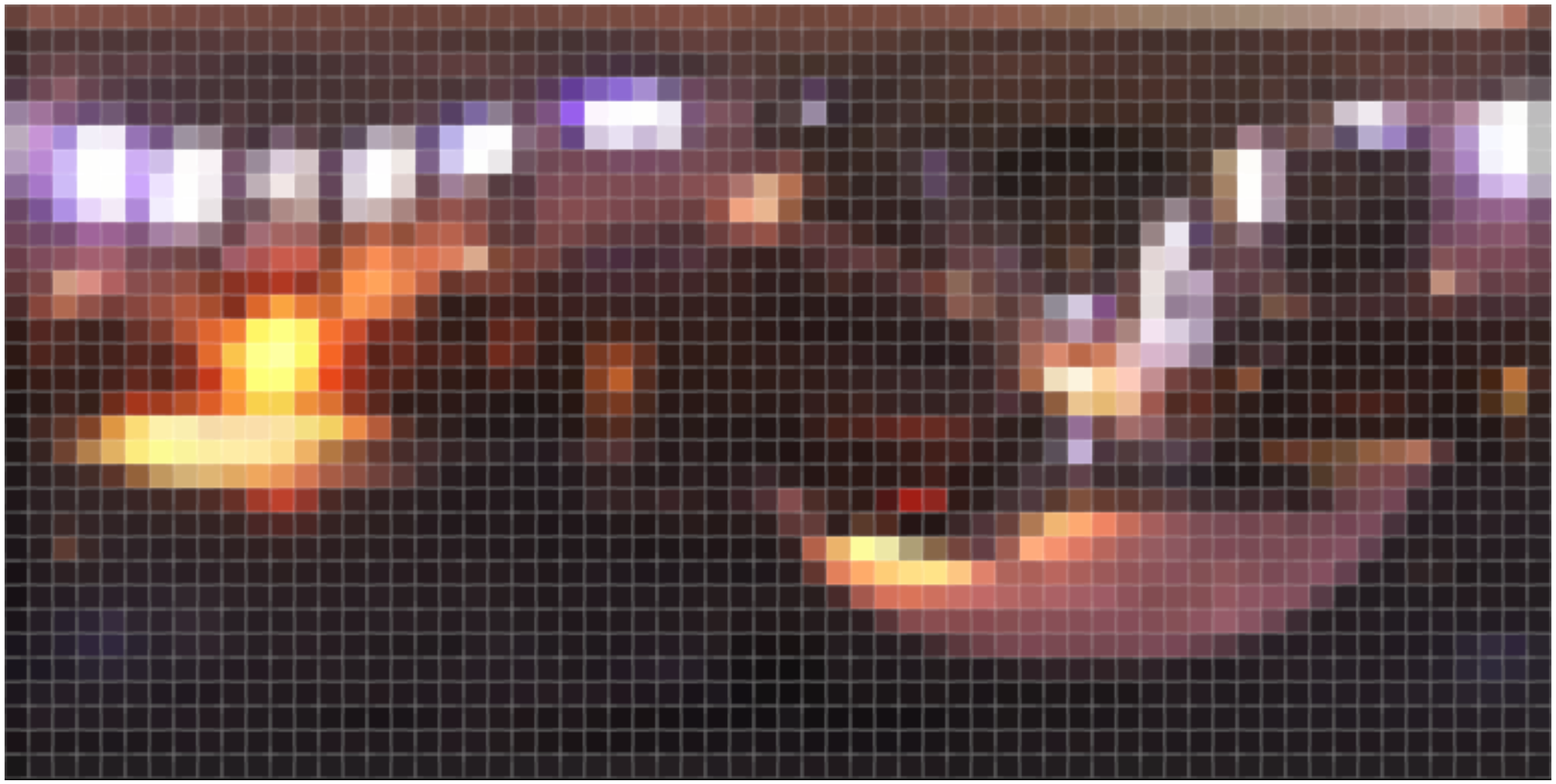
  - Step 3: compute conditional PDF

$$p(\phi|\theta) = \frac{p(\theta, \phi)}{p(\theta)}$$

  - Step 4: draw samples $\theta_i \sim p(\theta)$ and $\phi_i \sim p(\phi|\theta_i)$

Step 1: Scalar Importance Func.

Original environment map



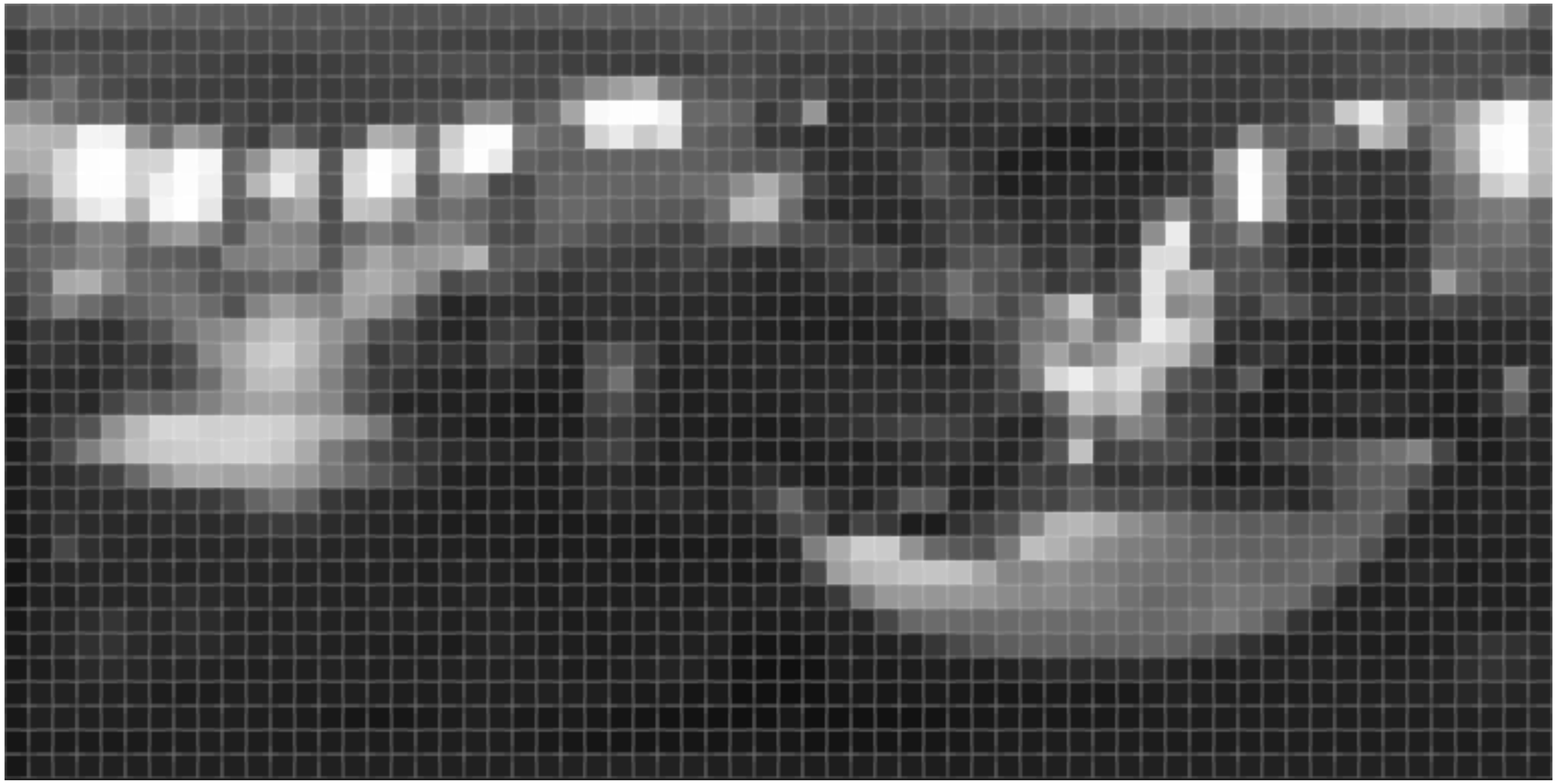
θ

ϕ

Step 1: Scalar Importance Func.

Scalar version

(average, max, or luminance of RGB channels)

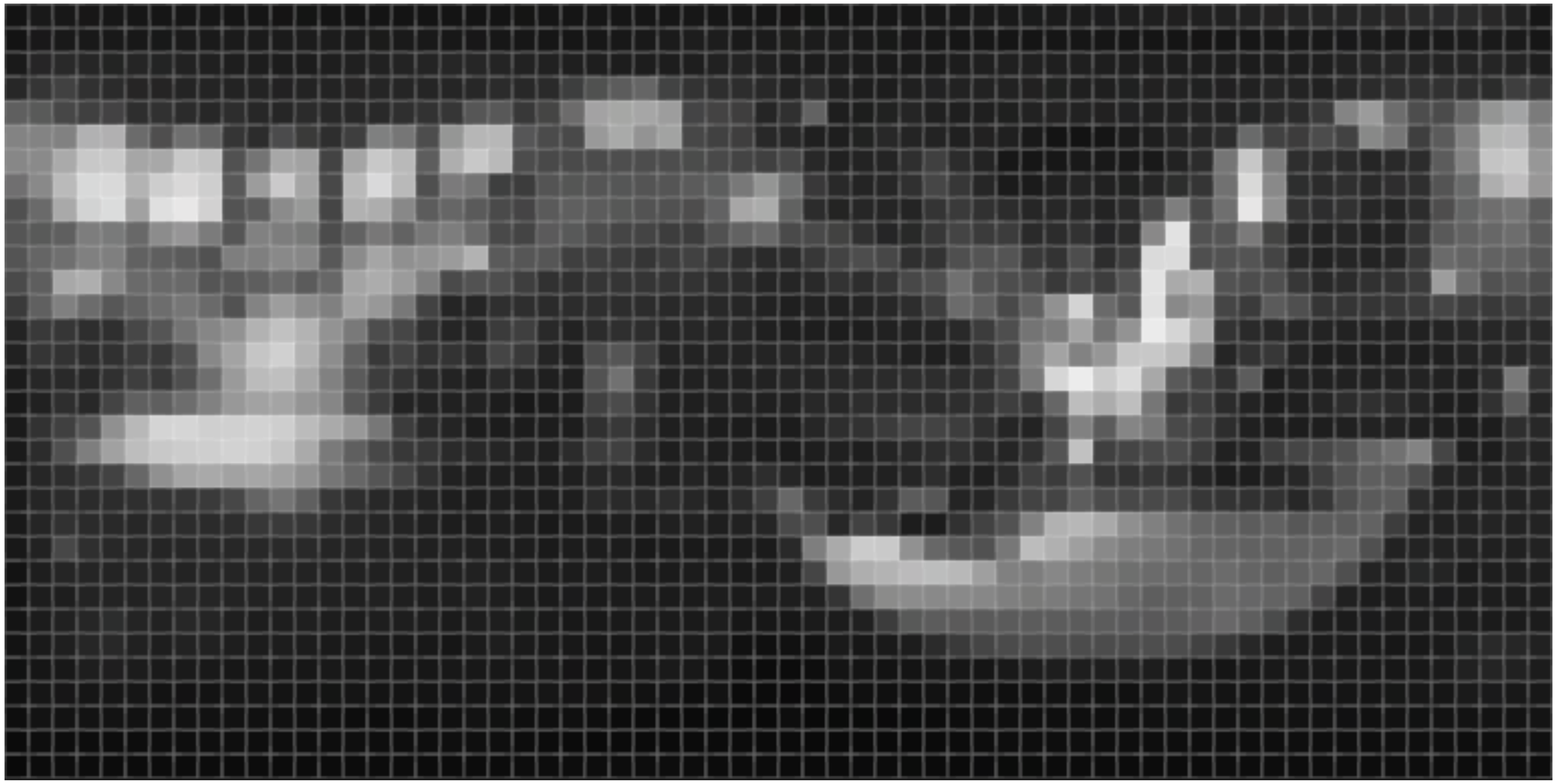


θ

ϕ

Step 1: Scalar Importance Func.

