Untitled

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Equations

Table 1. Population dynamics equations for species i and age j in each simulation year y. BT indicates the AFSC bottom trawl survey and EIT represents the echo-integrated acoustic- trawl survey. For all parameter definitions see Table 3.

Definition	Equation	
Recruitment	$N_{i1,y} = R_{i,y} = R_{0,i}e^{\tau_{i,y}}$	T1.1
Initial abundance	$N_{ij,1} = \begin{cases} R_{0,i}e^{(-jM1_{ij})}N_{0,ij} \\ R_{0,i}e^{(-jM1_{ij})}N_{0,i,A_i}/\left(1-e^{\left(-jM1_{i,A_i}\right)}\right) \end{cases}$	T1.2
Numbers at age	$N_{i,j+1,y+1} = N_{ij,y}e^{-Z_{ij,y}}$	T1.3
…	$N_{i,A_i,y+1} = N_{i,A_i-1,y}e^{-Z_{i,A_i-1,y}} + N_{i,A_i,y}e^{-Z_{i,A_i,y}}$	…
Catch	$C_{ij,y} = \frac{F_i j,y}{Z_{ij,y}}(1-e^{-Z_{ij,y}})N_{ij,y}$	T1.4
Total yield (kg)	$Y_{i,y} = \sum_{j}^{A_i} \frac{F_i j,y}{Z_{ij,y}}(1-e^{-Z_{ij,y}})N_{ij,y}W_{ij,y}$	T1.5
Biomass at age (kg)	$B_{ij,y} = N_{ij,y}W_{ij,y}$	T1.6
Spawning biomass at age (kg)	$SSB_{ij,y} = B_{ij,y}\rho_{ij}$	T1.7
Total mortality at age	$Z_{ij,y} = M1_{ij} + M2_{ij,y} + F_{ij,y}$	T1.8
Fishing mortality at age	$F_{ij,y} = F_{0,i}e^{\epsilon_{i,y}s_{ij}^f}$	T1.9
Weight at age (kg)	$W_{ij,y} = W_{\infty,iy}\left(1-e^{\left(-K_i(1-d_{i,y})(j-t_{0,i})\right)}\right)^{\frac{1}{1-d_{i,y}}}$	T1.10a
…	$d_{i,y} = e^{\left(\alpha_{d,i,y}+\alpha 0_{d,i}+\beta_{d,i}T_y\right)}$	T1.10b
…	$W_{\infty,iy} = \left(\frac{H_i}{K_i}\right)^{1/(1-d_{i,y})}$	T1.10c
BT survey biomass (kg)	$\hat{\beta}_{i,y}^{s} = \sum_{j}^{A_i}\left(N_{ij,y}e^{-0.5Z_{ij,y}}W_{ij,y}S_{ij}^{s}\right)$	T1.11

EIT survey biomass (kg)	$\hat{\beta}_y^{eit} = \sum_j^{A_1} \left(N_{1j,y} e^{-0.5 Z_{1j,y}} W_{1j,y} S_{1j}^{eit} q_1^{eit} \right)$	T1.12
Fishery age composition	$\hat{O}_{ij,y}^{f} = \frac{C_{ij,y}}{\sum_j C_{ij,y}}$	T1.13
BT survey age composition	$\hat{O}_{ij,y}^{s} = \frac{N_{ij,y} e^{-0.5 Z_{ij,y}} s_{ij}^{s}}{\sum_j \left(N_{ij,y} e^{-0.5 Z_{ij,y}} s_{ij}^{s} \right)}$	T1.14
EIT survey age composition	$\hat{O}_{1j,y}^{eit} = \frac{N_{1j,y} e^{-0.5 Z_{1j,y}} s_{1j}^{eit} q_1^{eit}}{\sum_j \left(N_{1j,y} e^{-0.5 Z_{1j,y}} s_{1j}^{eit} q_1^{eit} \right)}$	T1.15
BT selectivity	$s_{ij}^{s} = \frac{1}{1 + e^{\left(-b_i^s * j - a_i^s \right)}}$	T1.16
Fishery selectivity	$s_{ij}^{f} = \begin{cases} e^{\eta_i j} & j \le A_{\eta,i} \\ e^{\eta_{i,A_{\eta,i}}} & j > A_{\eta,i} \end{cases}$	T1.17
Proportion females	$\omega_{ij} = \frac{e^{-jM_{fem}}}{e^{-jM_{fem}} + e^{jM_{male}}}$	T1.18
Proportion of mature females	$\rho_{ij} = \omega_{ij} \phi_{ij}$	T1.19
Weight at age (kg)	$W_{ij,y} = W_{ij,y}^{fem} \omega_{ij} + \left(1 - \omega_{ij} \right) W_{ij,y}^{male}$	T1.20
Residual natural mortality	$M1_{ij} = M_{ij}^{fem} \omega_{ij} + \left(1 - \omega_{ij} \right) M_{ij}^{male}$	T1.21

