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SSIE 501 Introduction to Systems Science
Fall 2020
Test #3 Take-Home Exam
November 24 – December 3, 2020

Permitted: Calculator, computer, books, any notes you collected in class.
NOT Permitted: Consultation with anyone else other than the professor.
NOTES:

1. Be sure to explain in detail not just what your answer is, but also WHY you feel that is the correct answer.
2. Some of these questions are of an open-ended nature. If you cannot come to a definitive answer, just answer the best you can.
3. For some of these questions there may be more than one right way to answer the question.
4. If the meaning of a question is unclear to you, you can make your best effort to interpret the original intention. State your assumptions about the meaning of the question in detail.
5. Turn in your answers as a single legible file (pdf or MS Word) email attachment to hlewis@binghamton.edu no later than 1:15 PM Thursday, December 3.

Problem 1: (25 points) A chess club has 11 members, whose names (by an incredible coincidence) just happen to start with the first 11 letters of the alphabet—Arthur, Betty, . . . , Katherine. Thus, for the sake of convenience we will represent the members by their first initials, a through k. Over time each member has played other members a number of times. Furthermore, many members have held practice games against themselves on the days when not paired off against one of the other members. Let us define a relation to mean "has defeated". That is to say, for any two members x and y, (x, y) is an element of R_1 if and only if x has defeated y on at least one previous occasion. (Because of the practice games played against oneself, we will consider these to be a member defeating himself and therefore the relation is not anti-reflexive. Also, given any two members, they may have played each other more than once with the result sometimes the first player won and in other cases the other did, and for this reason, we cannot assume the relation to be anti-symmetric either.) The relation can be expressed by its characteristic matrix as shown below:

R_1	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	
a	1	1	0	0	0	0	0	0	1	0	0	3
b	0	1	1	1	0	0	0	0	0	0	0	3
c	0	0	1	0	0	0	1	0	1	0	0	3
d	0	1	0	1	0	0	0	0	0	0	0	2
e	0	0	1	0	1	0	0	0	0	1	0	3
f	1	0	0	0	0	0	0	1	0	0	1	3
g	0	0	1	1	0	0	0	0	1	0	0	3
h	0	0	0	0	0	1	0	1	0	0	1	3
i	0	0	1	0	0	0	1	0	1	1	0	4
j	0	0	0	0	1	0	0	1	0	1	0	3
k	1	0	0	0	0	1	0	1	0	0	1	4

Let the above set of members $T_1 = \{a, b, c, \ldots, k\}$ and R_1 as shown be the system S_1.

a. Given $S_2 = (T_2, R_2)$ where $T_2 = \{1, 2, 3, 4, 5\}$ and $R_2 = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1), (3, 3), (3, 5), (4, 1), (4, 2), (4, 4), (5, 4), (5, 5)\}$, demonstrate that S_2 is a strong homomorphism of S_1 including a clear statement of the function $h: T_1 \rightarrow T_2$.

b. Give an example of how if R_2 were to be modified slightly it would result in S_2 being a weak homomorphism, but not a strong homomorphism of S_1.

c. Why is it true that a system that is a strong homomorphism of the original system is a good simplifying model, but one that is a weak homomorphism is not? Difference between onto and one-to-one.

d. Sometimes we use systems that are isomorphic to the original system as system models, yet this does not lead to a simplification in the sense of the example above. List several reasons why we might want to use an isomorphic (and thus non-simplifying in the sense of system structure) model. (These reasons need not apply to the case of the chess club.)

Problem 2: (15 points) Consider a digital black box with n switches on the front (for some natural number n) and an output port that produces some real number output value based on the combination of the settings of each of the individual switches. Each of the switches has five settings, very low, low, medium, high, and very high. By "black box" we mean that the only way to know the output value under some possible combination of switch settings is actually to try out that combination. By "digital black box" we mean that the setting of the switches and the reading of the output value can be performed automatically by the computer. Assume that it is essential to find the combination of switch settings that produces the absolutely highest possible output value.

a. If it takes 0.001 second (one one-thousandth of a second) to try out one possible combination of switch settings, how long will it take to determine the highest possible output value when the number of switches is 5? 10? 15? 20? 25? 30?
b. Suppose that there are $10n$ (the number of switches times 10) bit evaluations necessary to try out a combination and see the output value. At what number of switches does the problem become transcomputational (in the sense of Bremermann's limit)?

Problem 3: (10 points) Suppose that for some event that has not yet occurred we can expect to get a value (outcome) for variable x and a value for variable y. The state space (the set of possible outcomes) for variable x is $X = \{\alpha, \beta, \gamma\}$ and that the state space for y is $Y = \{a, b, c, d\}$. We know the probabilities for each possible outcome for both x and y to be the following: $p(\alpha) = 0.20$, $p(\beta) = 0.30$, $p(\gamma) = 0.50$, $p(a) = 0.10$, $p(b) = 0.20$, $p(c) = 0.30$, and $p(d) = 0.40$.

a. What are the Hartley information measures $\mathrm{I}(X)$ and $\mathrm{I}(Y)$ in bits?
b. What are the Shannon information measures $\mathrm{H}(X)$ and $\mathrm{H}(Y)$ in bits?
c. Suppose that the joint probabilities of the outcomes are the following what is the information transmission $\mathrm{T}(X, Y)$?

		y			
	p (*x*, *y*)	*a*	*b*	*c*	*d*
	α	0.05	0.10	0.02	0.03
x	β	0.03	0.04	0.18	0.05
	γ	0.02	0.06	0.10	0.32

Problem 4: (25 points) You wish to study the training plan of a rival triathlete by understanding his pattern of distances he runs, swims, and cycles each week in miles. You didn't start collecting information on his training plan until week 5 (May 3 to May 9) of the 2020 training season.