Multiplied by $\sin\theta$

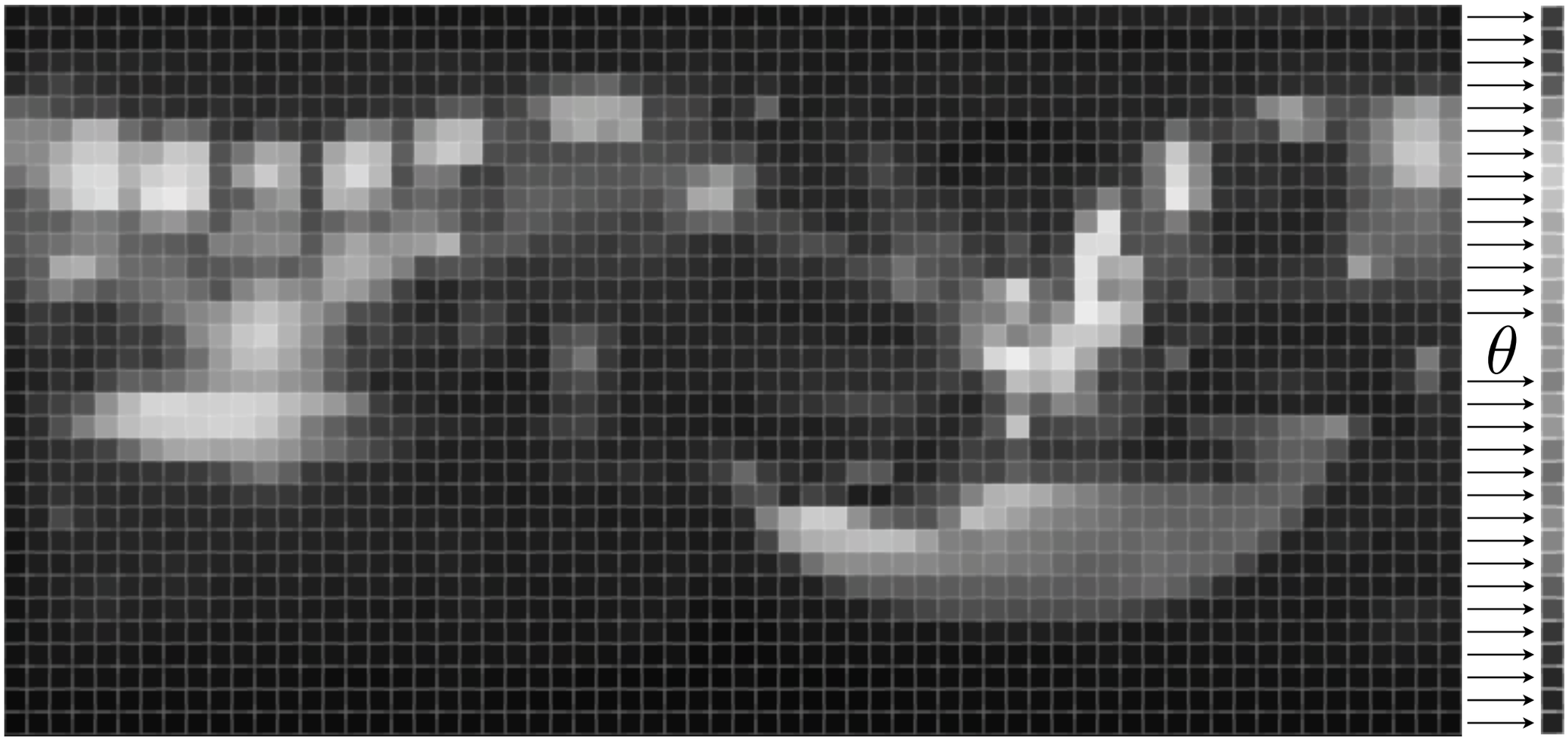


θ

ϕ

Step 2: Marginalization

$$p(\theta) = \int_0^{2\pi} L'(\theta, \phi)\, d\phi$$

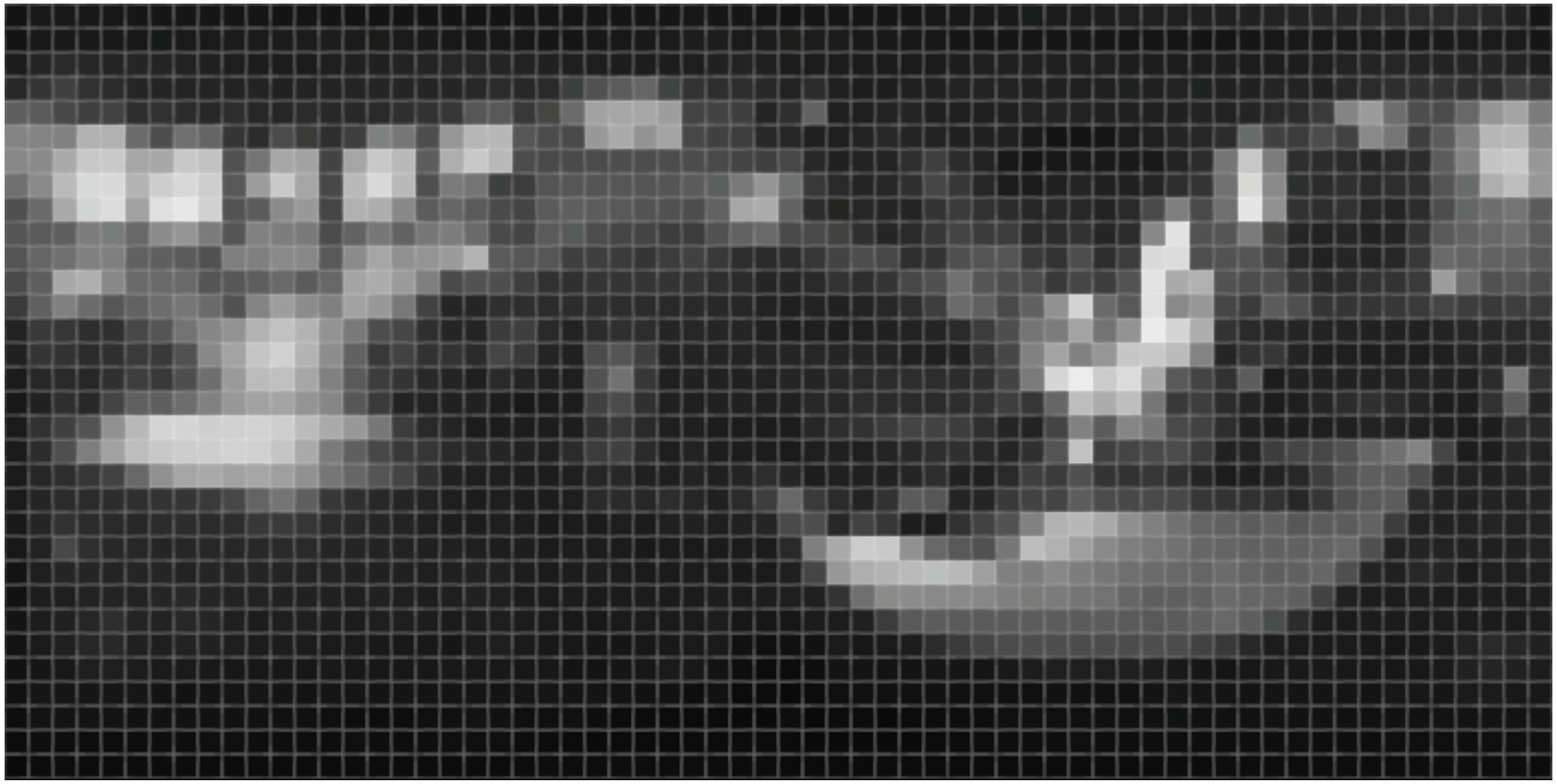


ϕ

Marginalize to get $p(\theta)$

Step 3: Conditional PDFs

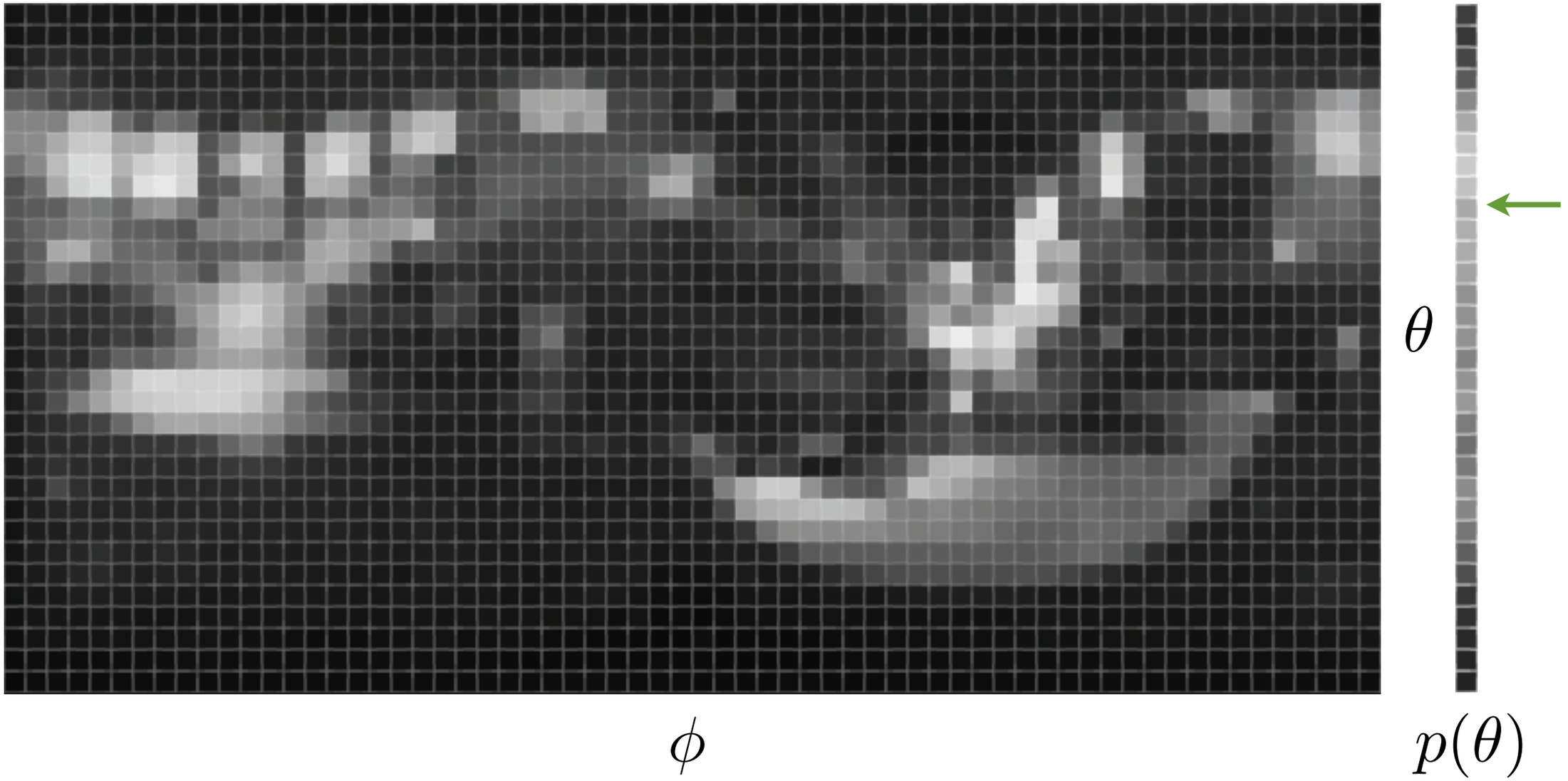
Once normalized, each row can serve as the conditional PDF



