Campaign Ads and Bayesianism

GOV 1347 Lab: Week VI

Matthew E. Dardet

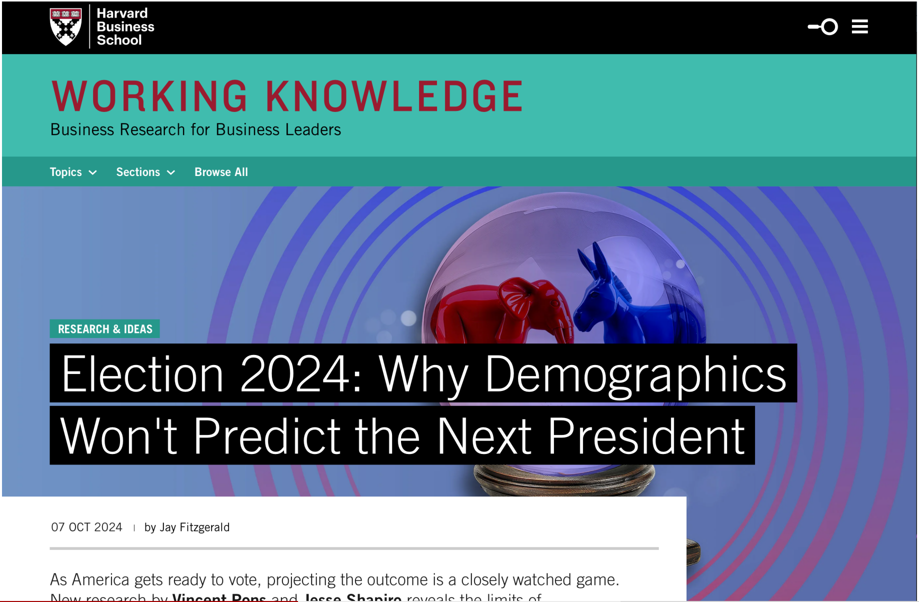
Harvard University

October 9, 2024

Check-In

- Any questions about the course so far?

Hot Off the Press?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RESEARCH & IDEAS

Election 2024: Why Demographics Won't Predict the Next President

07 OCT 2024 | by Jay Fitzgerald

As America gets ready to vote, projecting the outcome is a closely watched game. New research by **Vincent Pons** and **Jesse Shapiro** reveals the limits of

Calvo, Pons, & Shapiro, 2024

Pitfalls of Demographic Forecasts of US Elections
Richard Calvo, Vincent Pons, and Jesse M. Shapiro
NBER Working Paper No. 33016
October 2024
JEL No. C53, D72, J11, P0

ABSTRACT

Many observers have forecast large partisan shifts in the US electorate based on demographic trends. Such forecasts are appealing because demographic trends are often predictable even over long horizons. We backtest demographic forecasts using data on US elections since 1952. We envision a forecaster who fits a model using data from a given election and uses that model, in tandem with a projection of demographic trends, to predict future elections. Even a forecaster with perfect knowledge of future demographic trends would have performed poorly over this period—worse even than one who simply guesses that each election will have a 50-50 partisan split. Enriching the set of demographics available does not change this conclusion. We discuss both mechanical and economic reasons for this finding, and show suggestive evidence that parties adjust their platforms in accordance with changes in the electorate.

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This Week

1. **Describing Campaign Advertisements and Spending**
2. **Introduction to Bayesian Thinking & Statistics**
3. **Election Prediction via Bayesian Modelling**

Section 1

Campaign Advertisements and Spending

Campaign Ads and Spending

Visualizing spending using data from:

- Wesleyan Media Project — Ads Spending
  - `ad_campaign_2000-2012.csv`
  - `ad_creative_2000-2012.csv`
  - `ads_2020.csv`
  - `facebook_ads_2020.csv`: Link
  - `facebook_ads_biden_2020.csv`
- Federal Election Commission (FEC) — Campaign Spending
  - `FEC_contributions_by_state_2008_2024.csv`: Link

Section 2

Bayesian Thinking & Statistics

Bayesian Statistics: Motivations

- **Human-Behavioral Foundations:** Probability as numerical expression of subjective belief rather than objective reality.

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- Quantify uncertainty explicitly.
- Do not need p-values or large sample sizes.

Frequentist v. Bayesian Statistics

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- **Uncertainty (Credible intervals)**: The interval represents the probability that the parameter lies within it (e.g., 95% credible interval means a 95% chance the parameter is in this interval).

Bayes' Theorem

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- From the conditional probability formula, we have:

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Where:

$$P(B) = P(B|A)P(A) + P(B|\text{not } A)P(\text{not } A)$$

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- **Prior belief**: $P(A)$

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- **P(H)**: Prior probability, the initial probability of the hypothesis before seeing the evidence.
- **P(E)**: Marginal likelihood, the total probability of the evidence under all possible hypotheses.

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- **P(E | H)**: Likelihood, probability of a positive test if they have the disease.
- **P(E)**: Marginal likelihood, the probability of getting a positive test overall.

Bayes' Rule Example with Numbers

Suppose you are testing for a rare disease:

- The prior probability $P(H)$, or the probability that a randomly selected person has the disease, is 1% (0.01).
- The test has a sensitivity $P(E|H)$ of 90% (0.90), meaning 90% of those with the disease test positive.
- The test has a false positive rate, or $P(E|H^c)$, of 5% (0.05).