- Week 5, he runs 10 miles, swims 3.0 miles, and cycles 50 miles.
- Week 6, he runs 13 miles, swims 2.5 miles, and cycles 60 miles.
- Week 7, he runs 16 miles, swims 2.0 miles, and cycles 70 miles.
- Week 8, he runs 19 miles, swims 1.5 miles, and cycles 80 miles.
- Week 9, he runs 22 miles, swims 1.0 mile and cycles 90 miles.
- Week 10, he runs 0 miles, swims 3.0 miles, and cycles 100 miles.
- Week 11, he runs 19 miles, swims 2.5 miles, and cycles 30 miles.
- Week 12, he runs 22 miles, swims 2.0 miles, and cycles 40 miles.
- Week 13, he runs 25 miles, swims 1.5 miles, and cycles 50 miles.
- Week 14, he runs 28 miles, swims 1.0 miles, and cycles 60 miles.
- Week 15, he runs 6 miles, swims 3.0 miles, and cycles 70 miles.
- Week 16, he runs 9 miles, swims 2.5 miles, and cycles 80 miles.
- Week 17, he runs 12 miles, swims 2.0 miles, and cycles 90 miles.
- Week 18, he runs 15 miles, swims 1.5 miles, and cycles 100 miles.
- Week 19, he runs 34 miles, swims 1.0 miles, and cycles 30 miles.
- Week 20, he runs 12 miles, swims 3.0 miles, and cycles 40 miles.

a. How much do you predict he might run, swim, and cycle in Week 21?
b. How much would you retrodict that he ran in Weeks 4, 3, 2, and 1?
c. Completely state a source system according to Klir's epistemological hierarchy.
d. Completely state a data system.
e. Completely state a generative system.

Problem 5: (10 points) Given sets $A = \{1, 2, 3\}$, $B = \{\beta\}$

a. What is the number of relations that can be defined on $A \times B$? - 8 entries
b. What is $\mathscr{P}(A \times B)$ where $\mathscr{P}$ denotes power set? Write it out explicitly. - 8 entries
c. How are parts a and b of this problem related to each other?
d. What is $\mathscr{P}(\mathscr{P}(A))$? Write it out explicitly. - 256 entries
e. What is $\mathscr{P}(\mathscr{P}(\mathscr{P}(B)))$? Again, write it out explicitly. - 16 entries

Problem 6: (25 points)

Consider two systems, $\boldsymbol{S_1} = (T_1, R_1)$ and $\boldsymbol{S_2} = (T_2, R_2)$. Let T_1 represent a set of nurses who have worked in a particular department in a particular hospital. Here, we will just anonymously label them Nurse #1 through Nurse #8. The nurses' shift assignments vary from week to week, so that some nurses work on the same schedule one week and may work with other nurses the next. Let $R_1 \subseteq T_1^2$ be a relation defined such that for any $x, y \in T_1$, let $(x, y) \in R_1$ if and only if Nurse x has at some point worked on the same shift assignment as Nurse y. We are also provided with the following information.

Reflexive and symmetric.

Nurse #1 has worked a shift with Nurses# 2, 3, and 4.
Nurse #2 has worked a shift with Nurses# 3, 4, 5, and 6.
Nurse #3 has worked a shift with Nurses# 4, 5, 7, and 8.
Nurse #4 has worked a shift with Nurse# 6.
Nurse #5 has worked a shift with Nurses# 6, 7, and 8.
Nurse #6 has worked a shift with Nurse# 8.
Nurse #7 has worked a shift with Nurse# 8.

Let T_2 represent a set of 8 prescription drugs, and we'll just label these Drug *a* through Drug *h*. Let $R_2 \subseteq T_2^2$ be a relation defined such that for any $x, y \in T_2$, let $(x, y) \in R_2$ if and only if Drug x is taken together with Drug y by at least one patient. We also know the following.

Drug *a* is sometimes taken together with Drugs *c, d, f,* and *h*
Drug *b* is sometimes taken together with Drugs *c, e, f, g,* and *h.*
Drug *c* is sometimes taken together with Drugs *e* and *h.*
Drug *d* is sometimes taken together with Drugs *f* and *h.*
Drug *e* is sometimes taken together with Drugs *f* and *g.*
Drug *f* is sometimes taken together with Drugs *g* and *h.*

Is System $\boldsymbol{S_1}$ isomorphic to $\boldsymbol{S_2}$? Explain in detail. (Including the bijective function used.) YES

The End.
Thanks to all of you for a thoroughly enjoyable experience teaching this class.

b	f	h	D	G	E.	a.	c
2	3	5	7	1	4	8	6