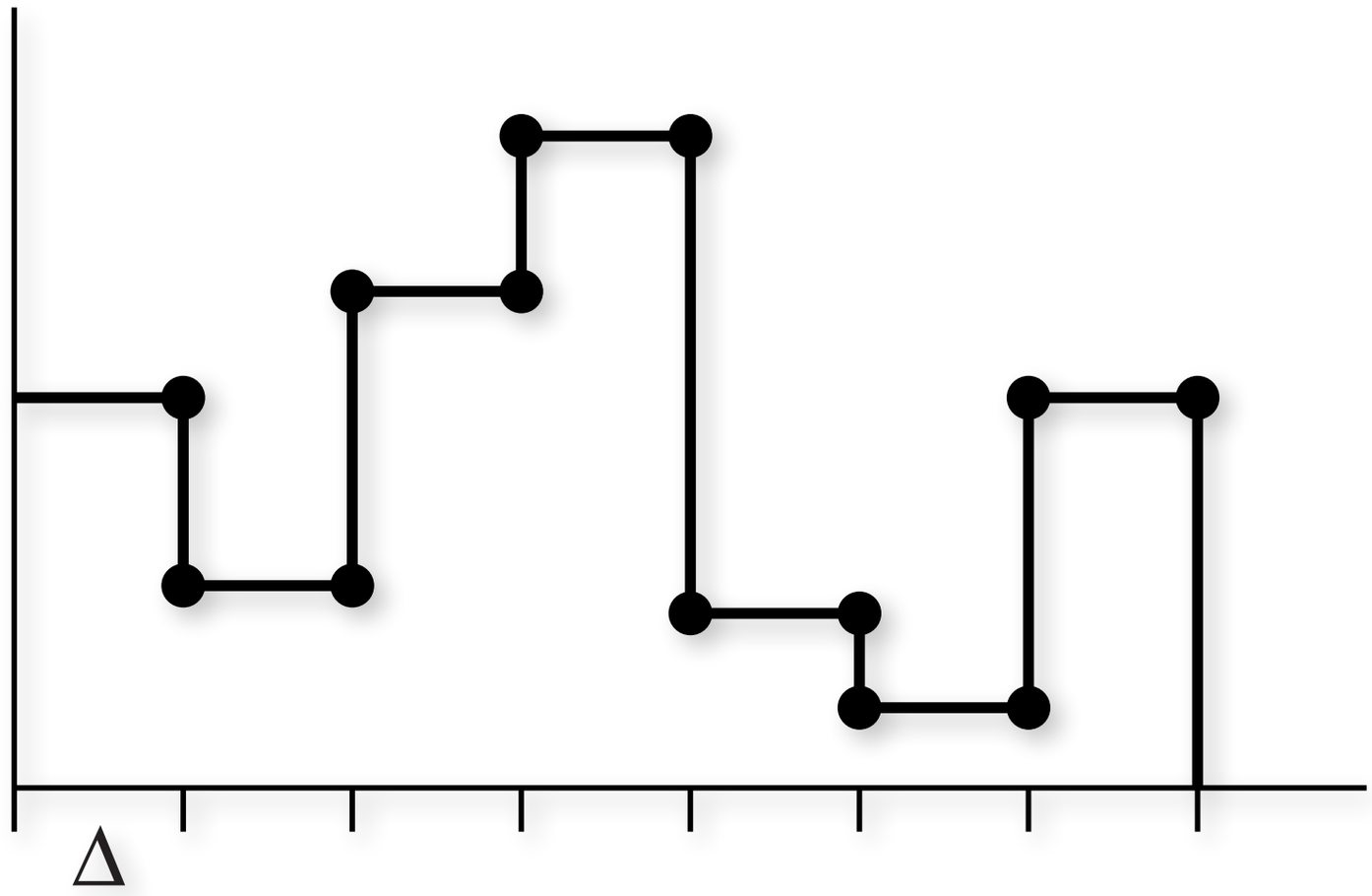
θ

ϕ

Step 4: Sampling

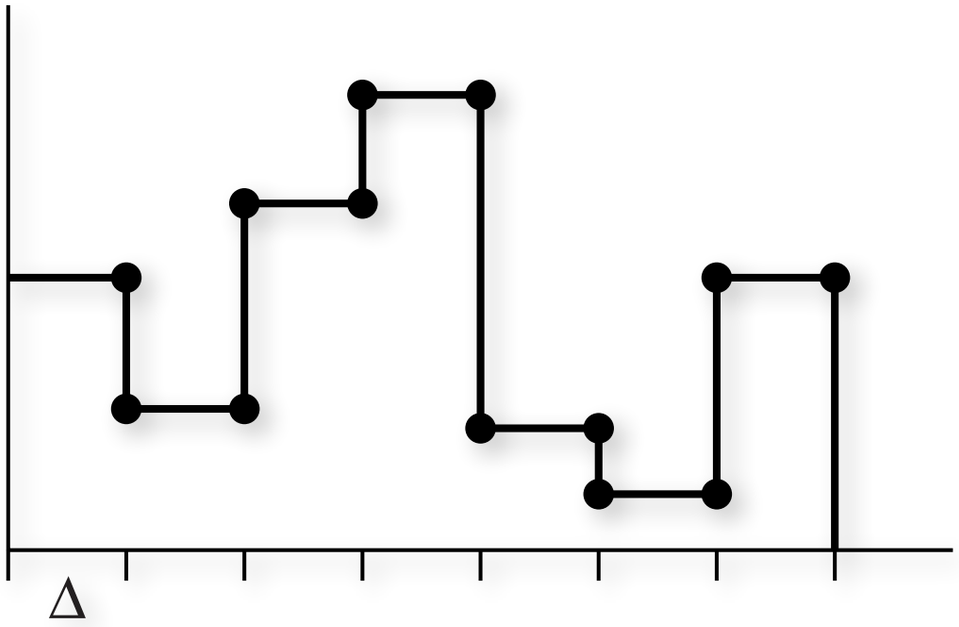


Sampling Discrete 1D PDFs

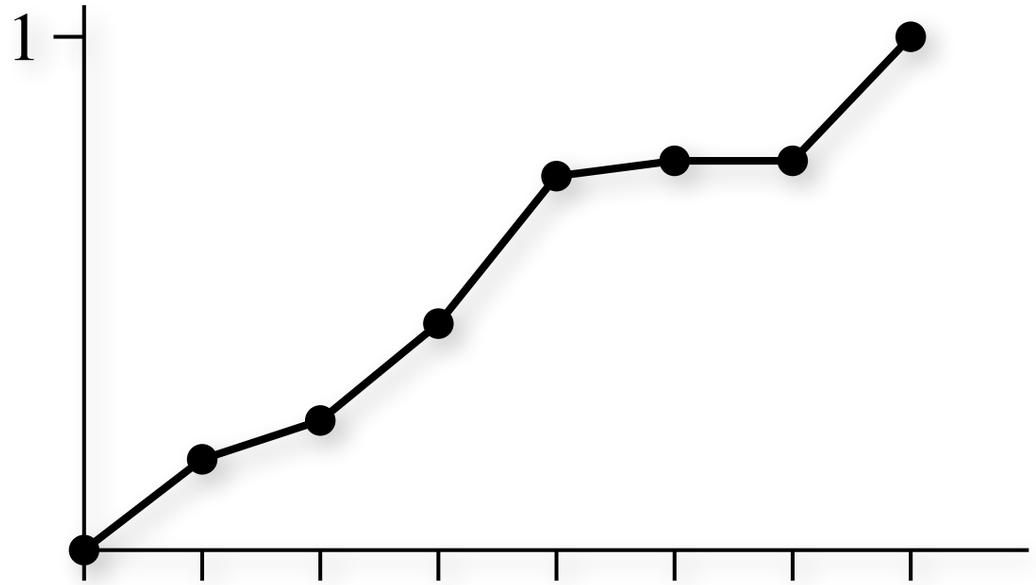


PDF

Sampling Discrete 1D PDFs



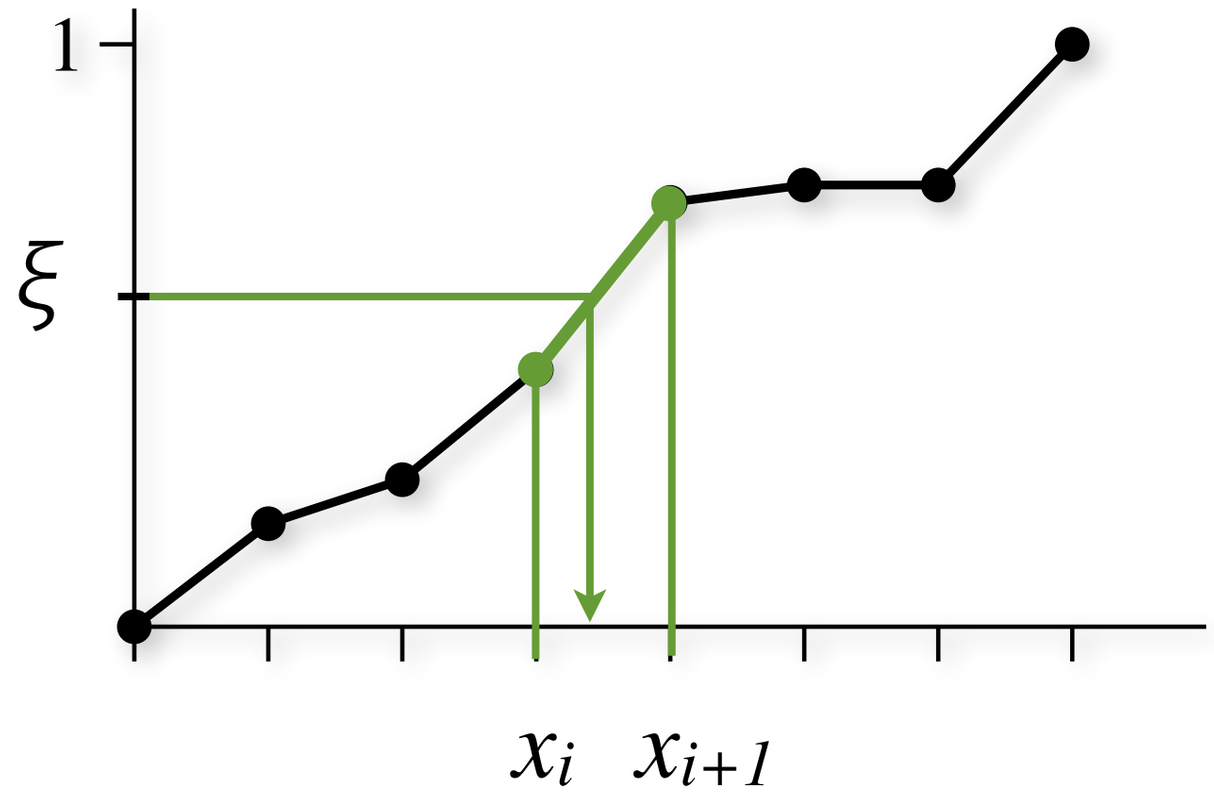
PDF



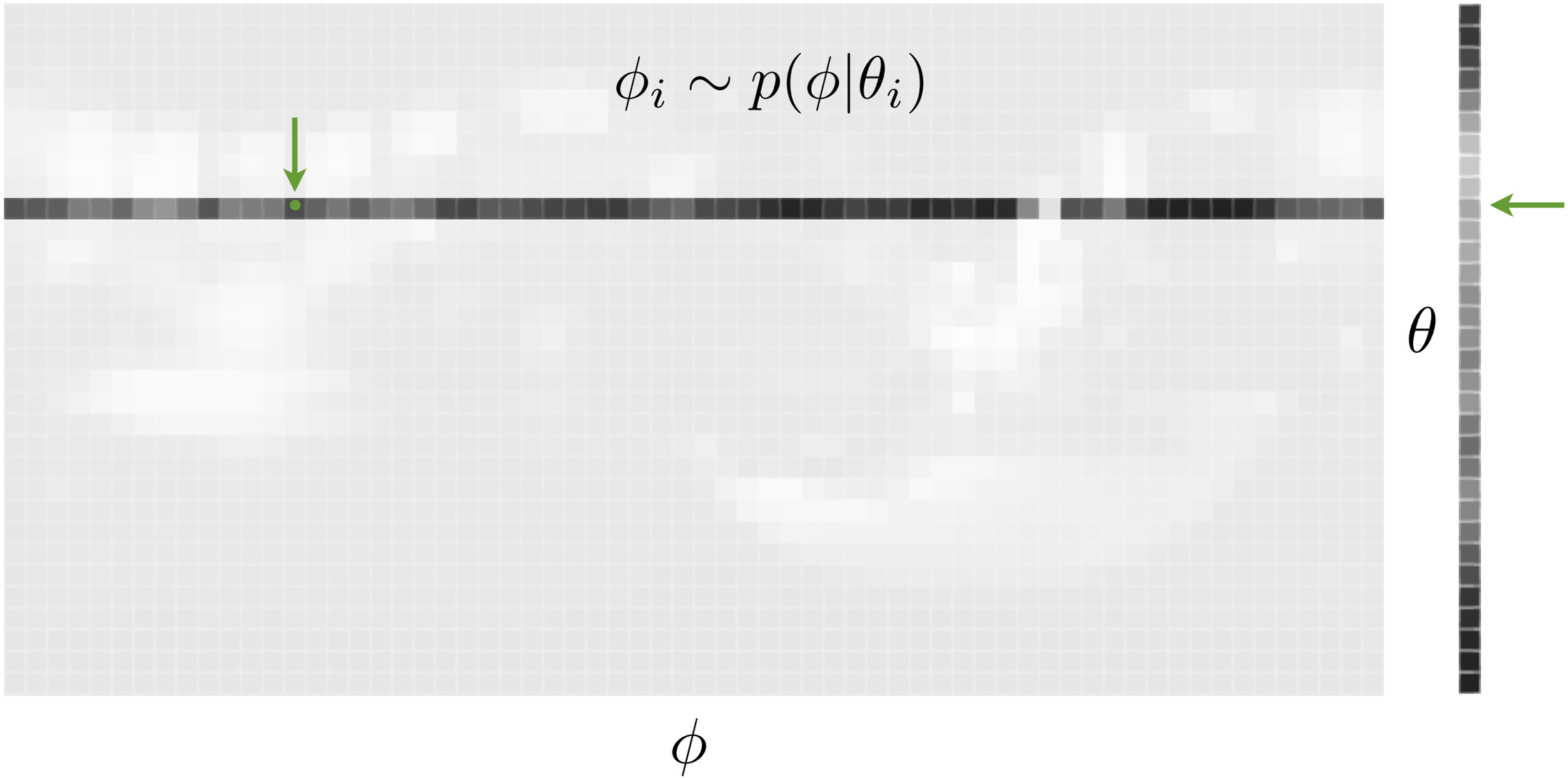
CDF

Sampling Discrete 1D PDFs

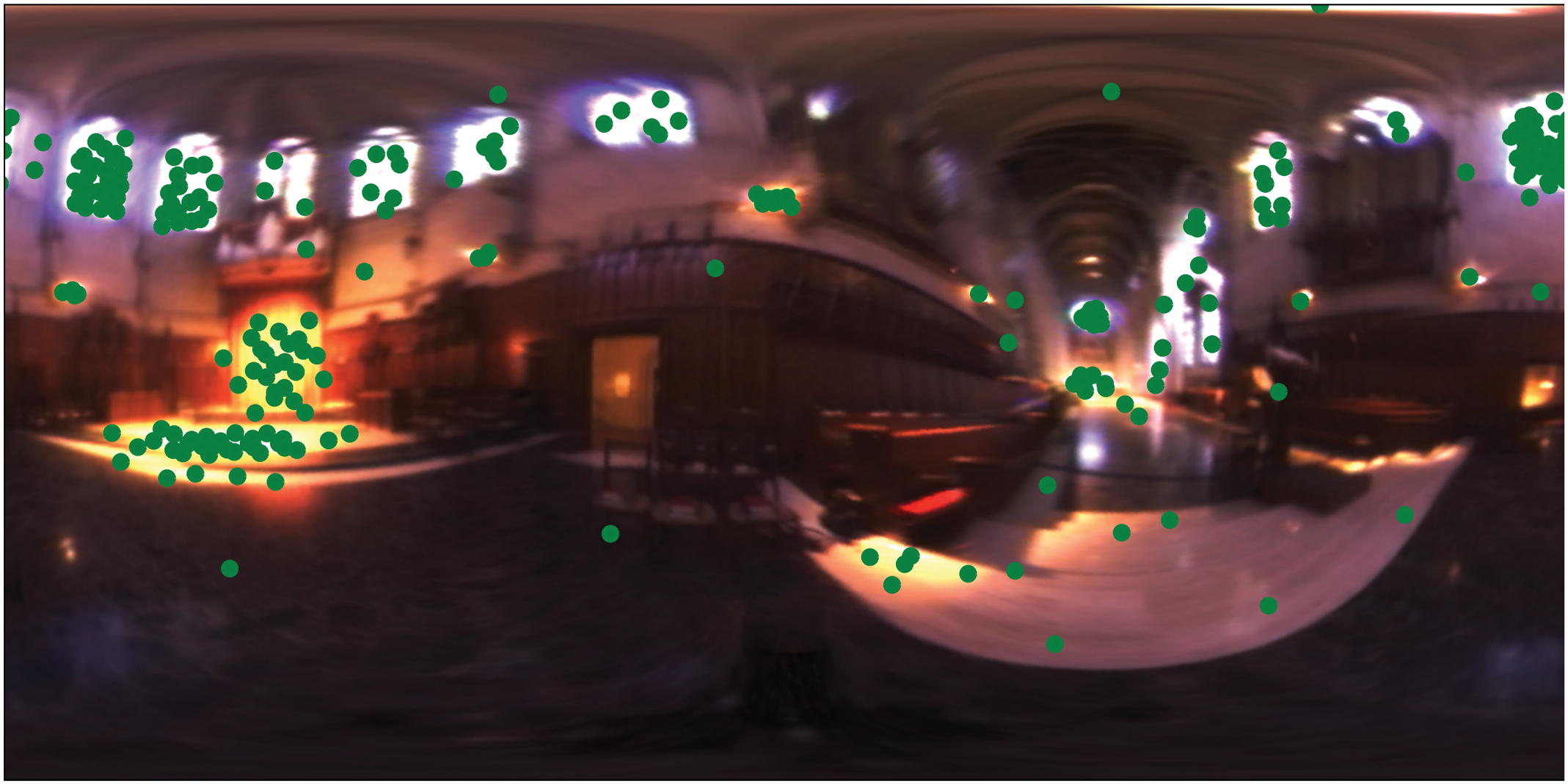
- Given a uniform random value ξ
- Using binary search find x_i and x_{i+1}
