Statistical Computer Vision (ECSE 626)
Assignment 1

Due: February 25^{th}, 2019 at 11:59PM

Please submit your assignment solutions electronically via the myCourses assignment dropbox. Attempt all parts of this assignment. The assignment will be graded out of total of **57 points**. Students are expected to write their own code. (Academic integrity guidelines can be found at `https://www.mcgill.ca/students/srr/academicrights/integrity`). Assignments received up to 24 hours late will be penalized by 30%. Assignments received more than 24 hours late will not be graded.

Submission Instructions

1. Title two scripts as (i) information_theory (ii) face_recognition.
2. Comment your code appropriately.
3. Do not submit given input images. Assume image folder and face dataset are kept in a same directory as the codes. Make sure that the submitted code is running without error. Add a README file if required.
4. Do not submit output images. You can include the output images in your report.
5. If external libraries were used in your code please specify its name and version in the README file.
6. Answers to reasoning questions should be comprehensive but concise.
7. Submissions that do not follow the format will be penalized 10%.

1 Information Theory

In this section, you will explore how information theory can be used to for the analysis of medical images. The image for analysis, I, can be seen in Figure 1. For all the questions below, you are permitted to use in-built functions, except when performing the information theoretic computations themselves (e.g. entropy).

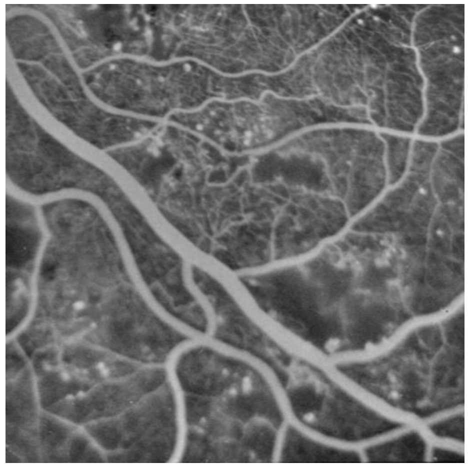


Figure 1: Image of a retina. courtesy: Ajit Rajwade.

1.1 Entropy of an image

Let X be a random variable defining the intensity of a pixel in an image and let $\{x_i | i = 1, 2, 3, \dots\}$ be the set of discrete values that X can take on. The entropy of X is given by:

$$H(X) = -\sum_i p(x_i) \log_2(p(x_i))$$

1. Compute the entropy of I. **(1 point)**

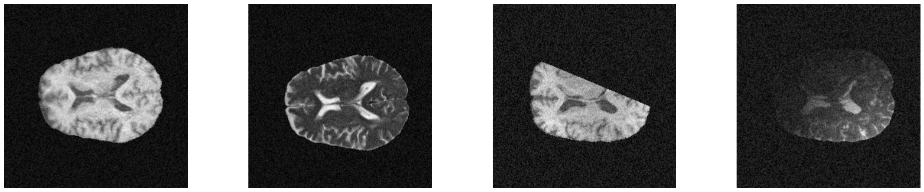
2. Generate an image $\mathcal{N}_A$ of the same size as I such that pixel intensities of $\mathcal{N}_A$ are drawn from $\mathrm{U}[-A, A]$(discrete uniform noise). Vary A from 0 to 200 in steps of 5 and generate instances of $\mathcal{N}_A$. Compute and plot in a single graph the entropy of: I, $\mathcal{N}_A$ and $I + \mathcal{N}_A$ for multiple values of A. Discuss the trend in the graph. **(5 points)**

1.2 Mutual Information and KL divergence

1. Generate the noise images, $\mathcal{N}_A$, by varying A from 0 to 200 in the steps of 5 as described in the previous question. Compute and plot the mutual information between the images I and $I + \mathcal{N}_A$ as a function of A. Discuss the trend in the graph. **(5 points)**

2. Generate a single noise image, $\mathcal{N}_{20}$, (i.e. $\mathcal{N}_A$ with $A = 20$). Compute the joint entropy of the image pair: $H(I,\, I + \mathcal{N}_{20})$. Also, verify numerically that:

$$H(I; I + \mathcal{N}_{20}) = H(I) + H(I + \mathcal{N}_{20}) - MI(I; I + \mathcal{N}_{20})$$



(a) (b) (c) (d)

Figure 2: Images to be registered. Images are titled as (a) I1_1 (b) I1_2 (c) I2_1 (d) I2_2.

