CSci 435: Formal Languages and Automata

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**Home Assignment 1: 113/110 points + 10 points (optional)**

Q1. [25/25] For Σ = {a, b}, construct the minimal DFA that accept the language consisting of

1. [8/8] all strings with an even number of *a*’s and an odd number of *b*’s.

*Ans:*

A drawing of a person

Description automatically generated

1. [8/8] every ‘*aa’* is followed immediately by a ‘*b’*. For example, the strings *aab*, *aaba*, *aabaabbaab* are in the language, but *aaab* and *aabaa* are not. Construct a DFA with 4 states.

*Ans:*

A drawing of a person

Description automatically generated

1. [9/9] L = {w | ( *na*(*w*) – *nb*(*w*) ) mod 3 = 0 }. Construct a DFA with 3 states.

*Ans:*

A picture containing drawing

Description automatically generated

Q2. [10/10] Show that the language L = { *a****n***| *n* ≥ 0, *n* ≠ 3 } is regular.

*Ans:*

A drawing of a person

Description automatically generated

Q3. [15/15] For the language L = {*an* | *n* ≥ 1 } ∪ {*bmak* | *m* ≥ 0, *k* ≥ 0}

1. [8/8] Construct an NFA with three states that accepts L.

*Ans:*

A drawing of a person

Description automatically generated

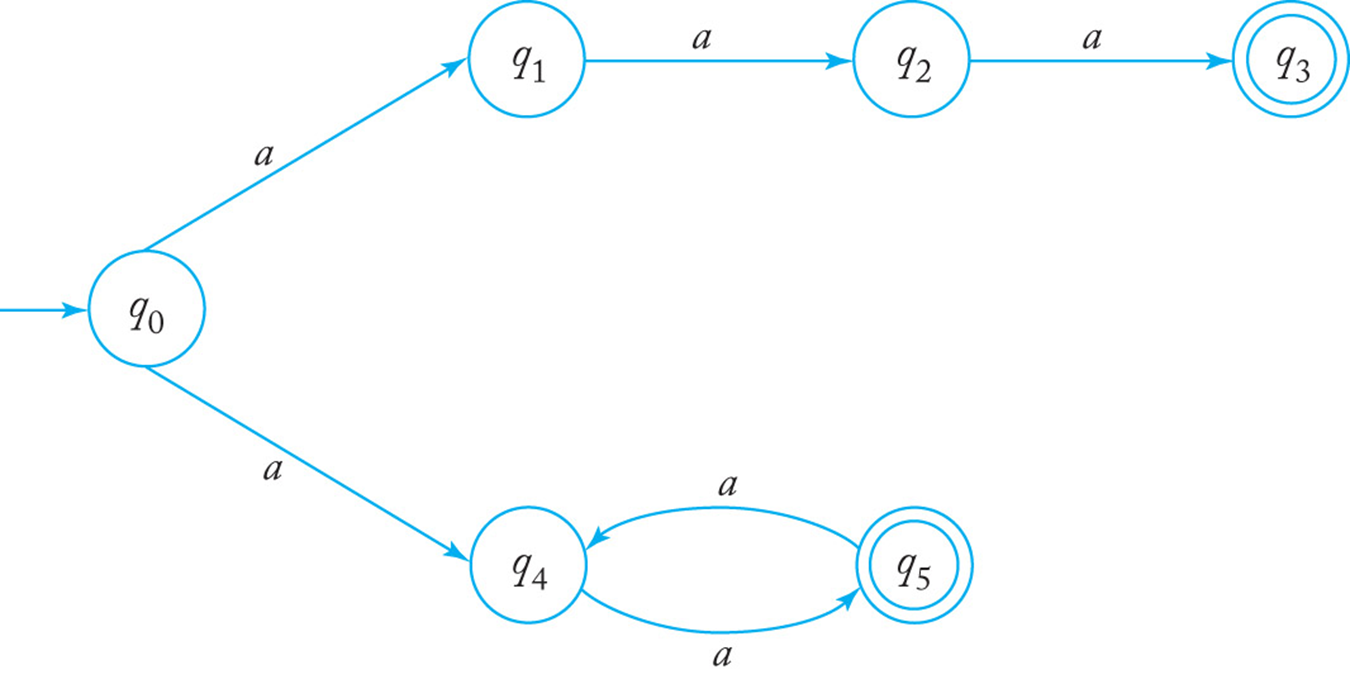
1. [7/7] Can you construct an NFA with the fewer states that accepts L? If so, construct it; otherwise, justify why your NFA in 1) is the minimal NFA.

