# Section 16.3 Homework

1-25 odd, 29–35 odd

**1** Since the gradient of $f$ is continuous, we know that the work line integral of the gradient is just the difference in $z$ values: $50 - 10 = 40$.

## 3-9

Determine whether or not $F$ is a conservative vector field. If it is, find a function $f$ such that $\mathbf{F} = \nabla f$.

**3**

$$\mathbf{F}(x,y) =< (xy + y^2), (x^2 + 2xy) >$$

**Solution**: From our notes, if the mixed second partials of the potential function candidate are equal (basically if the partials of the component functions of $\mathbf{F}$ are equal), then the vector field is conservative, and we can go about finding the potential function. Let $P(x,y) = xy + y^2$ and $Q(x,y) = x^2 + 2xy$. We have that

$$\frac{\partial P}{\partial y} = x + 2y$$
$$\frac{\partial Q}{\partial x} = 2x + 2y$$

The partials are not equal and thus $\mathbf{F}$ is not a conservative vector field.

**5**

$$\mathbf{F}(\mathbf{x},\mathbf{y}) =< y^2 e^{xy}, (1 + xy)e^{xy} >$$

**Solution**: Same as above, take partials of component functions of the vector field

$$\frac{\partial P}{\partial y} = 2ye^{xy} + y^2 x e^{xy}$$
$$\frac{\partial Q}{\partial x} = ye^{xy} + ye^{xy} + xy^2 e^{xy}$$

These partials are equal on their natural domains which is all of $\mathbb{R}^2$, a simply connected open region. Therefore a potential function exists. To find this function, we will start by integrating $P(x,y)$ against $x$:

$$f(x,y) = \int P(x,y)dx = \int (y^2 e^{xy})dx = ye^{xy} + g(y)$$

Take the derivative of $f$ with respect to $y$ and compare to $Q(x,y)$:

$$\frac{\partial f}{\partial y} = Q(x,y)$$
$$e^{xy} + xye^{xy} + g'(y) = e^{xy} + xye^{xy}$$
$$g'(y) = 0 \to g(y) = 0 + C$$

The general solution is $f(x,y) = ye^{xy} + C$.