CSci 435: Formal Languages and Automata

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**Home Assignment 2: 100 points + 10 points (optional)**

Q1. [10] Find all strings in L((*ab* + *b*)\* b (*a* + *ab*)\*) of length ***less than*** four.

*Ans*. *b, bb, ba, abb, bbb, baa, bab, bba.*

Q2. [10] Give a ***regular expression*** for the language

1. [10] L = {*anbm* | (*n*+*m*) is odd}.

*Ans.* (*aa*)\**b*(*bb*)\* + *a*(*aa*)\*(*bb*)\*.

1. [10, optional] L = {*w* ∈ {*a, b*}\* | ( *na*(*w*) - *nb*(*w*) ) mod 3 = 0}. Hint: Apply Thm 3.2. .

*Ans.* First we show the DFA for the above language.

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Then we reduce the automaton to a generalized transition graph.

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From the GTG we obtain the regular expression: (*ab*+(*b*+*aa*)(*ba*)\*(*a*+*bb*))\*.

Q3. [10] Using the construction in Theorem 3.1, construct an NFA that accepts the complement of the

Language L(*ab*\**aa* + *bba*\**ab*).

*Ans.* We first want to find the regular expression for the complement of the given language. To do this, we start by constructing an NFA for the given language.

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We then convert this NFA into an equivalent DFA.

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Next, we create the complement of this DFA by switching the accepting states to non-accepting states and vice versa.

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We then find the corresponding regular expression for this DFA, which is equivalent to the complement of our given regular expression.

Our new regular expression is L(*λ*+*a*+*aa*+*abb*\*+*abb*\**a*+*b*+*bb*+*bbaa*\*).

Finally, we follow Theorem 3.1 to systematically construct an NFA that accepts the above regular expression.

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Q4. [20] Construct a ***minimal DFA*** that accepts the following language

1. [10] L(*ab*(*a*+*ab*)\*(*a*+*aa*))

*Ans.* First we construct an NFA to represent the given regular expression.

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Then, we convert the NFA into an equivalent DFA.

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Description automatically generated

Finally, we prove our DFA is minimal by showing that no state is unreachable, and no pair of states are indistinguishable.

δ(*q0, a*) = *q1* and *a* is in L so *q1* is not unreachable.

δ(*q0, ab*) = *q2* and a*b* is in L so *q2* is not unreachable.

δ(*q0, aba*) = *q3* and *aba* is in L so *q3* is not unreachable.

δ(*q0, b*) = *q4* and *b* is in L so *q4* is not unreachable.

States *q0* and *q1* are distinguishable since δ\*(*q0, aba*) ∈ F but δ\*(*q1, aba*) ∉ F.

States *q0* and *q2* are distinguishable since δ\*(*q2, ab*) ∈ F but δ\*(*q0, ab*) ∉ F.

States *q0* and *q3* are distinguishable since δ\*(*q3, a*) ∈ F but δ\*(*q0, a*) ∉ F.

States *q0* and *q4* are distinguishable since δ\*(*q0, aba*) ∈ F but δ\*(*q4, aba*) ∉ F.

States *q1* and *q2* are distinguishable since δ\*(*q1, ba*) ∈ F but δ\*(*q2, ba*) ∉ F.

States *q1* and *q3* are distinguishable since δ\*(*q3, a*) ∈ F but δ\*(*q1, a*) ∉ F.

States *q1* and *q4* are distinguishable since δ\*(*q1, ba*) ∈ F but δ\*(*q4, ba*) ∉ F.

States *q2* and *q3* are distinguishable since δ\*(*q3, ba*) ∈ F but δ\*(*q2, ba*) ∉ F.

States *q2* and *q4* are distinguishable since δ\*(*q2, a*) ∈ F but δ\*(*q4, a*) ∉ F.

States *q3* and *q4* are distinguishable since δ\*(*q3, a*) ∈ F but δ\*(*q4, a*) ∉ F.

Therefore, the DFA is minimal.

1. [10] L((*aa*\*)\**b*)\*)

Hint: Start with constructing an NFA (by Theorem 3.1), convert it to DFA, then get the minimal DFA by mark & reduce procedures.

*Ans.* We start again by constructing an NFA.

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Then, we convert the NFA into an equivalent DFA.

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Finally, we prove our DFA is minimal by showing that no state is unreachable, and no pair of states are indistinguishable.

δ(*q0, a*) = *q0* and *a* is in L so *q0* is not unreachable.

δ(*q0, b*) = *q1* and *b* is in L so *q1* is not unreachable.

δ(*q0, ba*) = *q2* and *b* is in L so *q2* is not unreachable.

States *q0* and *q1* are distinguishable since δ\*(*q0, a*) ∈ F but δ\*(*q1, a*) ∉ F.

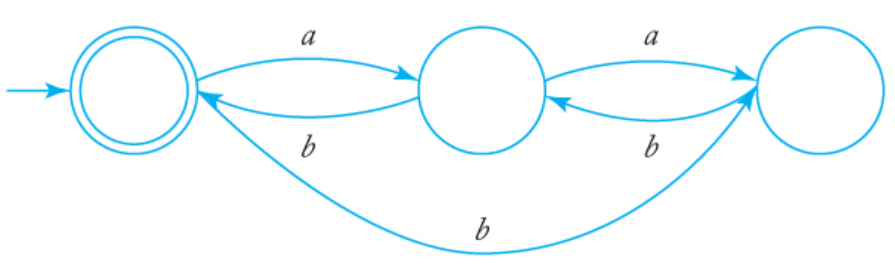
States *q0* and *q2* are distinguishable since δ\*(*q0, ab*) ∈ F but δ\*(*q2, ab*) ∉ F.

States *q1* and *q2* are distinguishable since δ\*(*q1, b*) ∈ F but δ\*(*q2, b*) ∉ F.

Therefore, the DFA is minimal.

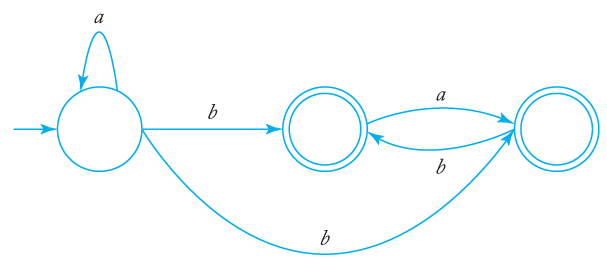
Q5. [20] Find ***regular expressions*** for the languages accepted by the following automaton.

1. [10]



*Ans.* *L*((*ab*+(*b*+*aa*)(ba)\*bb)\*)

1. [10]



*Ans. L*(*a*\*(*b*+*ba*+(*bb*+*bab*)(*ab*)\*(*λ*+*a*)))

Q6. [10] Construct a ***DFA*** that accepts the language generated by the *grammar*

S → *ab*S | B, A → *a*B | *bb,* B → *ba*A.

*Ans.*

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Q7. [20] Find a ***regular grammar*** that generates the language on Σ={a, b}

1. [10] *L*(*aa*\*(*ab*+*a*)\*)

*Ans.* S → *a*A , A → *a*A | *a* | *ab*B*,* B → A | *λ*.

1. [10] the language consisting of all strings with no more than two *a*’s.

*Ans.* S → *b*S | *a*A | *b* | *λ*, A → *b*A | *a*B| *b* | *λ,* B → *b*B | *b* | *λ*.