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ISYE 3133
Section MW3-4:15
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Project Part A Report

Decision variables:
$x_{i,j}$ = Amount of flow from node i to node j
y_j = Amount of energy used at node j

Other variables used:
$C_{i,j}$ = Capacity of arc i, j
d_j = Demand at each node j
A = Set of all arcs
N = Set of all nodes

Objective function:

$$max \sum_{\mathrm{j} \in \delta_1^+} x_{1,j}$$

Such that:

$x_{i,j} + x_{j,i} \leq C_{i,j}$	$\forall\ i, j \in A$	(1) # Capacity
$x_{i,j} \geq 0$	$\forall\ i, j \in A$	(2) # Non-Negativity
$\sum_{\mathrm{i} \in \delta_j^-} x_{i,j} = y_j + \sum_{\mathrm{i} \in \delta_j^+} x_{j,i}$	$\forall\ j \in N$	(3) # Net Flow Constraint
$y_j \leq d_j$		(4) # Maximum Energy Usage
$\sum_{\mathrm{j} \in \mathrm{N}} y_j = y_1 + \sum_{\mathrm{j} \in \delta_1^+} x_{1,j}$		(5) # Don't make or lose energy
$y_1 = d_1$		(6) # Assumption

Since part one requires a formulation of a linear programming model deciding how power should be distributed so that the most total demand is satisfied, the objective function is written to maximize the sum of the total energy flowing out of the generator (at station 1) to any stations connected to the generator. This maximizes the most total demand since no additional energy can be produced at another station, so the amount coming out of the generator is the total amount that can be distributed among the other stations. One of our decision variables, $x_{i,j}$, is representative of total amount of energy flowing from the node i to node j.The other decision variable, y_j, is the amount of energy used at each station j and is no longer transferrable. The first constraint (1) introduced is so that the total amount of energy flowing between two stations does not exceed the capacity of each conduit. The second constraint (2) is to ensure that the amount of flow between stations isn't less that 0 since this is not practical in terms of energy. The third constraint (3) is so the rule of net flow needing to be equivalent to 0 is not violated. In other words, the amount of energy that enters station j has to be equal to the amount of energy used at station j plus the amount that leaves station j. The fourth constraint (4) describes that the amount of energy used at the station has to be less than or equal to the amount demanded at that respective station, since it's not realistic to use more energy than necessary. The fifth constraint (5) states that the total energy used at each station j has to be equal to the total amount of energy used at the main generator in addition to the energy flowing from the main generator. In the final constraint (6), we made the assumption all the energy demanded at the main generator will be fulfilled since the demand is located at the same node as the generator and there is no given upper limit to energy the generator can produce.

Python Gurobi Glossary:
These are the variables used in our code, and related to our mathematical formula to answer part one-

- nodes- N
- demands[j]- $d_j \ \forall \ j \ \in N$
- arcs- A
- capacities[i, j]- $C_{i,j} \ \forall \ i, j \ \in A$
- x- $x_{i,j}$
- y- y_j

Findings of the Gurobi Solver:
For our optimal solution, Gurobi gave us the following flows, in a format such as:

energyFlow[i, j] = f

where f is the amount flow along arc[i,j] in the direction from node i to node j.

For example,
energyFlow[1,4] = 25
energyFlow[1,5] = 29
energyFlow[1,13] = 5
energyFlow[1,17] = 20
energyFlow[4,2] = 16
.
.
.
energyFlow[i, j] = f

for all possible flows. This means that Gurobi does NOT provide the net flow along an arc, but rather two outputs per arc, one flow for each direction of the flow along the arc.

This flow model gave us an optimal solution with a maximum flow of **103**. Another key assumption to note is that we included the energy used at station 1 in our maximum flow count, despite no energy needing to flow to be used at station 1.
It should also be noted that while the optimal solution to this problem will always remain the same; the net flows along each arc can differ in another Gurobi model but will always provide the same optimal value.

Part 2:

a) $z = \min\left(\frac{\Sigma_{j \in g} y_j}{\Sigma_{j \in g} d_j}\right) \; \forall g \; \in G$

Where G is the set of all groupids. In other words, z is the minimum of all groups' (energy used / energy demanded) ratios.

In our max flow solution, z= 0, because group 7 stations get 0 energy in total.

b) z will rate a scenario where multiple groups get ratio's of 1 and one group has a ratio of 0.5 as equal to a scenario where one group has a ratio of 1 and multiple groups have a ratio of 0.5. Obviously, it's more unfair for one group to get all their demand everyone else only gets half than for multiple groups to get everything they demand and one group half.

c) **New Decision Variables:**

z = our fairness metric for the system

New Other Variables:

G = the set of all groups

New Objective Statement:

Max z

New Constraints:

$z \geq 0$ 1) Nonnegativity Constraint

$z \; \leq \; \frac{\Sigma_{j \in g} x_j}{\Sigma_{j \in g} d_j} \; \forall g \in G$ 2) z = min ratio of all groups