(Note: Reuse $H(I)$, $H(I + \mathcal{N}_{20})$ and $MI(I; I + \mathcal{N}_{20})$ from the previous questions.) **(2 points)**

3. KL divergence is defined as follows:

$$D_{KL}(P||Q) = -\sum_i P(x_i)\log_2\left(\frac{Q(x_i)}{P(x_i)}\right)$$

Compute the forward and the backward KL divergences, i.e. $D_{KL}(P||Q)$ and $D_{KL}(Q||P)$ between: (i) I and the noise image $\mathcal{N}$ of the same size as I; where pixel intensities of $\mathcal{N}$ are drawn from U$[0, 255]$ (ii) I and $I + \mathcal{N}_{20}$ from the previous question. Discuss the values. (Note: For (ii), use Parzen window filtering on the histogram of I, and mention the parameters of the window.) **(4 points)**

1.3 Registration

A set of brain images are shown in Figure 2. The images consist of 2D slices of Magnetic Resonance Images (MRI) from patients with brain tumours. The images are taken from the MICCAI 2017 BRATS Challenge dataset [1, 2].

1. You will register pairs of images from Figure 2 using a simple transformation space: x-y translations. Integer values are sufficient. Consider two similarity metrics: mutual information and mean squared error. Report the optimal translation computed by these two metrics.

   Below are each of the image pairs to be registered. Consider the first of the pair to be the moving image and the second to be the fixed image.

   - I1_1 and I1_2 **(4 points)**
   - I2_1 and I2_2 **(4 points)**

2. Discuss the above results in terms of the assumptions inherent to the metrics. Describe the context in which each metric should be used. Support your arguments with an example or two. **(3 points)**

2 Face Recognition

You will explore and compare eigenfaces to probabilistic face recognition, based on face images from a subset of the publicly available Color FERET Database [3] provided with the assignment. Whereas the full database contains images from approximately 1000 different subjects, the given subset contains images from 52 subjects. The subset is arranged into several folders each containing images of a specific subject in different poses. The number of images per subject varies from 32 to 96. Each image is 768×512 pixels and the files are in PPM-format. You may choose to convert images to gray-scale and/or down-sample images by the scale which does not degrade the performance of the system.

You will design a face recognition system using this subset. Randomly separate the images into training and test sets as described below. A new random selection should be made by your program every time the system is retrained.

- **Train set.** For each subject, randomly select 80% of the total images given for that subject. This will be used as the training set. **(1 point)**
- **Test set.** All the remaining images from the given subset which are not used in the training set will be used as test images. **(1 point)**

2.1 Principal Components Analysis

1. The training set can be placed into a matrix $X = [\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_D]$ of size $N \times D$, N being the total number of pixels in an image $\mathbf{x}$ and D being the total number of training images. Compute the principal components using the snap-shot method. Display the mean face and the first 10 Eigenfaces. **(6 points)**
2. Decide on the number of principal components N_p required to represent the data. You can use either of the two methods for dimensionality estimation discussed in the class (refer PCA slides 69 and 72). Justify your choice and support it with an appropriate graph. **(3 points)**

2.2 Probabilistic Face Recognition

You will explore recognizing faces using Bayesian techniques. Let $\phi(\mathbf{x})$ be eigenspace representation of $\mathbf{x}$. Let y be the random variable describing class label i.e. the subject face identity.

1. Using Bayes rule, derive an expression for the posterior density of a subject label y for a given test image $\mathbf{x}_*$. **(1 point)**
2. Find eigen representation $\phi(\mathbf{x})$ for each training image $\mathbf{x}$ i.e project the image $\mathbf{x}$ on the first N_p Eigenfaces and find corresponding N_p coefficients. **(3 points)**
3. Build total 52 Gaussian density functions for the likelihood, one for each subject in the training set. **(3 points)**

4. Do the following for each test image $\mathbf{x}_*$.

   - Find eigen representation $\phi(\mathbf{x}_*)$ of $\mathbf{x}_*$. **(2 point)**
   - (Method 1) Assign the subject label to $\mathbf{x}_*$ based on the MAP. Display the posterior density function for one test image in the report. **(4 points)**
   - (Method 2) Use nearest neighbor classifier to assign the subject label to $\mathbf{x}_*$. **(2 points)**

5. Calculate the recognition rate as the ratio of correctly classified images to the total number of images for both the methods. Compare the results. **(3 points)**

References

[1] Menze, Bjoern H., et al. "The multimodal brain tumor image segmentation benchmark (BRATS)." IEEE transactions on medical imaging 34.10 (2015): 1993.

[2] Bakas, Spyridon, et al. "Advancing the cancer genome atlas glioma MRI collections with expert segmentation labels and radiomic features." Scientific data 4 (2017): 170117.

[3] Phillips, P. Jonathon, et al. "The FERET database and evaluation procedure for face-recognition algorithms." Image and vision computing 16.5 (1998): 295-306.