- Linearly interpolate to find x



Step 4: Sampling



Resulting Sample Distribution



Visual Break

openfootage.net

Hierarchical Sample Warping

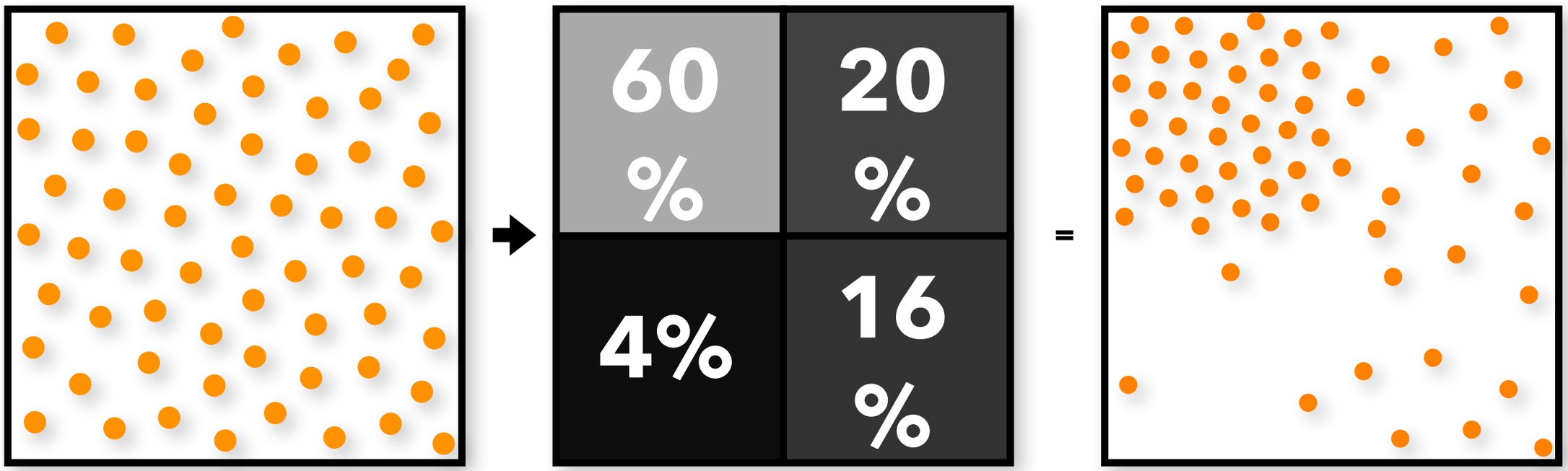
Clarberg, Jarosz, Akenine-Möller, Jensen. "Wavelet Importance Sampling," 2005.

- Given:
  - input point set, and
  - hierarchical representation of density function (mip-map)

Hierarchical Sample Warping

Clarberg, Jarosz, Akenine-Möller, Jensen. "Wavelet Importance Sampling," 2005.