Table 2. Predation mortality equations for predators p of age a, and prey i of age j

Definition	Equation	
Predation mortality	$M2_{ij,y} = \sum_{pa} \left(\frac{N_{pa,y} \delta_{pa,y} S_{paij}}{\sum_{ij} \left(S_{paij} B_{ij,y} \right) + B_p^{other} \left(1 - \sum_{ij} \left(S_{paij} \right) \right)} \right)$	T2.1
Predator-prey suitability	$\hat{S}_{paij} = \frac{1}{n_y} \sum_y \left(\frac{\frac{\overline{U}_{paij}}{B_{ij,y}}}{\sum_{ij} \left(\frac{\overline{U}_{paij}}{B_{ij,y}} \right) + \frac{1 + \sum_{ij} \overline{U}_{paij}}{B_p^{other}}} \right)$	T2.2
Mean gravimetric diet proportion	$\overline{U}_{paij} = \frac{\sum_y U_{paij,y}}{n_y}$	T2.3
Individual specific ration ($kg\ kg^{-1}\ yr^{-1}$)	$\delta_{pa,y} = \varphi_p \alpha_\delta W_{pa,y}^{(1+\beta_\delta)} f\left(T_y \right)_p$	T2.4
Temperature scaling algorithim	$f\left(T_y \right)_p = V^X e^{\left(X(1-V) \right)}$	T2.5
…	$V = \left(T_p^{cm} - T_y \right) / \left(T_p^{cm} - T_p^{co} \right)$	T2.5a

... $X = (Z^2(1 + (1 + 40/Y)^{0.5})^2)/400$ T2.5b

... $Z = ln(Q_p^c)(T_p^{cm} - T_p^{co})$ T2.5c

... $Y = ln(Q_p^c)(T_p^{cm} - T_p^{co} + 2)$ T2.5d

Table 3. Parameter definitions.

Parameter	Definition	Type	Model Object
Year	y	M	i
Predator	p	M	
Predator age (years)	a	M	
Prey	i	M	k
Prey age (years)	j	M	
Number of prey species	n_i	I	nspp
Number of predator species	n_p	I	
Number of prey ages	A_i	I	nages
Number of predator ages	A_p	I	
Number of simulation years	n_y	I	nyrs
Start year	y_0	I	styr
Annual relative foraging rate ($d\ yr^{-1}$)	$\hat{\varphi}_p$	I	
Intercept of the allometic maximum consumption function ($g\ g^{-1}\ yr^{-1}$)	α_δ	I	aLW
Allometric slope of maximum consumption	β_δ	I	bLW
Consumption maximum physiological temperature (°C)	T_p^{cm}	I	Tcm
Consumption optimum physiological temperature (°C)	T_p^{co}	I	Tco
Max consumption parameter	Q_p^c	I	Qc
Mean recruitment	$R_{0,i}$	E	
Annual recruitment deviation	$\tau_{i,y}$	E	rec_dev
Initial abundance	$N_{0,ij}$	E	
Mean fishing mortality	$F_{0,i}$	E	
Anuual fishing mortality deviation	$\epsilon_{i,y}$	E	
Fishery age selectivity coefficient	η_{ij}	E	
Survey age selectivity slope	b_i^s	E	
Survey age selectivity limit	a_i^s	E	
VBGF allometric slope of consumption	$d_{i,y}$	P	d