*Ans: Yes.*

A drawing of a person

Description automatically generated

Q4. [20/20] For a given NFA in the figure,



1. [10/10] Give a language *L* that is accepted by the NFA. Describe L in the proper mathematical format, not in the verbal English description. E.g.) L = { *a****n***| *n* ≥ 0, *n* ≠ 3 }

*Ans: L* = { *a2n* | *n* ≥ 1 } ∪ { *a3* }.

1. [10/10] Find a *DFA* that accepts the ***complement*** of the language defined by the NFA, i.e. .

*Ans: L* = { *a2n+1* | *n* ≥ 2 } ∪ { λ, *a*}.

A picture containing drawing

Description automatically generated

Q5. [10/10] Construct an NFA with the ***minimum*** number of states that accepts

*L* = { *an* | *n* ≥ 0 } ∪ { *bna* | *n* ≥ 1 }.

*Ans:*

A picture containing drawing

Description automatically generated

Q6. [10/10] Convert the NFA defined by the transitions below with the initial state *q0* and the final state *q2* into an *equivalent DFA*. Draw the transition graph of the DFA.

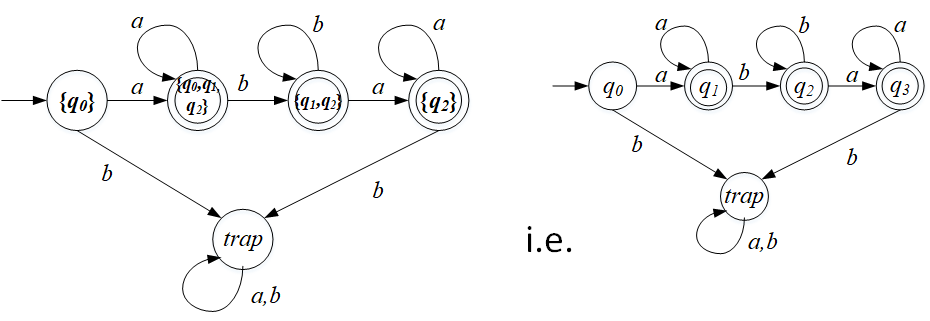
δ(*q0, a*) = { *q0, q1* }, δ(*q1, b*) = { *q1, q2* }, δ(*q2, a*) = { *q2* }, δ(*q1,* λ) = { *q1, q2* }.

*Ans:*

A drawing of a person

Description automatically generated

* You can connect q5 and q4 to one state.
* See the attached sample answer for reference



Q7. [20/20] For a given language, L = { *anb* | *n* ≥ 1 } ∪ { *bna* | *n* ≥ 1},

1. [10/10] Construct a *minimal DFA* with the minimum number of states that accepts L.

*Ans:*

A picture containing clock, drawing

Description automatically generated

1. [10/10] Prove that your DFA in 1) is minimal. Hint: Check if any pair of the states are indistinguishable to be merged in the same class so that the number of states are minimized

*Ans:* To prove a DFA is minimal, it suffices to show that no state is *unreachable* and no pair of states are *indistinguishable*.

δ(*q0, λ*) = *q0* and *λ* is in L so *q0* is not unreachable.

δ(*q0, a*) = *q1* and *a* is in L so *q1* is not unreachable.

δ(*q0, b*) = *q2* and *b* is in L so *q2* is not unreachable.

δ(*q0, ab*) = *q3* and *ab* is in L so *q3* is not unreachable.

δ(*q0, abb*) = *q4* and *abb* is in L so *q4* is not unreachable.

States *q0* and *q1* are distinguishable since δ\*(*q0, ba*) ∈ F but δ\*(*q1, ba*) ∉ F.

States *q0* and *q2* are distinguishable since δ\*(*q0, ab*) ∈ F but δ\*(*q2, ab*) ∉ F.