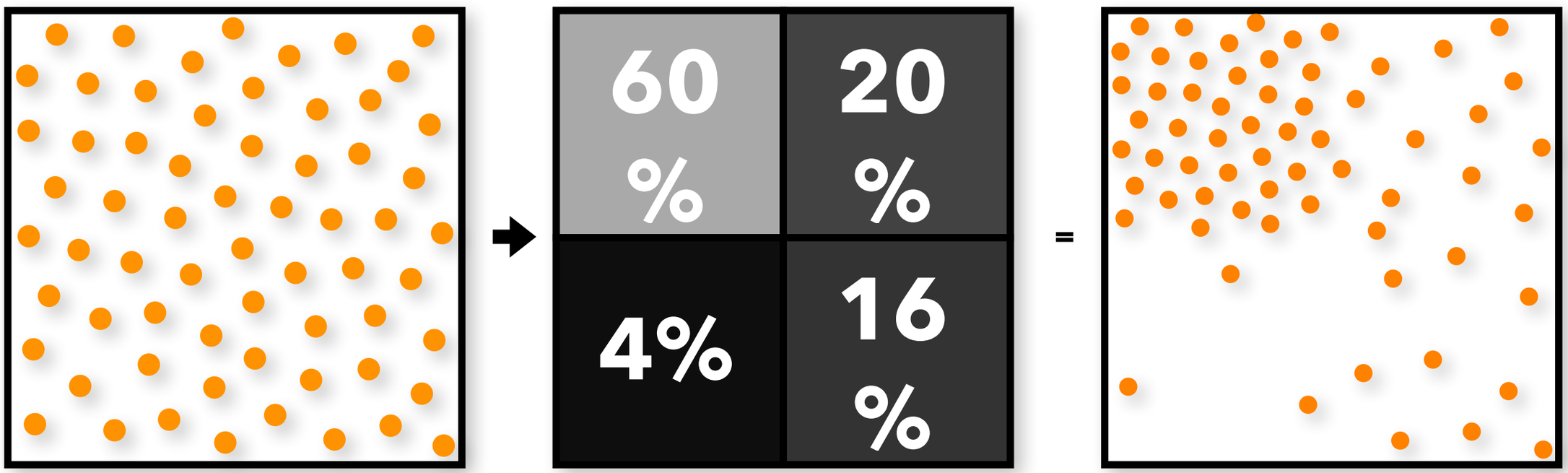
- Given:
  - **input point set**, and
  - hierarchical representation of density function (mip-map)
- Recursively warp point set to match the importance function



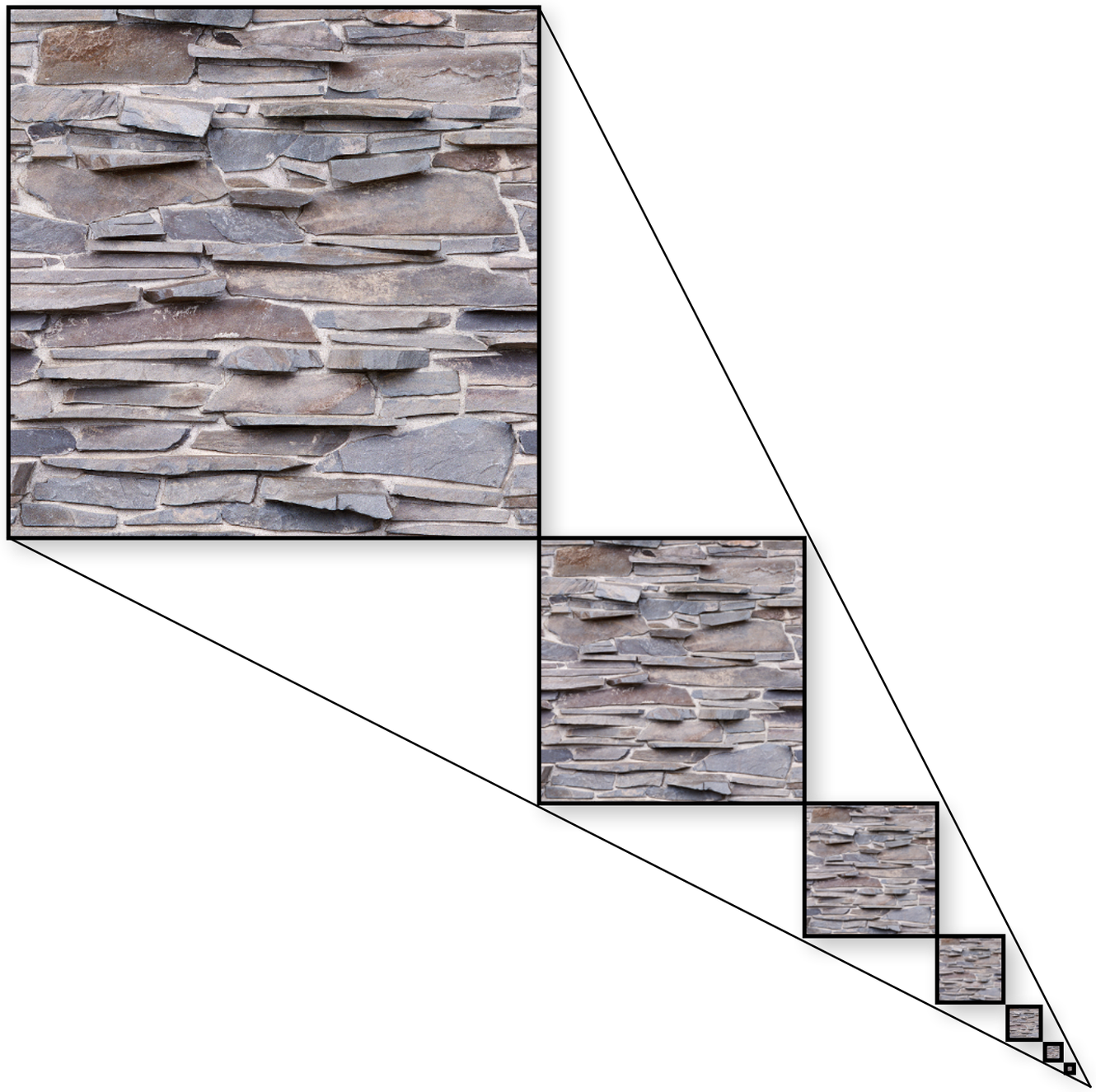
Hierarchical Sample Warping

Clarberg, Jarosz, Akenine-Möller, Jensen. "Wavelet Importance Sampling," 2005.

- Given:
  - input point set, and
  - **hierarchical representation of density function** (mip-map)
- Recursively warp point set to match the importance function



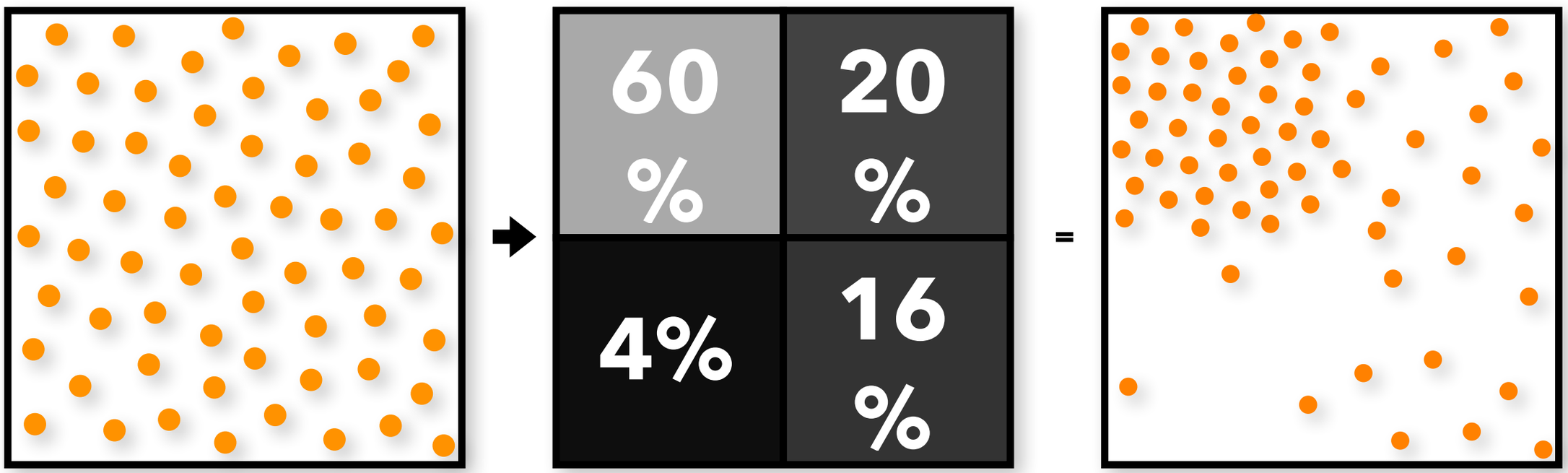
Mip-Map

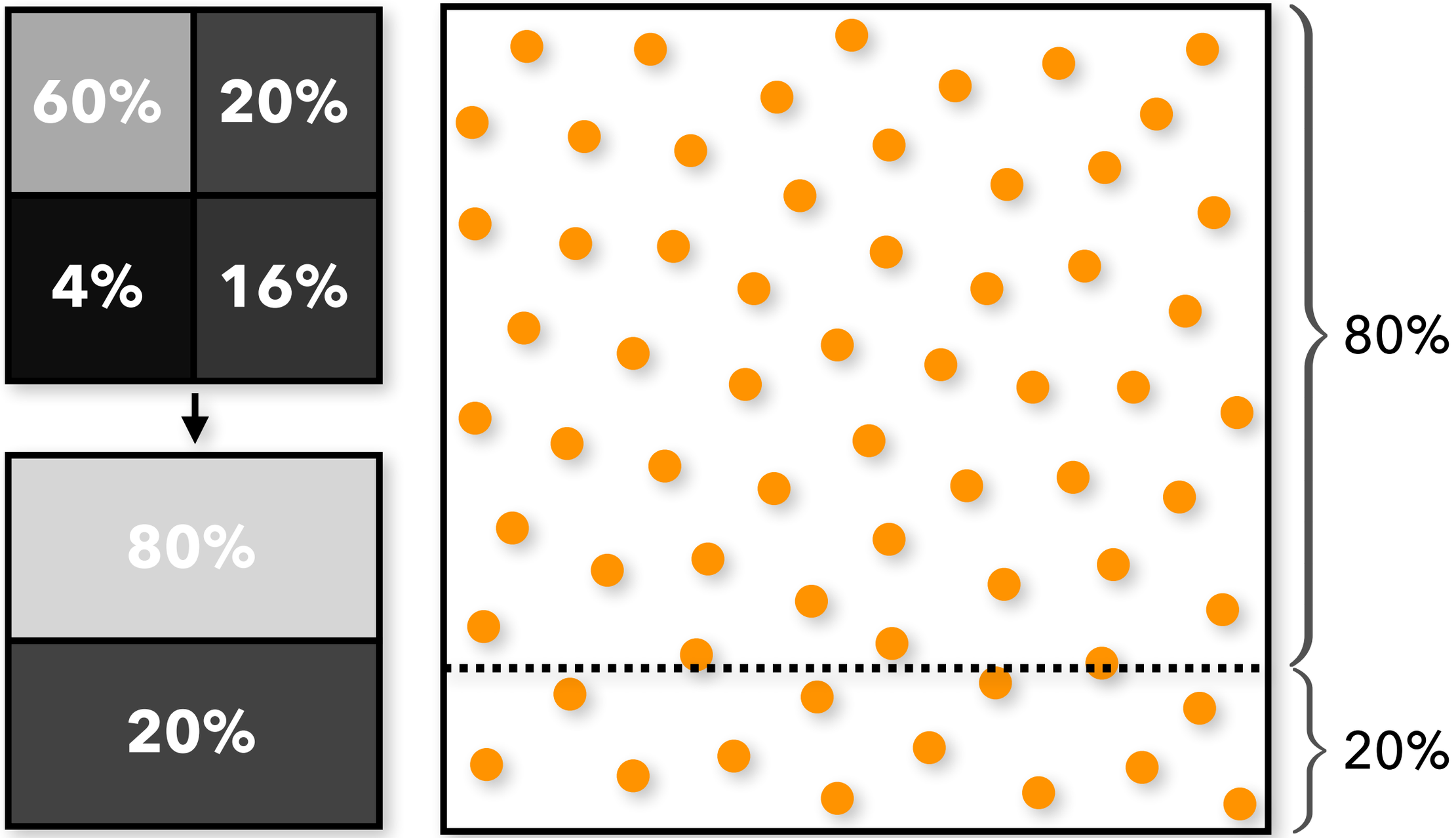


Hierarchical Sample Warping

Clarberg, Jarosz, Akenine-Möller, Jensen. "Wavelet Importance Sampling," 2005.