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Step 1: Compute the marginal likelihood $P(E)$

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- $P(H) = 0.01$ (Prior Probability)
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$$P(E) = (0.90)(0.01) + (0.05)(0.99) = 0.009 + 0.0495 = 0.0585$$

Bayes' Rule Example with Numbers

Step 2: Update the posterior probability $P(H|E)$

$$P(H|E) = \frac{(0.90)(0.01)}{0.0585} = \frac{0.009}{0.0585} \approx 0.154$$

Bayes' Rule Example with Numbers

Step 2: Update the posterior probability $P(H|E)$

$$P(H|E) = \frac{(0.90)(0.01)}{0.0585} = \frac{0.009}{0.0585} \approx 0.154$$

- The posterior probability that a person has the disease, given a positive test result, is approximately **15.4%**.

Bayesian Inference

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- **Marginal likelihood**: $\int p(Y|\theta)p(\theta)d\theta$

$$p(\theta|Y) \propto p(Y|\theta)p(\theta)$$

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- **Posterior predictive distribution**:

$$p(Y^{\text{new}}|Y) = \int p(Y^{\text{new}}|\theta)p(\theta|Y)d\theta$$

Many Frequentist Concepts Have Bayesian Analogues

Example: Linear Regression

Frequentist: $Y = X\beta + \epsilon$, where

$$\hat{\beta} = \arg\min_{\beta} \sum_{i=1}^{n} (y_i - X_i\beta)^2$$

Bayesian Linear Regression

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Model:

$$y = X\beta + \epsilon$$

Bayesian Linear Regression

Bayesian Approach:

- **Prior Distribution**: Assigns a prior belief to the parameters, e.g., $\beta \sim N(0, \sigma^2 I)$.

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$$y|X, \beta \sim N(X\beta, \sigma^2)$$

- **Posterior Distribution**: Combines the prior and likelihood using Bayes' theorem to update beliefs about β given the data.

$$p(\beta|y, X) \propto p(y|X, \beta)p(\beta)$$

Bayesian Linear Regression

Let's compare frequentist and Bayesian linear regression in `R`...

Using RStan.

Many Frequentist Concepts Have Bayesian Analogues

Example: Regularization

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- Referred to as Bayesian shrinkage.

Section 3

Bayesian Election Prediction

The Economists's Model

Putting the pieces together

After making all of these adjustments to polls' reported results, we are ready to use them to update our prior. Our method is an expansion of a technique first published by Drew Linzer, a political scientist, in 2013. It uses a statistical technique called Markov Chain Monte Carlo (MCMC), which explores thousands of different values for each parameter in our model, and evaluates both how well they explain the patterns in the data and how plausible they are given the expectations from our prior. For example, what would the election look like if all online pollsters over-estimated the Republicans' vote share by five percentage points? How about if all national polls over-estimated Democrats by two? If state polls of Michigan are oscillating by ten percentage points at a time, the model will incorporate more uncertainty in its prediction of the vote there—and in its predictions of the vote in similar states, such as Ohio.

FiveThirtyEight's Model

More recently, political scientist/survey statistician/pollster Drew Linzer combined aspects of these and other approaches in a Bayesian dynamic linear model of state and national polls of the 2012 general election. In 2016, Natalie Jackson at The Huffington Post also worked on a statistical forecasting model that ran on the Markov chain. And, as linked above, I worked with Gelman and statistician Merlin Heidemanns to add additional poll-level adjustments as well as other factors to model the 2020 general election in this way.

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- **Random-walk priors:** Smooth estimates over time and filter out day-to-day noise in the polls.
- **Daily tracking:** Provides continuous updates of voter preferences at both the state and national levels.
- **Electoral College Simulation:** Estimates the candidates' probability of winning a majority of votes.

The Dynamic Bayesian Approach

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- Polling gaps are filled using hierarchical modeling, allowing the model to "borrow strength" across states.

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$$\pi_{ij} = \text{logit}^{-1}(\beta_{ij} + \delta_j)$$

  - β_{ij}: State-level effect that captures long-term dynamics of voter preferences in state i
  - δ_j: National-level effect that detects systematic departures from β_{ij} on day j due to short-term campaign factors that act as "uniform swing" across all states.

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- Polls fluctuate due to sampling error and campaign events.
- Linzer's model smooths these variations using random-walk priors for the state and national-level effects.
- Start with informative Normal prior distribution based on state-level historical forecasts h_i:

$$\beta_{iJ} \sim N(\text{logit}(h_i), s_i^2)$$

Handling Poll Variability with Random Walk Priors

The dynamic nature of β_{ij} and δ_j is modeled as:

$$\beta_{ij}|\beta_{i,j+1} \sim N(\beta_{i,j+1}, \sigma_\beta^2)$$

Where the variance σ_β^2 captures the rate of daily change in β_{ij}.

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Likewise, for δ_j:

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- MCMC allows us to estimate expected values, posterior distributions, and other statistical quantities through iterative sampling.

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- Gibbs sampler is useful when these full conditionals are easy to sample from.

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Linzer (2013) uses this Bayesian model and estimates it using MCMC in a program called WinBUGS and `R`. Now we generally use `STAN` and `R`.

Blog Extensions (Optional)

1. **Campaign Spending.** How much have campaigns spent in 2024? How can campaign spending be used to predict the election outcomes?
2. **Social Media.** How much do campaigns spend on social media ads? Does social media influence election outcomes? How will this spending influence the 2024 elections? Many rich descriptive opportunities for this week!
3. **Bayesian/Probabilistic Election Prediction.** Implement a Bayesian model for your 2024 election prediction using the Linzer (2013) model or another probabilistic model such as the Binomial logistic model and explanations of *The Economist* or *FiveThirtyEight*'s Bayesian election models as guides.
   - Kremp's Implementation: https://www.slate.com/features/pkremp_forecast/report.html
   - Morris' Implementation: https://github.com/TheEconomist/us-potus-model/tree/master