States *q0* and *q3* are distinguishable since δ\*(*q0, ab*) ∈ F but δ\*(*q3, ab*) ∉ F.

States *q0* and *q4* are distinguishable since δ\*(*q0, ab*) ∈ F but δ\*(*q4, ab*) ∉ F.

States *q1* and *q2* are distinguishable since δ\*(*q1, b*) ∈ F but δ\*(*q2, b*) ∉ F.

States *q1* and *q3* are distinguishable since δ\*(*q1, b*) ∈ F but δ\*(*q3, b*) ∉ F.

States *q1* and *q4* are distinguishable since δ\*(*q1, b*) ∈ F but δ\*(*q4, b*) ∉ F.

States *q2* and *q3* are distinguishable since δ\*(*q2, a*) ∈ F but δ\*(*q3, a*) ∉ F.

States *q2* and *q4* are distinguishable since δ\*(*q2, a*) ∈ F but δ\*(*q4, a*) ∉ F.

States *q3* and *q4* are distinguishable since δ\*(*q3, λ*) ∈ F but δ\*(*q4, λ*) ∉ F.

Therefore, the DFA is minimal.

Q8. [3/10, optional] Prove or disprove the following conjecture: If L is regular, so is LR.

If it is true, construct a NFA MR s.t. L(M’) = LR , from a NFA M that accepts L, i.e. L(M) = L. Then, show that L(M’ ) = LR .

Otherwise, give a counter example.

*Ans:*

If we are given an NFA M that accepts L, we can create an NFA MR that accepts LR. To do this, we reverse all transitions for M, add new initial state q0’, and draw λ-transitions from q0’ to all the final states for M. Then, we switch the final states of M into ordinary states for MR and switch the initial state of M into a final state of MR.

* More explanation is expected

1. Since L is regular, there exists an NFA M that accepts L s.t. L = L(M) where M = (*Q*, Σ, δ, *q0*, *F* ),

To show LR is regular, let’s construct M’ that accepts LR as follows.

* + The start state *q0*, in *M* becomes the final state in *M’*.
  + Since there may be multiple final states in M, i.e. |F| ≥ 1, create a new start state p0  in M’ . Then, add a transition with λ from p0 to each of *qf* ∈ F.
  + The direction of all transition edges in *M* is reversed.
  + Thus, *M’* = (*Q*, Σ, δR, *p0’*, *q0* )

where ∃ (*qj, a*) = *qi* ∈ δR , ∀(*qi, a*) = *qj*∈δ

and (*p0,* λ) = *qf* for each *qf* ∈ F .

1. Then, show that L(M’) = LR .

→) Claim: For any *w∈ L(M’), w* ∈ *LR .*

Since *w∈ L(M’), w* is accepted by *M’,*

i.e. there is an transition from *p0* leading to the final state *q0* with *w* in M’ :

*δ R\* (p0, w) = δ R\* (p0, λw) = δ R\* (δ R (p0, λ), w) = δ R\* (qf* .*, w) = q0* for any *qf* ∈ F .

Since every transition in *M’* is the reverse of the transition in *M,*

for any *δ R\* (p0, w) = δ R\* (qf* .*, w) = q0 in M’,* there exists  *δ\* (q0, wR) = qf*  in *M.*

Thus, *wR ∈ L, i.e. w ∈ LR .*

←) Claim: For any *w* ∈ *LR , w∈ L(M’)*

For any *w* ∈ *LR , wR∈ L.*

Since *L* is a regular language accepted by *M, wR∈ L = L(M).*

So, there exists an extended transition *δ\* (q0, wR) = qf*  in *M.*

Since *M’* was defined with the reverse transitions of *M,*

*δ\* (q0, wRλ) =δ (δ\* (q0, wR ), λ)=δ (qf , λ)= δ\* (δ R (p0, λ), wR) = δ R\* (δ R (p0, λ), w)*

*=* *δ R\* (p0, λw) =* *δ R\* (p0, w) = q0 .* So, *w* ∈ *L(M’).*

Thus, L(M’) = LR .

Therefore, LR  is regular. Q.E.D.