- Given:
  - input point set, and
  - hierarchical representation of density function (mip-map)
- Recursively warp point set to match the importance function




60%
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16%
80%
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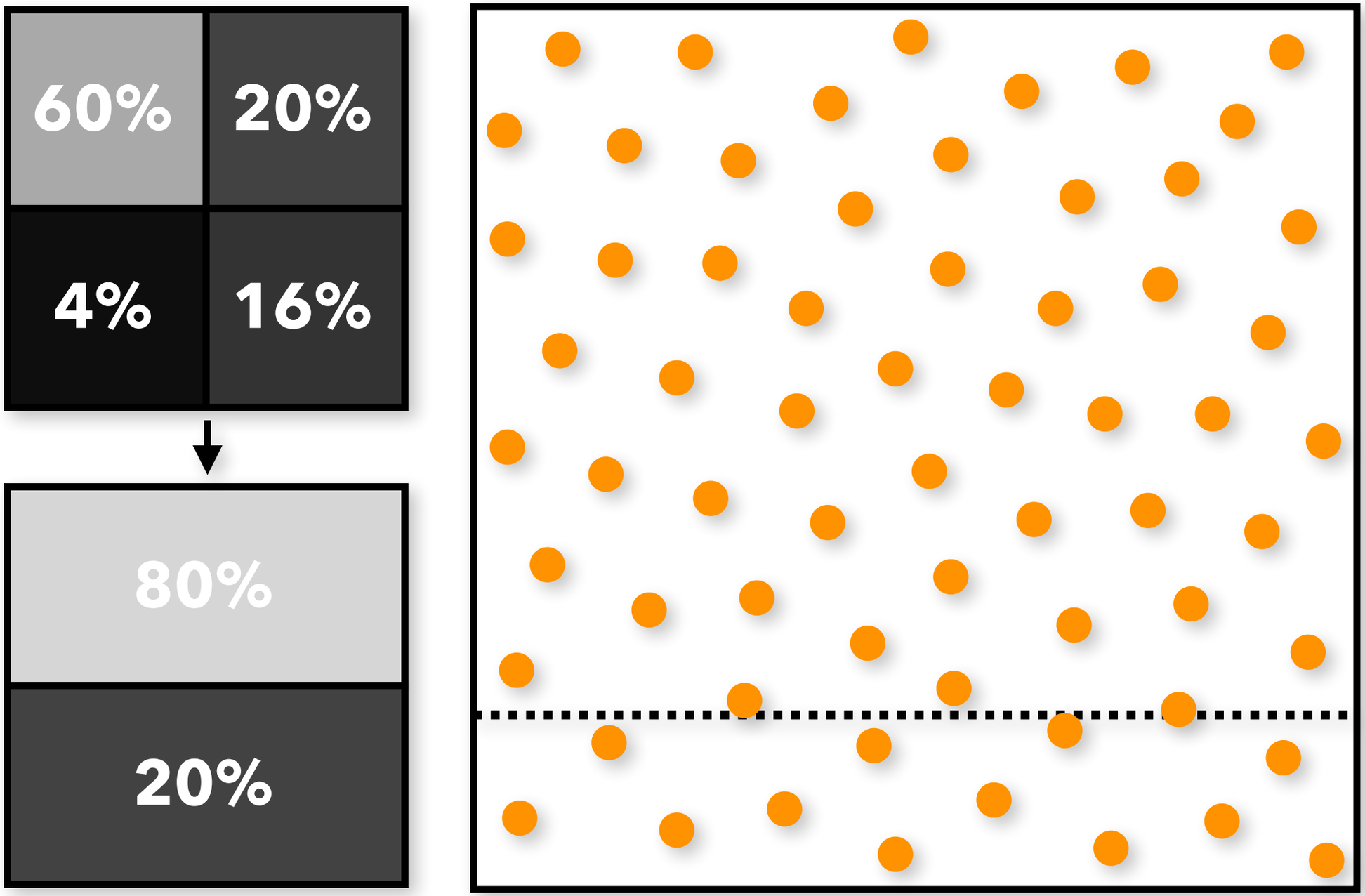
80%
20%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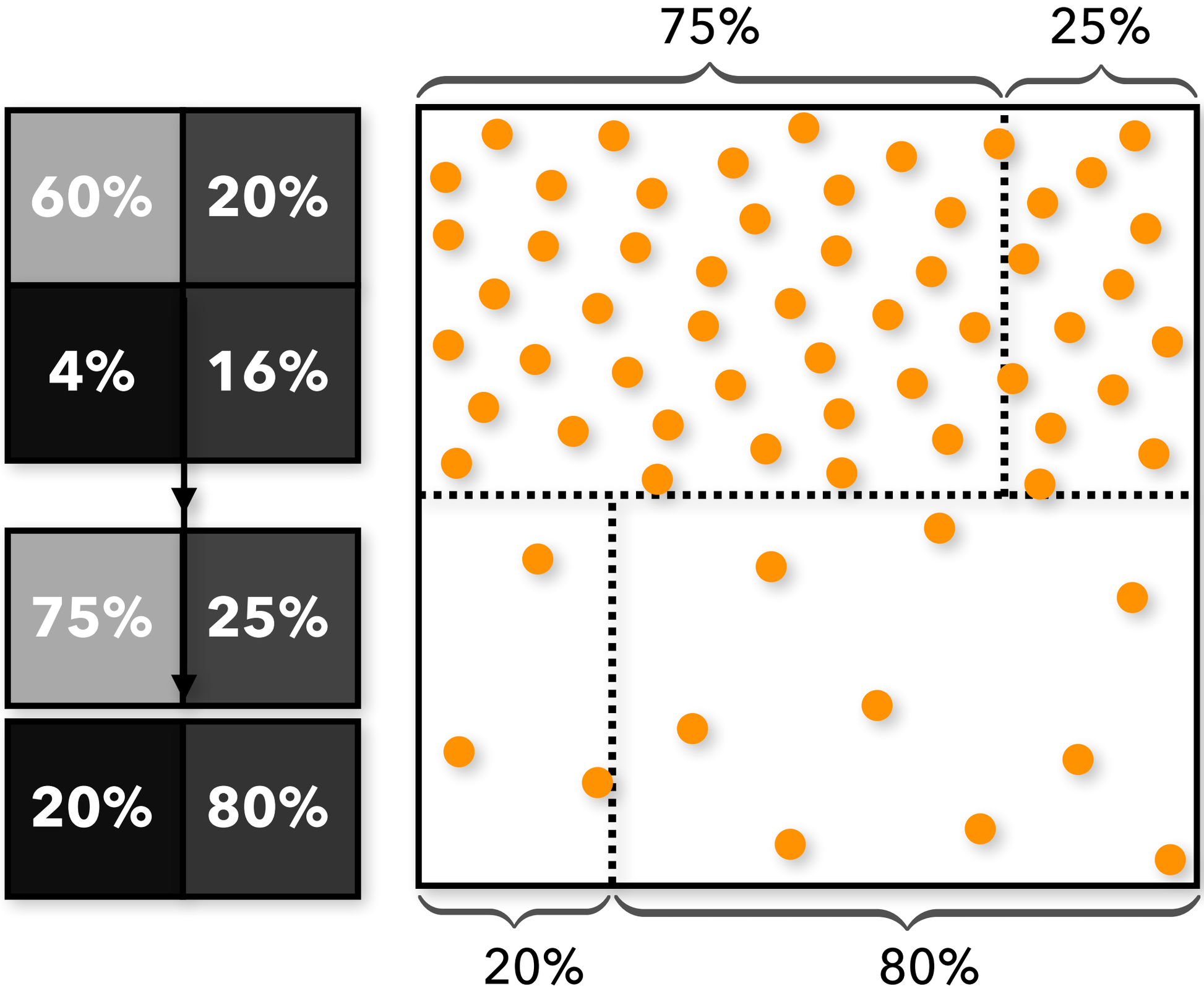
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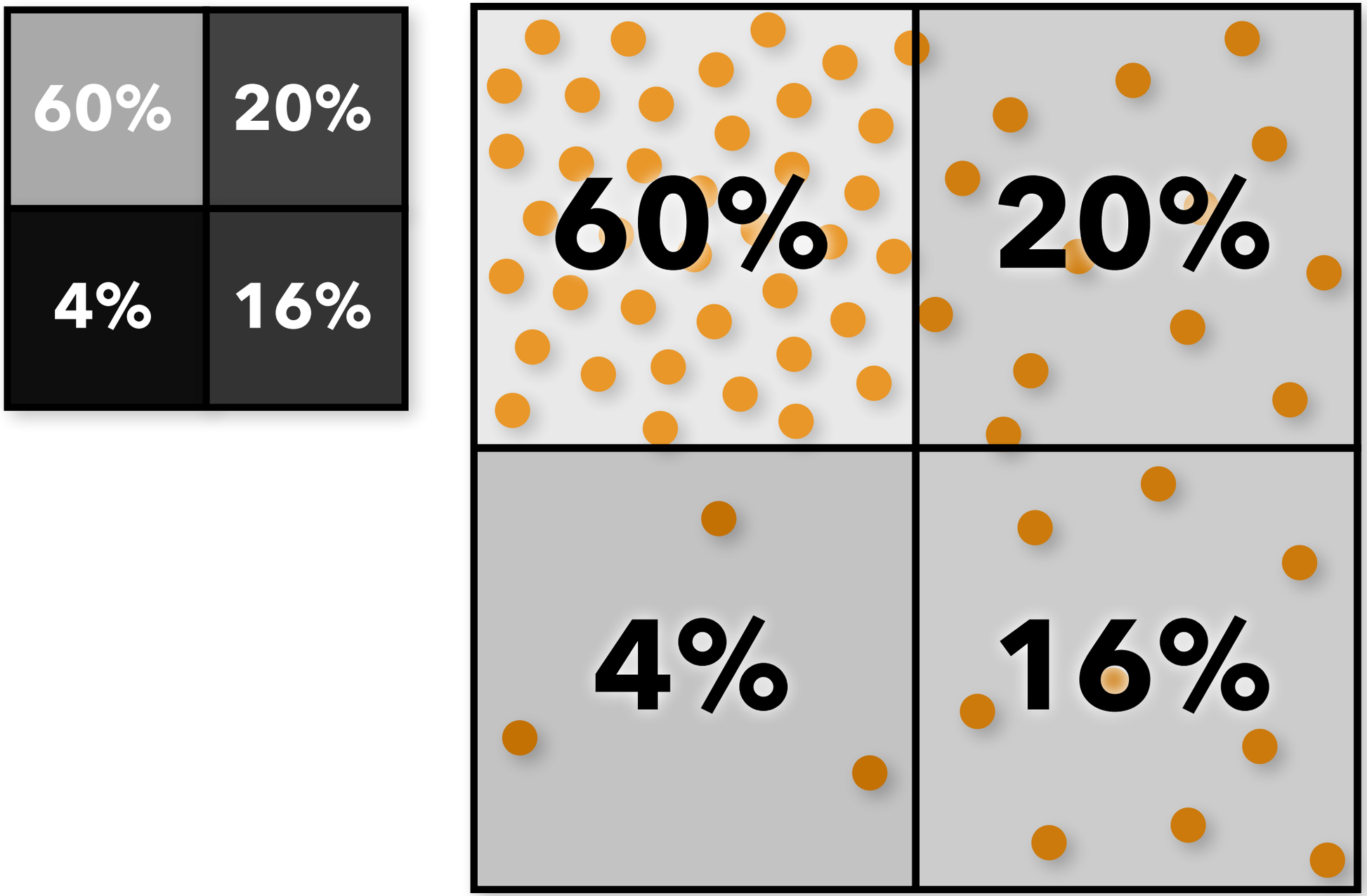