VBGF max asymptotic weight (kg)	$W_{\infty,iy}$	P	Winf
Proportion of mature females at age	ρ_{ij}	P	
Residual natural mortality	$M1_{ij}$	F	M1_base
Intercept for VBGF d parameter	$\alpha 0_{d,i}$	F	
Annual intercept for VBGF d parameter	$\alpha_{d,i,y}$	F	log_mean_d
Temperature covariate for VBGF d parameter	$\beta_{d,i}$	F	Tcoef
VBGF energy loss constant ($kg\ kg^{-1}\ yr^{-1}$)	K_i	F	logK
VBGF assimilation constant ($kg\ kg^{-1}\ yr^{-1}$)	H_i	F	logH
VBGF age when $W_{ij,y} = 0$ (years)	$t_{0,i}$	F	t0
EIT survey selectivity	S_{1j}^{eit}	F	
Female natural mortality	M_i^{fem}	F	
Male natural mortality	M_i^{male}	F	
Female proportion of population	ω_{ij}	F	
Age-specific maturity proportions	φ_{ij}	F	pmature
Observed total yield (kg)	$C_{i,y}^{*}$	D	tc_biom_obs
Observed fishery age comp.	$O_{ij,y}^{f}$	D	fsh_age_obs
Observed BT age comp.	$O_{ij,y}^{s}$	D	srv_age_obs
Observed EIT age comp.	$O_{ij,y}^{eit}$	D	obs_eit_age
Observed BT survey biomass (kg)	$\beta_{i,y}^{s}$	D	srv_bio
Observed EIT survey biomass (kg)	β_{y}^{eit}	D	obs_eit
Bottom temperature (°C)	T_y	D	TempC
Gravimetric proportion of prey in predator stomach	$U_{paij,y}$	D	
Biomass of other prey (kg)	B_p^{other}	D	other_food
Not in table 3			
Annual survey biomass error	$CV_{s,i,y}$	F	srv_Mean_CV
Number of years with total observed catch	$n_{Y,i}$	M	nyrs_tc_biom_obs
Years with total observed catch	$y_{Y,i}$	I	yrs_tc_biom_obs
Number of years in the fishery sp_age composition data	$n_{y,i}$	E	nyrs_fsh_comp
Number of estimation years	$n_{y,est}$	I	nyrs_est
End year	y_{n_y}	I	endyr
Number of years in the fishery age composition data	$n_{y_{of},i}$	I	nyrs_fsh_comp

Years in the fishery age composition data	$y_{O^{f},i}$	I	yrs_fsh_comp
Method of calculating fishery age		I	fsh_age_type
Number of fishery age bins	$n_{f,bin}$	I	fsh_age_bins
Number of years with weight-at-age data	$n_{y,W,i}$	I	nyrs_wt_at_age
Years with weight-at-age data	$y_{W,i}$	I	yrs_wt_at_age
Weight-at-age data	W_i	D	wt
Number of years in the BT survey data	$n_{y_{\beta^{s},i}}$	I	nyrs_srv_biom
Years in the BT survey data	$y_{\beta^{s},i}$	I	yrs_srv_biom
BT survey standard error	$\sigma_{s,i}$	F	srv_biom_se
Number of years in the BT survey age or length composition data	$n_{y_{O^{s},i}}$	I	nyrs_srv_age
Years in the BT survey age composition data	$y_{O^{s},i}$	I	yrs_srv_age
Method of calculating BT survey age type (age or length)		I	srv_age_type
Number of BT survey age bins	$n_{s,bin,i}$	I	srv_age_bins
Sample size for BT survey age composition multinomial	$n_{O^{s},i}$	I	srv_age_n
Observed survey BT size compositions	O_{ij}^{s}	I	srv_age_sizes
Age transition matrix		I	age_trans_matrix
Number of years in the EIT survey data	$n_{y_{\beta^{eit}}}$	I	n_eit
Years in the BT survey data	$y_{\beta^{eit}}$	I	yrs_eit
Sample size for EIT survey age composition multinomial	$n_{O^{eit}}$	I	eit_age_n
Number of years in the EIT selectivtiy data	$n_{y_{S^{eit}}}$	I	nyrs_eit_sel
Years in the BT selectivity data	$y_{S^{eit}}$	I	yrs_eit_sel
Sample size for EIT survey age composition multinomial	$n_{O^{eit}}$	I	eit_sel
Sex specific mortality and weight-at-age: 1 for combined, 2: for seperate		I	mf_type
Proportion	$n_{O^{eit}}$	I	propMorF
Observed catch-at-age	$C_{ij,y}$	I	obs_catch
Estimated catch-at-age	$\hat{C}_{ij,y}$	E	obs_catch_hat
Observed total catch	$C_{i,y}$	D	tc_obs
Estimated total catch	$\hat{C}_{i,y}$	D	tc_hat

