Math 488 Fall 2019 Problem Set #1 Wed 2018-08-28

(1) Evaluate $\displaystyle\int_{-2}^{-1} \frac{1}{\sqrt{-x^2-6x}}\, dx.$

(a) $\arcsin(2/3) - \arcsin(1/3)$

(b) $\arcsin(2/3) - \arccos(1/3)$

(c) $\arctan(2/3) - \arctan(2/3)$

(d) $\arctan(2/3) + \arctan(2/3)$

(e) $\arccos(2/3) + \arccos(1/3)$

(2) Let $f(x) = x^3 + 6x^2 - 32$. Define $T_{c,f}(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$ to be the Taylor series for the function f centered at c. For what values of C does $a_0 = 0$?

(a) $c = 3, 2$

(b) $c = -3, 2, -2$

(c) $c = 4, 2$

(d) $c = -4, 2, -2$

(e) $c = -4, 2$

(3) The value of the integral $\displaystyle\int_0^1 \sqrt{e^{2x} + e^{-2x} + 2}\, dx$ is

(a) $e - \frac{1}{e}$

(b) $e + \frac{1}{e}$

(c) $e - \frac{1}{e} + 1$

(d) $e + \frac{1}{e} - 1$

(e) $2e$

(4) For the function

$$f(x) = \begin{cases} \frac{x^2y}{x^4+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

which of the following is true?

I f is not continuous at $(0,0)$.

II f is differentiable everywhere.

III f has well-defined partial derivatives everywhere (i.e. $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ are both defined)

IV f is continuous at $(0,0)$ but not differentiable at $(0,0)$

(i) I only

(ii) II only

(iii) III only

(iv) IV only

(v) I and III only.

(5) Evaluate the limit: $\lim_{n\to\infty} (3^n + 5^n)^{2/n}$.

(a) 0

(b) 1

(c) 9

(d) 16

(e) 25

(6) Evaluate $\displaystyle\int_0^{\pi/2} \frac{1}{1+\tan(x)}\, dx$ (TRICKS INVOLVED!)

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi^2}{2}$

(e) $\frac{\pi^2}{4}$

(7) The series $1 - 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + \cdots$ converges to: [Hint: Start playing around with $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$.]

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{5}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi^2}{2}$