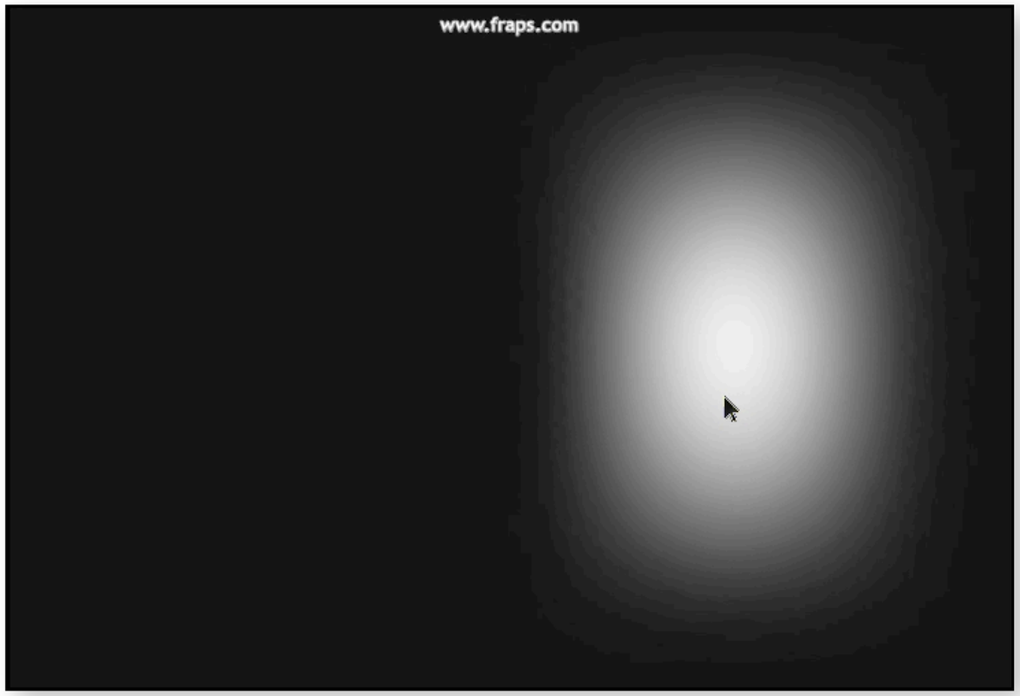
repeat on each quadrant recursively

Environment Lighting

$$
\begin{aligned}
L_r(\mathbf{x}, \vec{\omega}_r) &= \int_\Omega f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_i(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i \\
&= \int_\Omega f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_{\text{env}}(\vec{\omega}_i) V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i \, d\vec{\omega}_i \\
&\approx \frac{1}{N} \sum_{i=1}^{N} \frac{f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) L_{\text{env}}(\vec{\omega}_i) V(\mathbf{x}, \vec{\omega}_i) \cos\theta_i}{\text{pdf}(\vec{\omega}_i)}
\end{aligned}
$$

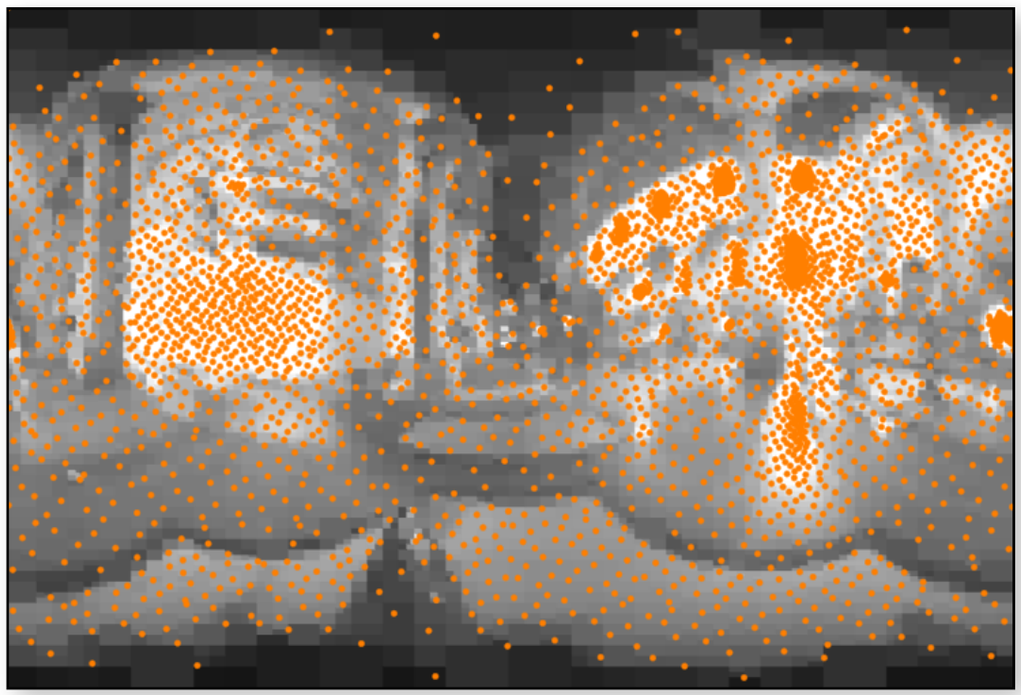
- What pdf should/can we use?
  - prop. to f_r
  - prop. to L_{env}

Sample Distributions



BRDF

or



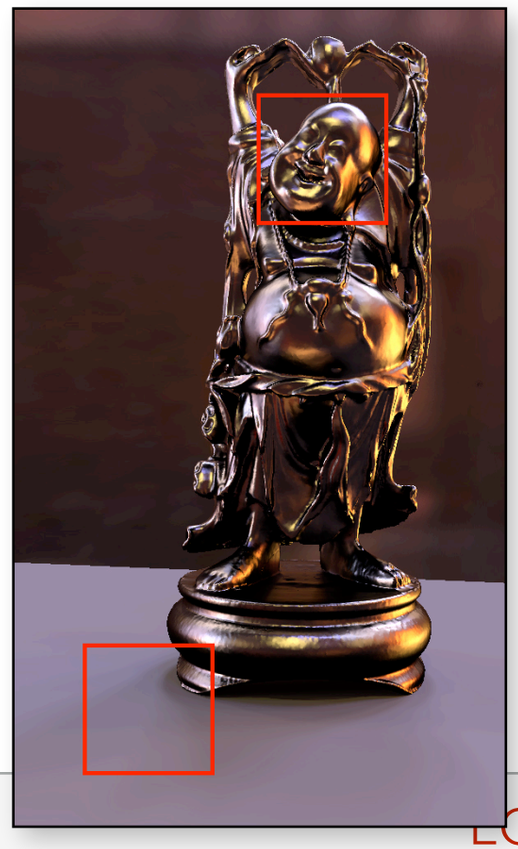
Lighting

BRDF

Lighting

1 3 10 30 60 100

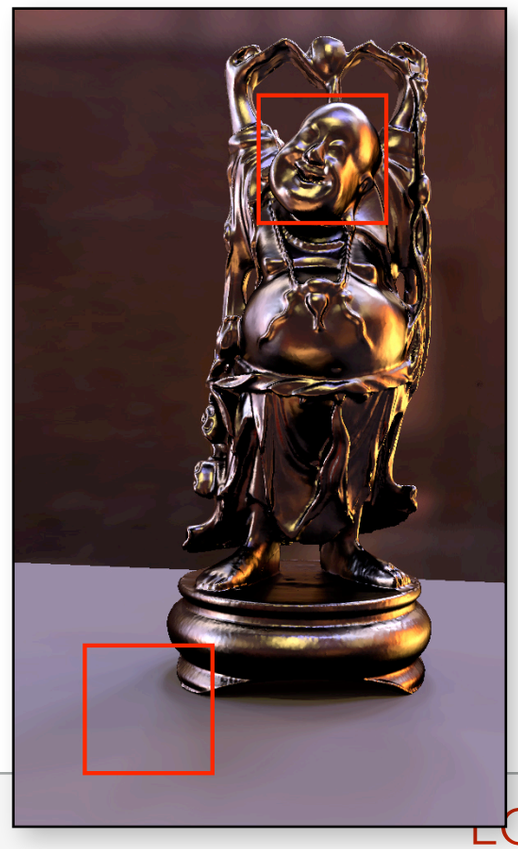
Samples / Pixel



BRDF

Lighting

1 3 10 30 60 100



Samples / Pixel

Product Sampling

Environment Lighting

- Could use MIS
  - pdf is a linear combination of f_r and L_{env}
- Can we sample the *product* $(f_r \cdot L_{env} \cdot \cos\theta_i)$ directly?

★ Clarberg, Jarosz, Akenine-Möller, Jensen. “Wavelet Importance Sampling,” 2005.

★ Subr and Arvo. “Steerable Importance Sampling,” 2007.

★ Jarosz, Carr, Jensen. “Importance Sampling Spherical Harmonics,” 2009.

★ many more...