Estimated total yield	$B_{ij,y} = N_{ij,y}W_{ij,y}$	E	tc_obs
Observed fishery age composition	$O^{f}_{ij,y}$	I	fsh_age_obs
Estimated fishery age composition	$\hat{O}^{f}_{ij,y}$	E	fsh_age_hat

Table 4. Components of the likelihood function for each species i of age j in year y.

Description	Equation	Data source	
Data components			
BT survey biomass	$\sum_{i}\sum_{y}\frac{\left[ln(\beta^{s}_{i,y}) - ln\left(\hat{\beta}^{s}_{i,y}\right)\right]^{2}}{2\sigma^{2}_{s,i}}$	NMFS annual EBS BT survey	T4.1
BT survey age composition	$-\sum_{i} n_i \sum_{y}\sum_{j}\left(O^{s}_{ij,y} + v\right)\ ln\left(\hat{O}^{s}_{ij,y} + v\right)$	NMFS annual EBS BT survey	T4.2
EIT survey biomass	$\sum_{i}\sum_{y}\frac{\left[ln(\beta^{eit}_{y}) - ln\left(\hat{\beta}^{eit}_{y}\right)\right]^{2}}{2\sigma^{2}_{eit}},\ \sigma_{eit} = 0.2$	Pollock acoustic trawl survey	T4.3
EIT survey age composition	$-n\sum_{y}\sum_{j}\left(O^{eit}_{1j,y} + v\right)\ ln\left(\hat{O}^{eit}_{1j,y} + v\right)$	Pollock acoustic trawl survey	T4.4
Total catch	$\sum_{i}\sum_{y}\frac{\left[ln(C^{*}_{i,y}) - ln\left(\hat{C}^{*}_{i,y}\right)\right]^{2}}{2\sigma^{2}_{C}},\ \sigma_{C} = 0.05$	Fishery observer data	T4.5
Fishery age composition	$-\sum_{i} n_i \sum_{y}\sum_{j}\left(O^{f}_{ij,y} + v\right)\ ln\left(\hat{O}^{f}_{ij,y} + v\right)$	Fishery observer data	T4.6
Penalties			
Fishery selectivity	$\sum_{i}\sum_{j}^{A_i-1}\chi\left[ln\left(\frac{\eta^{f}_{ij}}{\eta^{f}_{ij+1}}\right) - ln\left(\frac{\eta^{f}_{ij+1}}{\eta^{f}_{ij+2}}\right)\right]^{2},\ \chi = \begin{cases}20,\ if\eta^{f}_{ij} > \eta^{f}_{ij+1}\\ 0,\ if\eta^{f}_{ij} \le \eta^{f}_{ij+1}\end{cases}$		T4.7
Priors			
	$\sum_{i}\sum_{j}\left(\tau_{i,y}\right)^{2}$		T4.8
	$\sum_{i}\sum_{j}\left(N_{0,ij}\right)^{2}$		T4.9

$$\sum_i \sum_j \left(\varepsilon_{i,y}\right)^2 \qquad \text{T4.10}$$

$$v = 0.001$$

References

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