CSci 435: Formal Languages and Automata

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**Home Assignment 5: 100 points + 15 points (optional)**

In any (N/D)PDA, assume that a start stack symbol z is already in the stack; so, you don’t have to insert z into the stack at the beginning of transition.

Q1. [20] For a given language L = { *w* | *na*(*w*) + *nb*(*w*) = *nc*(*w*) } where Σ = Γ = {*a*, *b, c*}

1. [10] Construct a PDA M that accepts L with Σ = Γ = {*a*, *b, c*}

*Ans*.

Diagram

Description automatically generated

1. [10] Show the sequence of instantaneous descriptions for the acceptance of *acacbcbc* by M in 1).

*Ans.*

(*q0*, *acacbcbc*, Z) ⊢ (*q0*, *cacbcbc*, *a*Z) ⊢ (*q0*, *acbcbc*, Z) ⊢ (*q0*, *cbcbc*, *a*Z) ⊢ (*q0*, *bcbc*, Z) ⊢

(*q0*, *cbc*, *b*Z) ⊢ (*q0*, *bc*, Z) ⊢ (*q0*, *c*, *b*Z) ⊢ (*q0*, *λ*, Z) ⊢ (*q1*, *λ*, *λ*)

1. [10, optional] Give a CFG G that generates L, L(G) = L.

*Ans*. S → A*c* | *c*A | *λ*, A → *a*S | *b*S

Q2. [20] Construct an NPDA for the following languages.

1. [10] L1 = {*bba*\**bab*\* }

*Ans*.

A picture containing chart

Description automatically generated

1. [10] L2 = {*bbb\*aba* }

*Ans*.

A picture containing diagram

Description automatically generated

1. [5, optional] L4 = L2 – L1.

*Ans*. The only string that is in L2 that is also in L1 is *bbaba*. So, we simply need to add an additional state between *q1* and *q2* with another transition from input *b* to require one additional *b*.

A picture containing chart

Description automatically generated

Q3. [10] Give the language that is accepted by the NPDA M in a formal expression (including a regular expression) where M = ({*q0, q1, q2*}, {*a, b*}, {*a, b*, z}, δ, *q0*, z, { *q0* , *q1*, *q2*}), with transitions

♦ δ(*q0*, *a*, z) = {(*q1*, *a*), (*q2*, λ)},

♦ δ(*q1*, *b*, *a*) = {(*q1*, *b*)},

♦ δ(*q1*, *b*, *b*) = {(*q1*, *b*)},

♦ δ(*q1*, *a*, *b*) = {(*q2*, λ)},

*Ans*. L = {*a*} ∪ {*abna* | *n* ≥ 0},

L(*a*+*ab*\**a*)

Q4. [20] (A) Construct a NPDA that accepts the language defined by the given grammar and (B) give the language in a formal expression (including a regular expression).

1. S → *ab*S*b* | λ.

*Ans*.

Begin by converting the given grammar into Greibach Normal Form, after eliminating the λ-transition:

S → *a*BSB | *a*BB, B → *b*.

From here, we follow Theorem 7.1 to obtain a NPDA for our grammar.

A picture containing chart

Description automatically generated

The language accepted by the above NPDA is L = {(*ab*)*nbn* | *n* ≥ 1}.

The language accepted is not regular, thus it cannot be represented by a regular expression.

1. S → AA | *a*, A → SA | *ab*.

*Ans*.

Convert to GNF:

S → AA | *a*, A → SA | *a*B, B → *b*

Follow Theorem 7.1 to obtain a NPDA for our grammar.

A picture containing chart

Description automatically generated

The language accepted by the above NPDA is L = {*a*} ∪ {(*ab*)*n*(*aab*)*m* | *n* ≥ 1, *m* ≥ 0}.

The regular expression is L(*a*+(*ab*)\*(*aab*)\*).

Hint: Convert the grammar into Greibach Normal Form, then apply Thm. 7.1.

Q5. [20] Find a (minimal) Context-Free Grammar that generates the language accepted by the NPDA M where M = ({*q0, q1*}, {*a, b*}, {*A*, z}, δ, *q0*, z, {*q1*}), with the transitions

♦ δ(*q0*, *a*, z) = {(*q0*, *Az*)},

♦ δ(*q0*, *b*, *A*) = {(*q0*, *AA*)},

♦ δ(*q0*, *a*, *A*) = (*q1*, λ).

Simplify the production rules by eliminating the useless variables and productions.

*Ans*. We first create a new final state *q2* so we can be sure the final state can only be entered when the stack is empty. This gives us the following two new transitions:

♦ δ(*q1*, *λ*, *A*) = {(*q1*, *λ*)},

♦ δ(*q1*, *λ*, z) = {(*q2*, *λ*)}.

The first two transitions give us the following productions:

(*q0zq0*) → *a*(*q0Aq0*)(*q0zq­0*) | *a*(*q0­Aq1*)(*q1zq0*) | *a*(*q0Aq2*)(*q2zq0*),

(*q0zq1*) → *a*(*q0Aq0*)(*q0zq­1*) | *a*(*q0­Aq1*)(*q1zq1*) | *a*(*q0Aq2*)(*q2zq1*),

(*q0zq2*) → *a*(*q0Aq0*)(*q0zq­2*) | *a*(*q0­Aq1*)(*q1zq2*) | *a*(*q0Aq2*)(*q2zq2*),

(*q0Aq0*) → *a*(*q0Aq0*)(*q0Aq­0*) | *a*(*q0­Aq1*)(*q1Aq0*) | *a*(*q0Aq2*)(*q2Aq0*),

(*q0Aq1*) → *a*(*q0Aq0*)(*q0Aq­1*) | *a*(*q0­Aq1*)(*q1Aq1*) | *a*(*q0Aq2*)(*q2Aq1*),

(*q0Aq2*) → *a*(*q0Aq0*)(*q0Aq­2*) | *a*(*q0­Aq1*)(*q1Aq2*) | *a*(*q0Aq2*)(*q2Aq2*),

The remaining transitions give the productions:

(*q0Aq1*) → *a*,

(*q1Aq1*) → *λ*,

(*q0zq2*) → *λ*.

By eliminating useless variables and productions we obtain the final Context-Free Grammar with start variable (*q0zq2*):

(*q0zq0*) → *a*(*q0Aq0*)(*q0zq­0*),

(*q0zq1*) → *a*(*q0Aq0*)(*q0zq­1*),

(*q0zq2*) → *a*(*q0Aq0*)(*q0zq­2*),

(*q0Aq0*) → *a*(*q0Aq0*)(*q0Aq­0*),

(*q0Aq1*) → *a*(*q0Aq0*)(*q0Aq­1*),

(*q0Aq2*) → *a*(*q0Aq0*)(*q0Aq­2*),

(*q0Aq1*) → *a*,

(*q0zq2*) → *λ.*

Q6. [10] Construct a Deterministic-PDA that accepts L= { *anbm* | 0 ≤ *m* < *n* } to show L is a Deterministic-CFL.

*Ans.*

Diagram

Description automatically generated