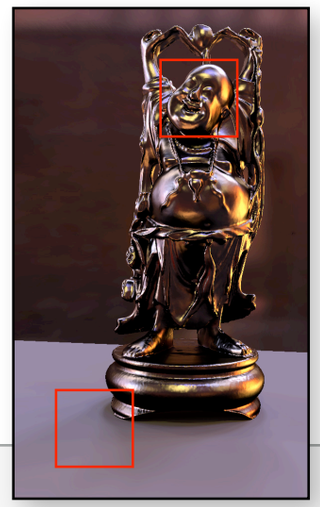
BRDF

Lighting

Product

1 3 10 30 60 100

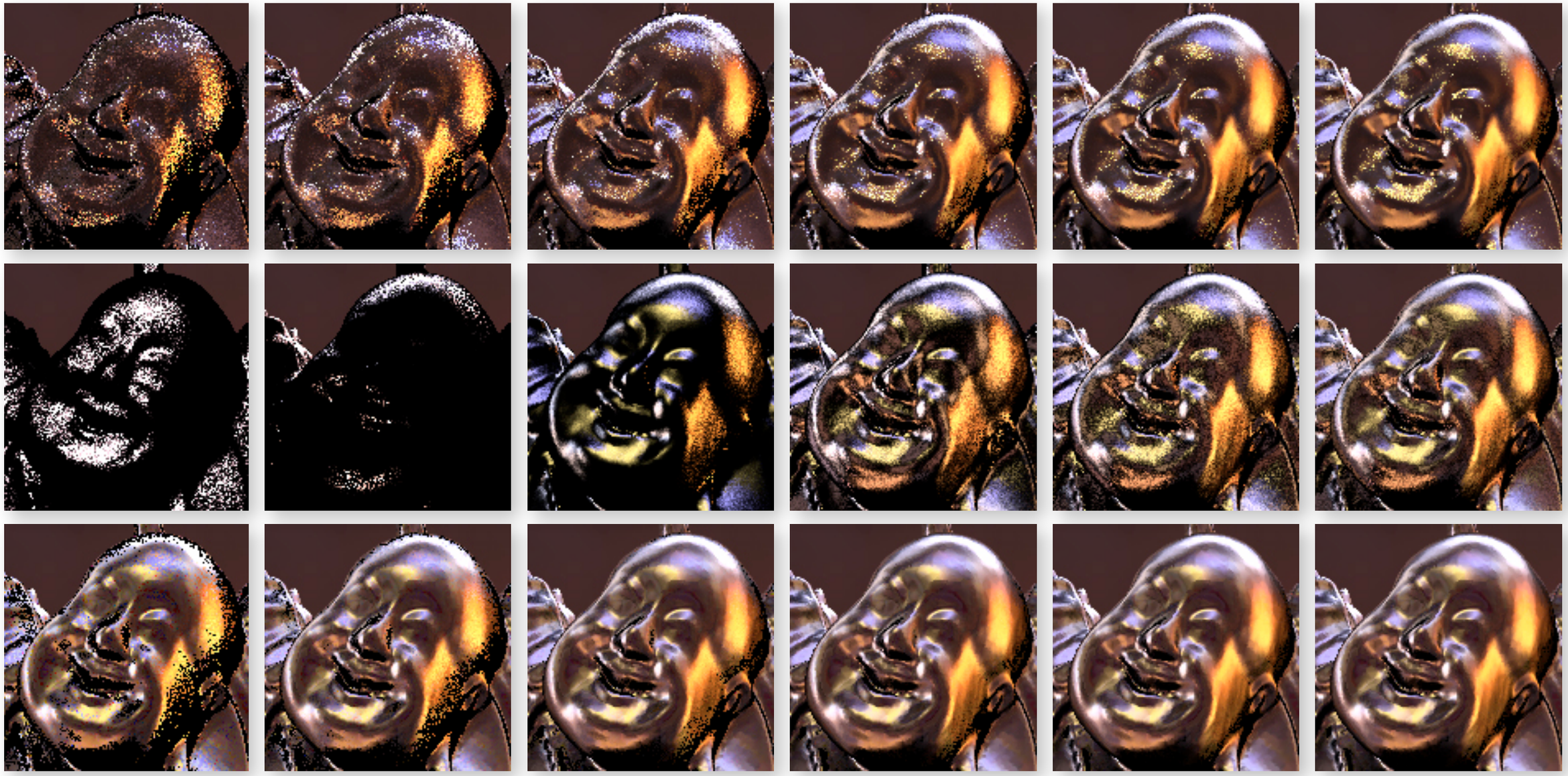
Samples / Pixel



BRDF

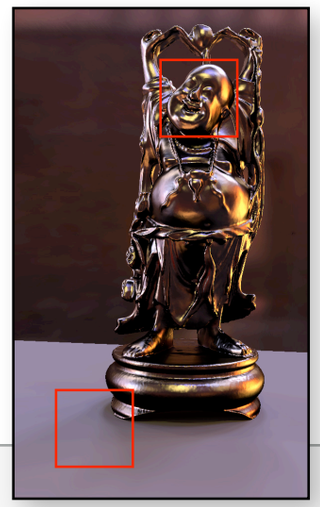
Lighting

Product

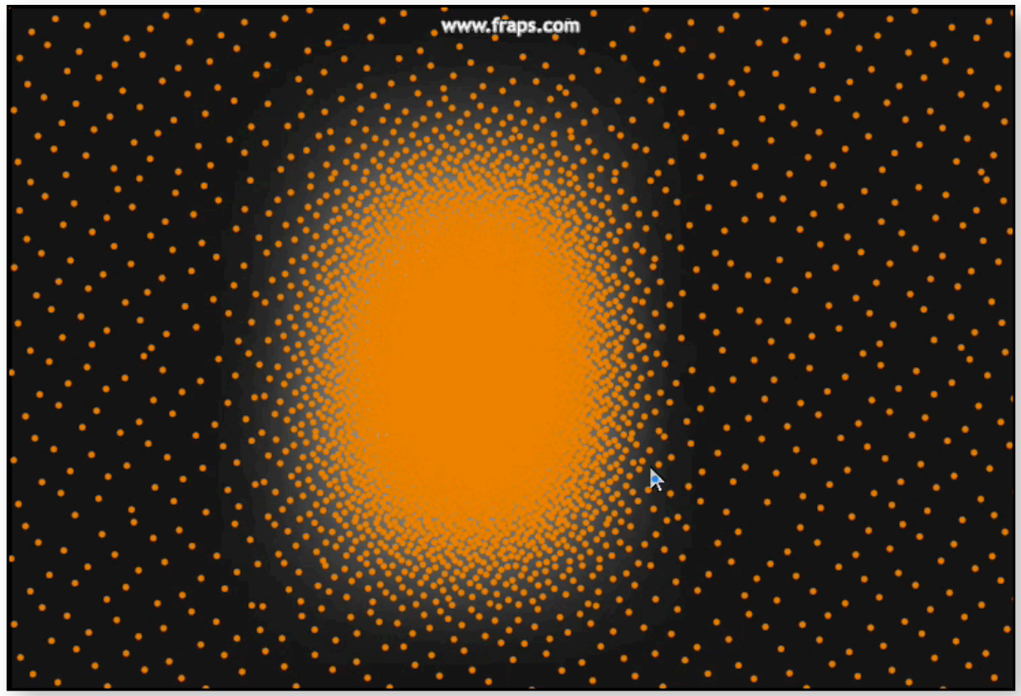


1 3 10 30 60 100

Samples / Pixel

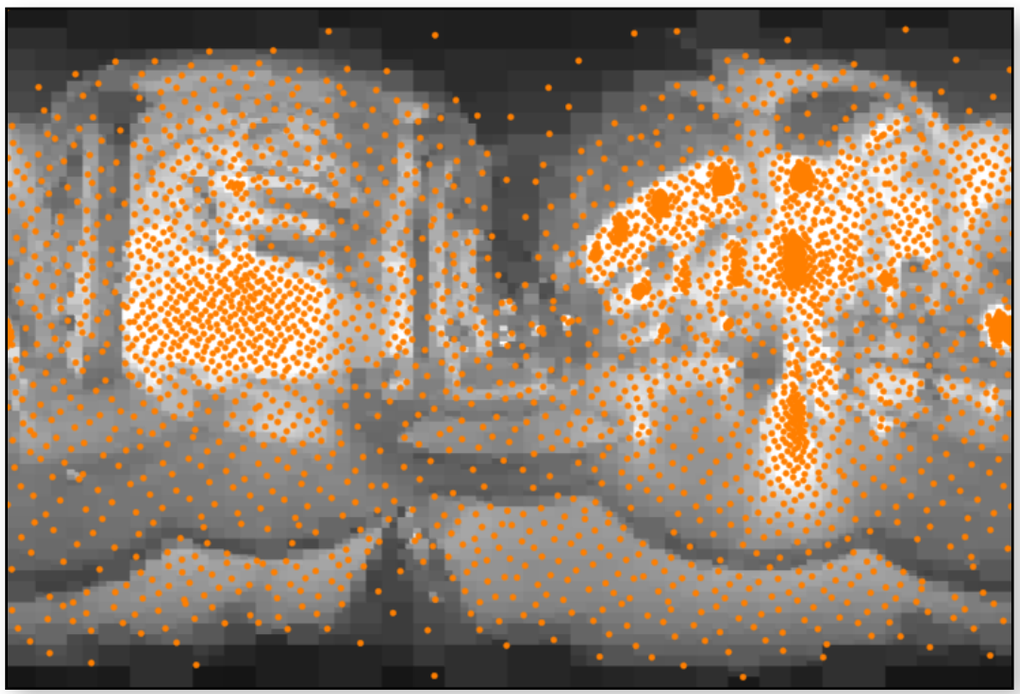


Sample Distributions



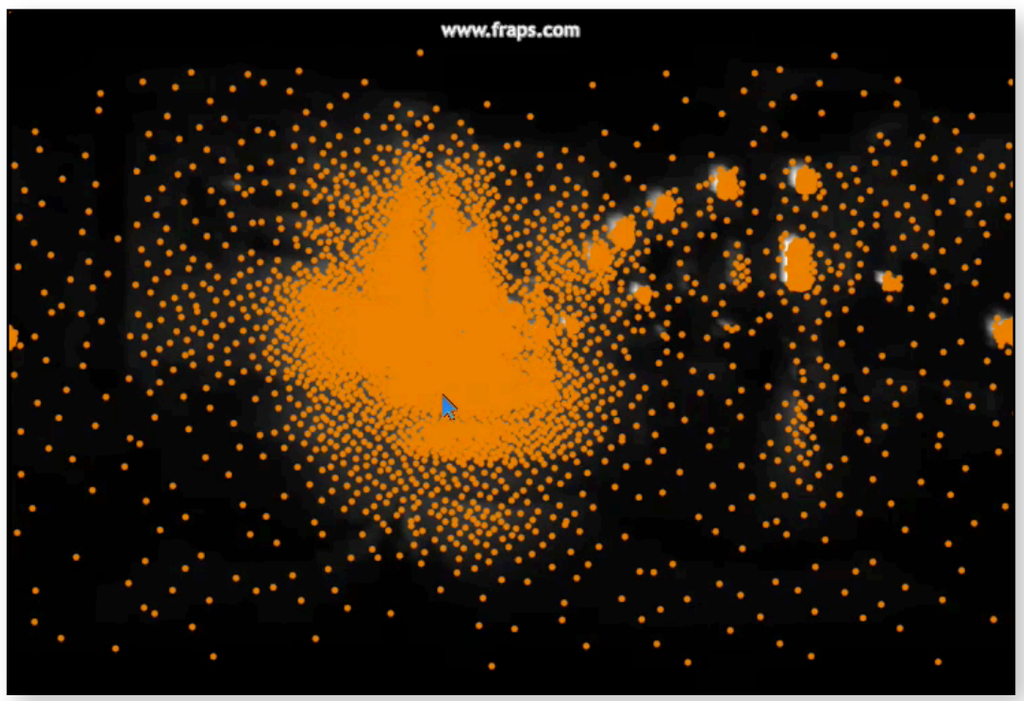
BRDF

x



Lighting

=



BRDF x Lighting

Clarberg, Jarosz, Akenine-Möller, Jensen. “Wavelet Importance Sampling,” 2005.

Questions?