Quadratic Form Minimization

Summary of https://youtu.be/oaiiyIsbNdI

Why do we care?

- Solving Ax=b is the same as solving the function:

$$f(x) = \frac{1}{2}x^T Ax - b^T x + c$$

- Sparse matrix problem

How to fix?

- Instead of solving Ax=b with gaussian elimination we minimize. Each step should reduce the size of the solution eventually yielding zero:

$$f(x) = \frac{1}{2}x^T A x - b^T x + c$$

Using Derivative to Find Minima

- What is the minimum value of ax^2-bx?
- What is the minimum value of 1/2ax^2-bx
- The connection of the minimum of 1/2ax^2-bx and Ax=b
  - If we find the minimum of 1/2ax^2-bx it *is* Ax=b. They're the same!

Proving the equations are the same

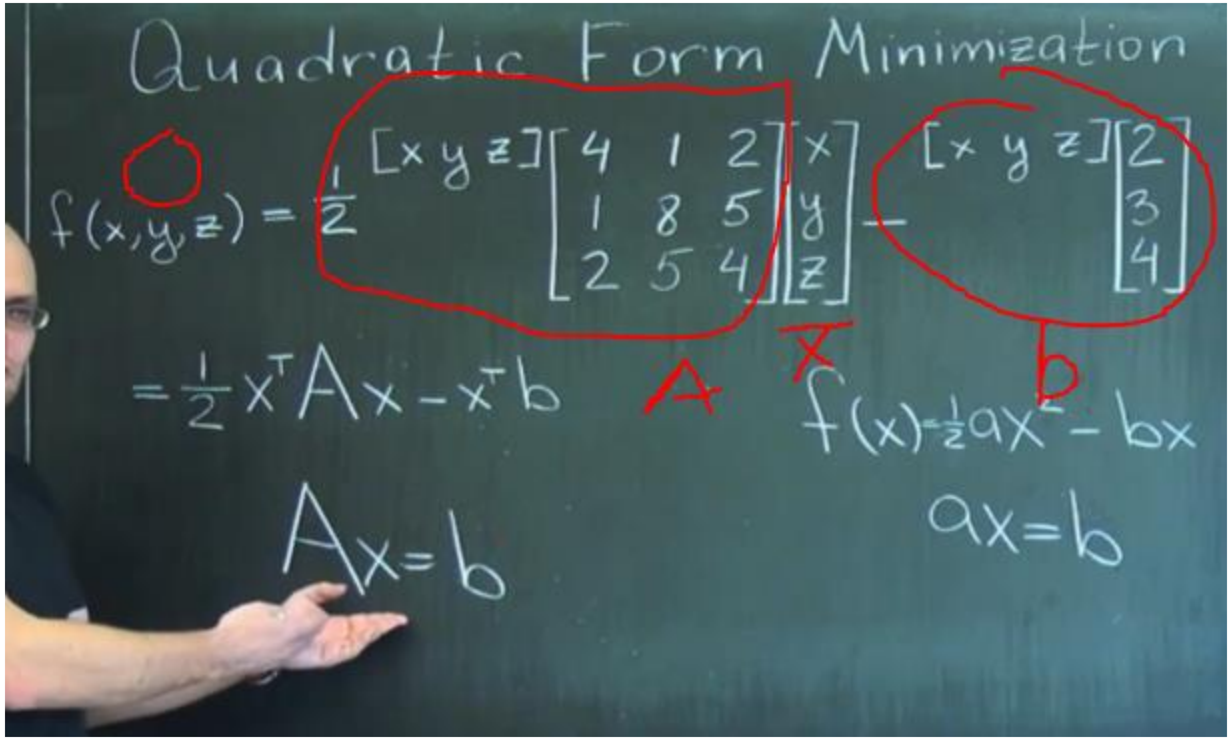
$$f(x,y,z) = \frac{1}{2}\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 4 & 1 & 2 \\ 1 & 8 & 5 \\ 2 & 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$f(x,y,z) = 2x^2 + 4y^2 + 2z^2 + xy + 5yz + 2zx - 2x - 3y - 4z$$

$$\begin{cases} f_x = 4x + y + 2z - 2 & = 0 \\ f_y = x + 8y + 5z - 3 & = 0 \\ f_z = 2x + 5y + 4z - 4 & = 0 \end{cases}$$

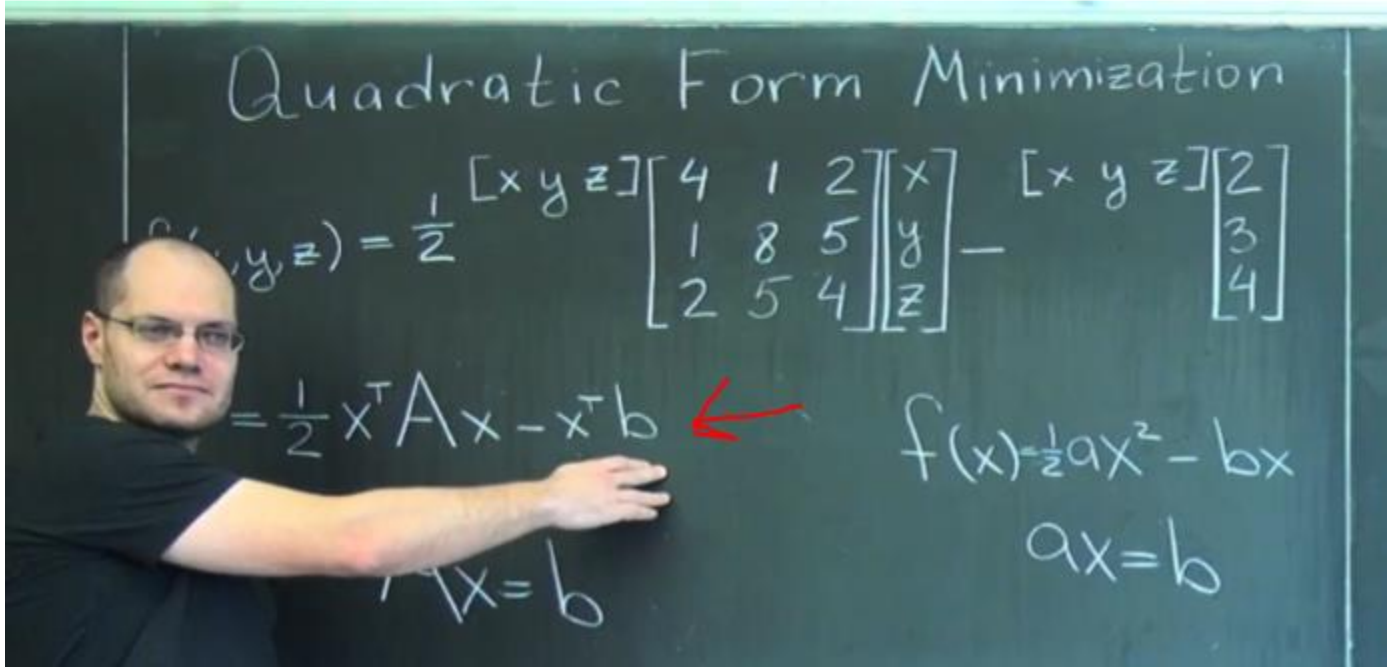
$$\begin{bmatrix} 4 & 1 & 2 \\ 1 & 8 & 5 \\ 2 & 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

Why the same as Ax=b?



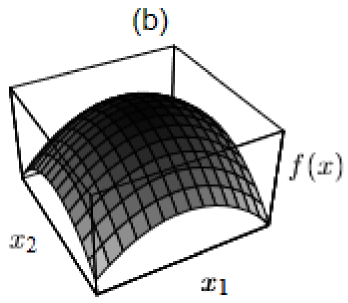
Gradient Decent

Show how gradient descent works and how it relates to this
Here the gradient is Ax. Our gradient steps will be x->x-alpha*Ax



Why does it have to be positive definite/symmetric?

- If it were not positive definite you could end up with stuff like:



- If it weren’t symmetric then you don’t get a parabola you may get some wonky thing and instead end up finding local minimas. Proof at end of paper.